RAILWAY ELECTRIC TRACTION

BY

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PREFACE

In writing this book the author set himself a twofold task: firstly, to discuss the methods of electric traction, as applied to Railways; secondly, to expound methods of technical calculation applicable to the subject. In the pursuance of the first objective he propounded to himself, and endeavoured to answer, the questions: What constitutes good practice, and why? Except where needed for purposes of illustration, descriptive matter has been avoided not only as being outside the scope of the book, but also as being available in full measure in the technical press, and in the publications of manufacturers. The methods of calculation described in the later chapters of the book are for the most part the author's own, and do not exceed in refinement what he has found necessary in dealing with the subject.

The author wishes to express his indebtedness to the British Thomson-Houston Co., Ltd., for permission to use a quantity of data and diagrams without which his effort would have been far less satisfying to himself. He also takes this opportunity of thanking the International General Electric Company, and the Westinghouse Electric and Manufacturing Company, for a number of views and particulars of locomotives, the J. G. Brill Co. for the drawing of fig. 15, and the Societa Italiana Westinghouse for fig. 202.

It only remains to add that British units have been used throughout the book, the ton representing 2,240 lb.

F. W. C.

RUGBY, 1922.

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CHAPTER I

INTRODUCTORY

It is characteristic of industrial progress to replace methods of working which make use of simple tools and individual effort, by methods which use costly and elaborate plant actuated by collective effort. In this manner labour is saved at the expense of machinery ; and the great production which is characteristic of modern industry becomes possible. In the realm of transportation, the railway furnishes an example of a highly capitalized but efficient substitute for the more primitive methods of earlier times. The use of electricity as the motive power of railways may be regarded as an extension of the same tendency. It involves great outlay for operating plant ; which it seeks to justify by offering improved facilities and reduced running costs.

In the working of railways by means of steam locomotives, the driving axles are actuated directly by the prime movers. In electrical working, however, there intervenes in general between prime movers and driving axles, the generators, the transmission lines, the converting plant, the distribution lines and the train motors, besides a large quantity of protective apparatus and controlling gear whose functions are ancillary to the transmission of the power. When the whole of this plant is thus taken into account, it is inevitable that the capital invested in motive apparatus should be much greater for electrical operation than for steam operation.

The chief obstacle to the general use of electrical working being the economic one of the great initial expense, it is natural that, in its inception, it should have been limited to very busy lines concerned with special classes of traffic. It has needed the development of a quarter of a century to demonstrate its technical applicability to all classes of traffic with which a

railway is concerned; but the economic advantage resulting from its adoption is still in most cases a matter to be determined by investigation of the particular circumstances.

Abnormal Traffic Conditions.—Electrical operation has indeed been described, not without justification, as an expedient for overcoming abnormal difficulties of working, or for fulfilling the conditions imposed by special circumstances. For the ordinary inter-urban railway, in country of no more than ordinary difficulty, where fuel is plentiful, the question of electrical operation has hardly been considered. It has hitherto always been some special consideration that has determined the decision to electrify.

URBAN RAILWAYS. When tramway experience suggested the electrification of urban railways, and the developments effected in pursuit of this object were found to result in methods of working the traffic which far surpassed the best efforts of steam operation in efficiency and economy, electrical operation was soon recognized as by far the most effective way of overcoming the peculiar difficulties of urban and suburban railway working. The capacity of the lines is greatly increased, the operation is faster, and the working of terminal traffic is much simplified by electrical operation. This application indeed now hardly needs the assurance given by an economic investigation to justify it.

TUNNELS.—On certain railways, the existence of a long tunnel has restricted the traffic under steam operation, the accumulation of noxious gases from the engines limiting the size and frequency of the trains. In a number of cases electrical operation has been adopted with a view to removing the restriction. A pioneer example of this application is that of the Baltimore belt line tunnel of the Baltimore & Ohio Railway, electrified in 1895; but the Detroit River tunnel electrification, the Hoosac tunnel electrification, the Cascade tunnel electrification, the Simplon tunnel electrification, and many others have been undertaken in order to overcome the special difficulties of tunnel working. The New York Central Terminal electrification, and indeed that of all lines entering the city, was insisted upon by the competent authorities, largely as a result of a tunnel accident attributed to an accumulation of gases. The Underground Railways of London furnish another

example in which the obnoxious conditions of tunnel service compelled electrification.

GRADIENTS.—On other railways, the gradients by their length and steepness impose a limit on the traffic under steam operation, which is removed or considerably ameliorated by electrical operation. The electrification of the Norfolk & Western Railroad, a heavy goods line, which includes a considerable length of 2 per cent. gradient, was determined by this consideration. The Chicago Milwaukee & St. Paul Railway, which crosses the Rocky Mountains and a number of other ranges, with many long gradients up to 2 per cent. in steepness, was electrified, as to the divisions affected, largely for the same reason.

LACK OF FUEL.—In some regions locomotive fuel is scarce or non-existent as a local product, although the natural resources of water-power may be ample for the purpose of working the railways electrically, and where these conditions exist there is a great inducement to electrify. The development of electric railways in Switzerland, in Sweden, in Bavaria and in Italy is to be attributed largely to this state of affairs. The Chicago Milwaukee & St. Paul Railway also operates in a region where water-power is plentiful, but where fuel has to be brought from a distance, and this no doubt had large influence on the decision to electrify.

The history of existing schemes of electrical operation, therefore, appears to support the conclusion that electrification is a device for special circumstances. The inference is, however, hardly justified. A more powerful and flexible agent having been discovered, it is in the natural order of development that it should be applied first where steam operation has been found wanting, and its success under these circumstances is not evidence of its inadvisability under more normal conditions. At the same time, where the requirements as regards transportation are met without difficulty by steam operation, the justification of electrical operation must be sought in its economy rather than in improvement in the service rendered. Some increase of facilities is doubtless to be expected, but there is not scope for the significant improvement that has generally been found under the abnormal conditions enumerated above. The reason for this has less to do with

electrical operation than with the nature of normal railway working.

Improvements in Passenger and Goods Traffic Working.—Passenger service is improved by running more frequent and faster trains. Experience indeed has shown that an enormous increase of revenue results from these causes in the case of railways working short haul traffic in urban districts, and the gain is considerable also for railways which provide inter-urban service in well-populated regions. Long distance service, however, does not appear to offer scope for the improvement possible in short haul service. Here, increase of speed is apt to be limited by considerations of roadbed, curves, etc., which have nothing to do with the system of operation; and the reasons for travelling long distances are not generally such as frequency of service would affect. Improved branch-line services, by feeding the main lines more efficiently, would doubtless lead to some increase in travel; but on the whole it appears that, apart from urban and certain localized inter-urban railways, no very great increase in revenue can be foreseen as the result of electrical operation. It is, however, largely a question of psychology on which experience is not yet available; and it is worthy of remark that, in the past, the increase of revenue resulting from improved facilities has usually exceeded all reasonable estimates.

It does not appear that goods traffic would in general be increased by electrification, for no question of psychology comes into the transportation of goods; and it is only occasionally that a sensible improvement in facilities could be offered. The most desirable improvements in the working of goods traffic are not usually within the power of the Railway Company to effect, and certainly not such as electrification would influence. There are, however, certain incidental advantages in the electrical operation of goods traffic, arising primarily from the great power on which a locomotive can draw. This permits greater rapidity in train working; and in congested districts allows more efficient use to be made of the lines.

Economic Aspect.—It may be concluded, therefore, that apart from conditions of special difficulty, and from traffic of special nature, electrical working must be justified for the

most part by its economy rather than by the improved facilities it can offer. In countries where water power is abundant and locomotive fuel has to be imported, the justification is in general present in the saving of fuel, particularly if an industrial load can be used to aid in the development of the water powers. Where, however, coal is abundant, it is more difficult to justify the electrical working of normal railroads. It is true that such working results in a great saving of fuel, variously estimated from a half to two thirds of the consumption of the steam-worked trains. It is true also that as a result of the war, the value of coal is likely to remain high as compared with other commodity values, so that the saving is of increased significance. But it is nevertheless very doubtful whether the saving would in general be sufficient in itself to justify the electrification of the railways. In densely populated countries, however, there are likely to be many regions of the kind referred to above, in which electrical operation is justified, not on account of economy of working, but on account of the increase in revenue, which results from improved traffic facilities. Where such conditions exist, the balance may readily be turned in favour of the electrical working of the whole railway system; for, with many sections of the line suitable for electrical operation, the additional outlay required to work traffic, which considered by itself would be uneconomical, is smaller than if this traffic were so considered, whilst the economy of the working is at least as great. Indeed where any considerable section of a locomotive division is electrified for sufficient reasons, it is usually justifiable to electrify the whole division for the sake of the saving which results; and if one class of traffic is worked electrically, all classes using the lines may be so worked with advantage.

Technical View of Electrical Operation. From the standpoint of technical engineering, electrical operation is distinguished essentially as employing centralized power generation, as against the distributed power generation of steam operation. This is at once a strength and a weakness of the system. On the one hand, it enables power to be concentrated where it is most needed, thus making it possible to work heavy trains on steep gradients with economy, and to give a high rate of acceleration and high average speed to trains

engaged in suburban service. On the other hand, a breakdown at a vital point may stop all traffic throughout an extended area; and it is necessary to exercise the greatest care in the engineering, and to expend a large amount of capital in standby plant, and devices whose sole purpose is that of minimizing the chances of serious breakdown.

Adaptation of Methods to Agent.—It is a mistake to view electrical operation of railways simply as a question of the supersession of the steam locomotive by the electrical locomotive; for the steam-worked railway has grown up around the steam locomotive, and the whole method of working the traffic accords with the limitations and characteristics of this machine. Electrical operation should, in like manner, be conducted to suit the characteristics of the electric locomotive, and indeed of the whole plant. It is accordingly unfair to electrical operation to judge it as limited by the methods of steam-operation; and more or less onerous to mingle the two methods of operation. On the other hand, confidence may be felt that economic estimates based on present methods of working will be improved upon as more appropriate methods are adopted.

In general, the great power available at any point of an electrically worked railway, and the long continued duty of which electrical apparatus is capable, remove limitations under which steam operation suffers; and thereby gives greater freedom to the traffic managers in dealing with the work of their departments. At the same time, if the best results are to be obtained, certain limitations of electrical operation should be recognized. Chief among these is perhaps the desirability of spreading the whole effective load as uniformly as is practicable, both in time and space, thus making efficient use of the generating and substation plant, and reducing the investment therein. Close association is desirable between goods and passenger departments, in order that their respective load-variations may be made, as far as may be practicable, complementary to one another. The Chicago Milwaukee & St. Paul Railway takes power for working its Rocky Mountain divisions from the Montana Power Company, which operates a large number of hydro-electric plants in the region; and the railway company has contracted to pay for power in the

following manner. If the load factor in any month is less than 60 per cent., payment to be made at a definite price (5·36 mils. per unit) for a uniform load equal to 60 per cent. of the maximum 5-minutes peak; if the load factor exceeds 60 per cent., payment is made at the same rate for the actual k.w. load. By means of an efficient system of train-dispatching, and with the aid of a number of automatic and hand devices for reducing the substation voltage when the current is excessive, the peaks of load are kept down and the load factor raised, so that in practice it nearly attains the 60 per cent. for which payment is made. As regeneration is a feature of this road, it is probable that, without these devices, and with trains worked without reference to the supply conditions, the load factor would not have exceeded 25 per cent.; and the amount of generating plant required to have been kept at the disposal of the railway company would have been more than doubled. In this case the train dispatcher is given a wide discretion as regards some of the traffic, and he is able to use it greatly to the advantage of the railway. Although a control so highly centralized may not always be practicable, it is very desirable in the interests of economy that there should be very intimate co-operation between the traffic and operating departments.

Electric and Steam Locomotives.—The nature of the steam-locomotive places it under disabilities from which the electric locomotive is happily free. It consumes fuel as long as it is in commission, whether it is in the shed or out, whether it is hauling a train or standing. A large fraction of its life is consumed in tube-cleaning, oiling, and overhauling. The electric locomotive, on the other hand, consumes power only when running, and the time spent in inspection, overhauling and cleaning is insignificant. Much greater service can accordingly be got from the electric machine in the course of a year, and the number of locomotives required to work a given traffic is correspondingly smaller. In wintry weather particularly the steam locomotive suffers in efficacy, but the electric locomotive retains and indeed increases its service capacity. Experience has shown that, in general, half the number of electric locomotives is more than equivalent to a given number of steam locomotives in service capacity.

The electric locomotive is, however, not the full equivalent

of the steam locomotive, in that the latter is a power generator as well as a power consumer. To put steam and electrical operation on a comparable basis, the whole chain of appliances from power generating plant to wheels should be brought under review. The essential feature of electrical operation is centralized power-generation, with distribution to the power-consuming locomotives. Great power is therefore at the disposal of every locomotive on the railway, and it is this, in great measure, that enables electrical operation to deal with conditions of abnormal difficulty.

In the chain of appliances which constitute the working plant of an electric railway, the locomotive must be regarded as the weakest link, in the sense that it is less amenable to adjustment than other parts of the plant. The reason behind this is that the locomotive can only provide limited space for the apparatus, which is therefore restricted in design, besides having to stand continual vibration; the stationary plant on the other hand is not tied for space, and rests on firm foundations. Given a satisfactory locomotive (using the term to mean the motor-carrying vehicle whatever form it may take), a reliable power supply and distribution system can be devised to suit; but if from any cause the locomotive is unsatisfactory, no merit in the rest of the plant can make up for the deficiency. It is therefore the best practice which chooses methods of operation strong at the locomotive end of the chain, and adjusts the remainder of the plant to suit the locomotive.

System of Operation.—The nature of the plant which connects the prime movers with the locomotive wheels depends in large measure on the system of operation adopted. The technical characteristics of the several systems in use are described in a later chapter; and it is these that ultimately determine the economic results by which the systems are to be judged. These results depend also on the circumstances of the case; and, it may be added, on the time; for the Great War has changed this, like so many other things. Neither present nor pre-war costs give a reliable indication to future costs. The relation between the burden of capital expense and operating cost is particularly conjectural at present.

The question of system of operation must be viewed broadly, if it is to be decided judicially; for concentration on special

features is apt to mislead the judgment. Every system has advantages which are more or less valuable according to the circumstances of the particular case. For purposes of general railway operation, however, two systems only need be considered, viz., the continuous current system and the single-phase system. For heavy urban passenger service, the former has natural advantages ; and it would not be difficult to propose a service of this nature which the latter system would practically be unable to undertake. Apart from such special problems, however, it may be said that either system could be used for the operation of a railway at some cost ; and the question of selection resolves itself into the determination of the least costly, having regard both to initial and operating expenses, with due allowance made for indirect advantages or disadvantages.

The adoption of partial views by a few prominent engineers has resulted in somewhat acrimonious discussion of the subject of systems of operation, both in England and in America. Even before a single-phase railway motor had been developed, the system had been proclaimed the only possible one for railway working.* The development of the motor was therefore hailed with an enthusiasm which its technical qualities by no means merited. Cautious designers, studying to perfect it, saw that fundamentally it was inferior to the continuous current motor ; and had doubt whether the advantages of the system were sufficient to warrant its use in the exacting conditions of railway service, where the locomotive motor was already tried to the utmost. However, the advertisement of the system continued, and many engineers looked forward to its universal adoption. The author was apparently among the first publicly to dissociate himself from this view, showing in the course of a paper read in 1906 † that for suburban service the single-phase system compared unfavourably with the continuous current system. This conclusion, now regarded as commonplace, was strenuously contested at the time. Mr. H. M. Hobart, who was among the earliest advocates of a high voltage continuous current system, did much to propagate sound views on the subject, insisting on economic as well as technical comparison of the two systems.

* See *Minutes of Proc. Inst. C.E.*, vol. 49, p. 40.
† See *Journ. of Proc. Inst. E.E.*, vol. 36, p. 231.

Unsuitability of Single-phase System for Urban Railways.—Time and experience being on the side of good engineering, the enthusiasm for the universal use of the single-phase system waned as its characteristics became better known. It was, however, an episode in connection with the electrification of the Victorian Railways that finally demolished its pretensions as applied to suburban service. The scheme was perhaps the most extensive that has ever been undertaken at a single venture, including in its scope more than three hundred miles of trackwork in the neighbourhood of Melbourne.

TABLE I

	Direct Current Scheme.	Single-Phase Scheme.
POWER PLANT.	£	£
Turbo-Alternators, step-up transformers, auxiliary transformers and condensing plant	222,151	241,377
Power station switchgear, auxiliary motors, wiring and yard locomotives	46,956	46,667
High Tension Transmission :		
20,000 V. underground cables .	106,368	110,015
20,000 V. overhead lines .	61,495	21,368
Substations	378,605	41,905
Electrical Equipment of Permanent Way :		
(*a*) Trolley wires (material only) :		
For running roads .	142,165	47,806
For sidings	4,264	4,264
(*b*) Erection of conductors, provision and erection of structures, insulators, connecting cables, section switchgear and track bonding	415,851	359,846
Alterations to Ways and Works :		
(*a*) Increasing height under bridges	4,350	24,800
(*b*) Altering telegraph and telephone wires	20,000	181,632
Rolling Stock :		
Electrical equipment of 500 motor coaches and 450 trailer coaches .	947,232	1,977,344
	£2,349,437	£3,057,024

and involving an expenditure of several millions sterling. The service contemplated was fairly heavy, but by no means approached the limit of practicability. The engineers in charge of the work considered it a case for the use of the continuous current system; but strong influence was brought to bear in the interest of the single-phase system. In order to satisfy their clients, therefore, the engineers called for complete tenders and guarantees for the work in both systems, the service required being exactly defined. Tenders were obtained from all parts of the world, and the results of the enquiry are sum-

TABLE 2

	Direct Current Scheme.	Single-Phase Scheme.
ELECTRICAL ENERGY.		
Variable power house charges, including coal, water, stores and wages of coal and ash handling staff	£ 59,700	£ 58,400
Inspection and maintenance of high tension transmission lines	3,190	2,169
Operation and maintenance of substations	12,456	1,940
Maintenance and renewal of track conductors	18,051	18,051
Maintenance of coach equipments:—		
(*a*) Inspection, cleaning, stores and small repairs	22,900	30,800
(*b*) Repairs and renewals	43,900	91,100
Interest charges on capital expenditure	93,977	122,281
	£254,174	£324,741

marized in tables 1 and 2,* which, however, include only the items affected by the question of system. The tables, although applying to a particular scheme, are in their main features characteristic of suburban service. By far the largest item of expenditure for plant is in any case that for train-equipments; and this is at least doubled in the single-phase system as compared with the continuous current system. Although the other items, in their sum, show a balance in favour of this

* *Times* Engineering Supplement, Nov. 20, 1912.

system, it is insignificant compared with the adverse balance on the equipment. The running costs moreover are governed in large measure by the item for maintenance of coach equipments, and, since this is also about doubled in the single-phase system, the total shows a considerable balance in favour of the continuous current scheme.

DISTRIBUTION OF CAPITAL COSTS IN URBAN ELECTRIFICATION.—The prices at which the contracts in connection with the Victorian Railways electrification were given out may here be cited (Table 3), not indeed as having any absolute value at this date, but as being fairly representative of the proportions of the various items in suburban electrification. For this reason, the several items have been expressed also as percentages of the whole, both with generating plant and transmission lines included and with these items excluded.

TABLE 3

Item.	£	%	%
Boiler house equipment and structural steel work for generating station	427,720	18·8	—
Turbo-alternators and transformers	182,046	8·0	—
Condensing plant	85,251	3·8	—
Substation equipments	201,624	8·9	16·5
Switchgear, power station and substations	140,070	6·2	3·8
High Tension feeder cables	259,121	11·4	—
Overhead track equipment	278,286	12·3	22·8
Bonds	20,000	·9	1·6
Train equipments	676,180	29·8	55·3
	2,270,298	100·1	100·0

Table 3 may be compared with table 4, which gives relative items of expense for the electrification of the Rocky Mountain divisions of the Chicago Milwaukee and St. Paul Railroad.*

* Given by R. Beeuwkes, Electrical Engineer, C.M. & S.P. Ry., in Report of Committee on Electrification of Steam Railroads, National Elec. Light Association, 1920. See *Electric Railway Journal,* 29 May, 1920, page 1103.

These figures may be taken as typical of main line railways using the high voltage continuous current system for working heavy traffic.

TABLE 4

CHICAGO MILWAUKEE AND ST. PAUL RAILWAY.

ELECTRIFICATION COSTS:

Route miles railway	438
Mileage transmission lines	364
No. of substations	14
Total capacity substations (k.w.)	59,500
No. of road locomotives	42
No. of switching locomotives	2

	Cost $	Cost %
Trolley system, complete	3,675,000	28·4
Transmission system	1,030,000	8·0
Substation buildings	620,000	4·8
Substation plant	2,030,000	15·7
Engineering and miscellaneous	340,000	2·6
Locomotives, road	5,150,000	39·9
Locomotives, switching	75,000	·6
Total	$12,920,000	100·0%

American Experience. Reverting to the question of systems of electrification, the controversy in America was as vigorous as that in this country, and similarly based on preconceptions. Time, however, has dissolved the illusions and little of the controversy now remains. A number of sections of American railroad use the single phase system of operation, but nowhere with outstanding success. Indeed American experience shows nothing to justify the system even when used for trunk line service, remote from suburban territory.

THE SINGLE-PHASE LOCOMOTIVE MOTOR. The weakness of the single-phase system, as disclosed by American experience, lies principally in the locomotive motor, which is costly both in manufacture and maintenance. The later defenders of the system, indeed, are disposed to throw over the single-phase motor, and to rely on the induction motor and phase converter, or on the continuous current motor and rectifier to justify their position; although the practice of carrying the sub-

TABLE

GENERAL DATA AND MAINTENANCE COST OF MAIN LINE ELECTRIC
SION AND ANNUAL REPORTS

ALTERNATING.

	Boston & Maine Railroad (Hoosac Tunnel.)		
	1916.	1917.	1918.
No. of Elec. Locomotives	5	7	7
Avg. Loco. Wgt. Tons—2,000 lbs.	—	130	—
Date of First Installation	—	—	—
Route Miles—Electrified Track	—	—	—
Miles—Single Track Basis	—	—	—
Maintenance of Elec. Locos.—			
Repairs in Dollars	25,104	59,673	89,543
Depreciation in Dollars	7,360	7,673	11,215
Elec. Loco. Mileage—			
Freight Revenue Miles	160,626	171,663	—
Passenger Revenue Miles	56,411	43,381	—
Switching Revenue Miles	—	—	—
Mixed and Special Revenue Miles	401	590	—
Total Revenue Miles	217,438	215,634	212,553
Non-Revenue Miles	2,661	1,511	3,548
Total Locomotive Miles	220,099	217,145	216,101
Maintenance per Locomotive Mile in Cents (not including retirements and depreciation)	11·4	27·5	41·43

DIRECT

	Baltimore & Ohio Railroad (Baltimore Tunnels).		
	1916.	1917.	1918.
No. of Elec. Locomotives	10	10	9
Avg. Loco. Wgt. Tons—2,000 lbs.	—	—	98
Date of First Installation	—	—	—
Route Miles—Electrified Track	—	—	—
Miles—Single Track Basis	—	—	—
Maintenance of Elec. Locos.—			
Repairs in Dollars	15,161	12,501	18,117
Depreciation in Dollars	6,372	5,978	5,662
Elec. Loco. Mileage—			
Freight Revenue Miles	169,940	162,492	137,852
Passenger Revenue Miles	67,272	71,104	86,960
Switching Revenue Miles	9,180	9,210	9,080
Mixed and Special Revenue Miles	—	—	—
Total Revenue Miles	246,392	242,806	233,892
Non-Revenue Miles	—	—	—
Total Locomotive Miles	246,392	242,806	233,892
Maintenance per Locomotive Mile in Cents (not including retirements and depreciation)	6·15	5·15	7·74

* 3 40-Ton tractor trucks

	New York Central Railroad.		
	1916.	1917.	1918.
No. of Elec. Locomotives	63	73	73
Avg. Loco. Wgt. Tons—2,000 lbs.	—	118	—
Date of First Installation	—	—	—
Route Miles—Electrified Track	—	—	—
Miles—Single Track Basis	—	—	—
Maintenance of Elec. Locos.—			
Repairs in Dollars	64,950	87,280	116,111
Depreciation in Dollars	51,330	68,572	79,763
Elec. Loco. Mileage—			
Freight Revenue Miles	2,473	2,560	2,702
Passenger Revenue Miles	1,387,569	1,401,567	1,163,430
Mixed and Special Revenue Miles	710	466	189
Switching Revenue Miles	759,024	758,431	681,527
Total Revenue Miles	2,149,776	2,163,024	1,847,848
Non-Revenue Miles	11,549	11,125	6,660
Total Locomotive Miles	2,161,325	2,174,149	1,854,508
Maintenance per Locomotive Mile in Cents (not including retirements and depreciation)	3·00	4·01	6·26

LOCOMOTIVES FROM REPORTS OF INTERSTATE COMMERCE COMMIS-
CALENDAR YEARS 1916–1917–1918

URRENT.

GREAT NORTHERN RAILWAY (CASCADE TUNNEL).			NEW YORK, NEW HAVEN & HARTFORD RAILROAD.			NORFOLK & WESTERN RLY.		
1916.	1917.	1918.	1916.	1917.	1918.	1916.	1917.	1918.
4	4	4	100	102	103	12	12	12a
—	—	—	—	110	110	270	—	—
—	—	—	—	—	—	1915	—	—
—	—	—	—	79	79	30	—	—
—	—	—	—	550	550	85	—	—
6,278	5,436	9,269	448,554	612,641	815,368	107,257	166,249	225,347
3,220	3,220	3,220	76,810	77,000	77,470	28,800	28,800	28,800
46,608	38,128	49,056	734,558	720,233	703,987	342,265	378,523	478,318
31,944	32,904	26,040	3,619,465	3,514,637	3,406,549	18,451	22,542	24,122
—	—	—	1,107,714	1,060,660	1,000,171	19,335	25,538	32,047
—	—	24	960	1,126	775	28	28	0
78,552	71,032	75,120	5,462,697	5,296,656	5,111,482	380,079	426,631	534,487
—	—	—	24,116	20,327	31,359	160	24	190
78,552	71,032	75,120	5,486,813	5,316,983	5,142,841	380,239	426,655	534,677
7·99	7·65	12·34	8·18	11·5	15·85	28·21	38·96	42·15

URRENT.

BUTTE, ANACONDA & PACIFIC RAILWAY.			C. M. & ST. P. RAILWAY.			MICHIGAN CENTRAL RAILROAD (DETROIT RIVER TUNNEL).		
1916.	1917.	1918.	1916.	1917.	1918.	1916.	1917.	1918.
24	28*	28*	20	44	45	10	10	10
80	80	—	290	290	290	—	—	—
—	—	—	1915	—	—	—	—	—
—	—	—	220	440	447	—	—	—
114	—	—	300	590	600	—	—	—
49,811	55,846	60,295	90,961	220,526	236,906	8,022	16,474	41,312
24,143	36,695	38,132	33,909	77,134	85,209	14,208	14,219	14,222
506,162	412,509	438,033	718,461	1,289,167	1,253,850	134,774	144,158	172,334
100,290	94,659	80,020	335,557	785,703	672,929	43,642	43,227	40,020
404,356	367,690	414,158	18,063	106,143	89,765	35,802	68,862	65,804
616	456	—	451	960	1,585	—	16	8
.011,424	875,314	932,211	1,072,532	2,181,973	2,018,129	214,218	256,263	278,166
2,477	10,796	3,072	35,537	111,198	161,108	—	435	373
.013,901	885,110	935,283	1,108,069	2,293,171	2,179,237	214,218	256,698	278,539
4·91	6·3	6·45	8·21	9·62	10·87	3·74	6·42	14·83

so in operation.

PENNSYLVANIA RAILROAD.		
1916.	1917.	1918.
34	35	35
—	156	—
—	—	—
—	—	—
—	—	—
41,772	63,352	121,273
75,021	71,636	70,731
873	79	4,657
559,660	613,956	664,257
504	18	9
407,202	441,655	452,360
968,239	1,055,708	1,121,283
41,036	41,776	24,398
)09,275	1,097,484	1,145,681
4·14	5·77	10·59

station plant on the locomotive hardly gives promise of great advantage, either in first cost or running cost.

The comparison of costs of different railway systems, unless conditions of operation are similar, and similar methods of accounting are employed, is apt to be misleading and should not be given undue weight. Statistics compiled to meet statutory requirements however have value, and if used with sagacity, justify general conclusions. Table 5 gives statistics of maintenance costs of the electric locomotives used on a number of American railways, the figures being taken from the Reports of the Interstate Commerce Commission. In this table, the New York Central, the Pennsylvania, and the New Haven electrifications may be considered in a general way comparable, as being terminal electrifications of New York City. The Baltimore and Ohio, the Great Northern, the Michigan Central, and the Boston and Maine electrifications are comparable in being local tunnel systems. The Chicago Milwaukee and St. Paul, the Butte, Anaconda and Pacific and the Norfolk and Western electrifications are comparable as dealing with heavy trains on steep gradients. Incidentally the figures of table 5 show the effect of the war in increasing running expense.

INTERURBAN PASSENGER SERVICE. — Interurban electric passenger service, as conducted in America, was for some years regarded as particularly suited to take advantage of the merits of the single-phase system, and a number of roads were so electrified. The service is generally intermittent and the distances considerable. The traffic is for the most part worked by single cars of great weight. The stops being few, a high rate of acceleration is not essential. The distribution and substation costs in such service are proportionately much greater than in city service. Unfortunately, the inferiority of the single-phase motor has proved the obstacle to success, even here. The system shows no advantage over the continuous current system in such service; and indeed many of the roads in question have, after more or less extended experience of the former system, changed to the latter.* Such action on the part of a railway would indicate a saving in operating

* *E.g.*, Anderson Traction Co., Annapolis Short Line, Atlanta and Marietta, Illinois Traction System, Milwaukee Electric Railway, Piedmont and Northern, Pittsburgh and Butler, Toledo and Chicago,

expense by the conversion sufficient to justify the heavy additional burden of capitalization involved in the change.

A comparison was made some years ago, of operating conditions and expenses of single-phase interurban roads with those of high voltage (1,200 volts) continuous current roads. The roads of the two kinds were chosen to be as nearly as possible comparable in service conditions, and represented more than 40 per cent. of the interurban mileage of the classes in question existing in the United States at the time. The investigation established that the average number of men employed in car-barns and substations, taken together, was 3·2 per car in service for roads worked by the single-phase system, and 1·6 per car in service for the continuous current roads. The combined running expenses of car-barns and substations, together with maintenance of overhead lines, was found on the average to be 5·42 cents per car-mile for the single-phase system, and 2·44 cents for the continuous current system.* The particular case of the Washington Baltimore and Annapolis Railroad,† a road which has had experience in both systems, may be cited. In 1909, under 6,600 volt single-phase operation, the number of cars was 23, and the number of men employed in the car-barns 63, the car-barn expenses amounting to 3·72 cents per car-mile. In 1911, under 1,200 volt continuous current operation, the number of cars was 44, the number of men employed in the car-barns 27, and the car-barn expenses 1·37 cents per car-mile. It is not remarkable therefore that for this class of service the single-phase system is no longer considered.

Frequency in Single-phase Operation. American experience, therefore, justifies the opinion of those who favour the continuous current system; and it is worthy of notice that of all countries the United States alone has had extended experience of both systems under railway conditions, including the heaviest classes of service. Continental engineers, however, explain the unfavourable results obtained with the single-phase system in America as being due principally to the general use of a frequency of 25 cycles per second instead of about 15 cycles, as required for the successful operation of the single-

Warren and Jamestown, Washington, Baltimore and Annapolis, York and Hanover.

* *Times* Engineering Supplement, Sept. 27, 1911.

† *General Electric Review*, vol. 15, p. 464.

phase motor. It must be admitted that there is some justification for the contention: the single-phase motor is severely limited; and the lower the frequency of supply, the more successfully can it be designed, until at zero frequency it merges in the continuous current motor itself. This is well understood by designers, and indeed the first proposal of single-phase operation in America was at $16\frac{2}{3}$ cycles.* The Visalia Electric Railway is moreover operated single-phase at 15 cycles; and the Pennsylvania experiments were conducted at the same frequency.† Nevertheless the additional expense and other grave disadvantages of generation at the low frequency have caused the higher frequency to be generally accepted by American engineers as the lesser evil.

The Continental Development.—The single-phase system has been developed rationally in Germany and other continental countries. The limitations of the locomotive motors have been duly recognized, and the whole installation designed in conformity therewith. Motors of large capacity are employed, the motive power of the locomotive being concentrated in one or two motors only; although this involves the use of side rod types of locomotive. Power is supplied at a frequency of 15 or $16\frac{2}{3}$ cycles per second; and is, in general, generated at the frequency of supply; so that the use of rotating machinery between generators and trains is avoided. Under these conditions it is claimed that disabilities under which the single-phase system has been found to suffer elsewhere, are no longer oppressive; and the simple distribution arrangements accordingly restore the balance in favour of the system. Unfortunately no adequate statistics are available in support of the claim. Unfortunately also, comparable experience with the rival system, under railway conditions, is almost lacking in the countries where the single-phase system has reached its highest development.

Bureaucratic Engineering.—To the seeker after truth it is a little disconcerting to find different communities arriving at different conclusions on fundamental matters of fact. In details such differences are to be expected; they are accounted for by differences in labour costs, in the general level of skill and education of the workmen, in temperament of public and

* *Transactions A.I.E.E.*, Vol. 20, page 15.

† *Ibid.*, Vol. 26, page 1385.

staff, and in other such national characteristics. There are, however, no differences apparent in operating conditions in normal civilized countries sufficient to account for an entire change in economic values such as is indicated by a difference in system of operation. It might be imagined that the matter was really a somewhat indifferent one, were it not that, where direct comparison has been made between the systems, the results have always proved decisive. There is, however, an aspect of the matter which should be kept in mind. The management of a railway is a large and complex organism, with a natural tendency to bureaucracy; and its efficiency is usually most in evidence when it is able to keep in a familiar groove. In breaking new ground, it is as apt as any other human institution to be carried away by the most confident of its advisers. But once having decided upon a course, and involved itself in great expense, it can rarely reconsider the matter; for, besides the economic, there are very human issues involved. In the case of railway companies fettered only by commercial considerations, economic pressure may in the long run be relied upon to exert some righting effort; but where the railways are owned and operated by the State even this influence will be of little avail. Under such auspices indifferent engineering is likely to be perpetuated; for there is none to question it. But it is the Prussian State Railways that have led the development of the single-phase system in Europe; and it is not too much to say that the opinion of its engineer has been largely responsible for the Continental development of the system. The biased attitude of the German authorities towards the question, may be judged from the following recent statement,* reported to be official: "The German Federal Railway Administration has always considered the single-phase system as the only one possible for its main lines and has never, even temporarily, considered any other."

Apart from the State Railways there is no unreserved acceptance of the single-phase system in Germany. The Hamburg Elevated and Underground Electric Railway is an urban and suburban line which parallels in parts the single-phase Blankenese line, and deals with a similar class of traffic. It is, however, of more recent installation than the State line, having been opened for traffic in 1912. The concession for

* *Deutsche Allgemeine Zeitung*, May 14, 1921, quoted from *Electric Railway Journal*, Vol. 58, page 14, July 2, 1921.

equipping and operating the railway was granted to Siemens and Halske and the A. E. G. jointly ; and it is worked by the continuous current system at 800 volts.

However the main Continental development ended with the outbreak of war, and much water has flowed under the bridges since then. Outside of Germany there is generally a wholesome tendency to investigate the merits of the rival systems before declaring in favour of either. The French Government recently appointed a Commission to consider the question as regards its own railways ; and this Commission, after very careful and full investigation, reported strongly in favour of a continuous current system at moderately high voltage.* The Belgian and the Netherlands Governments have also investigated the matter, and have come to a like decision. The Swedish Government has the matter under consideration at the time of writing. The conditions in Sweden, it may be remarked, and particularly in the Northern provinces, favour the single-phase system ; and the decision to extend the electrification of the Riksgräns line to Lulea on this system, has all the appearance of sound engineering, quite apart from the interest vested in the system by the existing electrification. It is, however, the lines of the Southern provinces that are at present under consideration ; and here, although the population is sparce by comparison with the countries of Western Europe, it is sufficiently dense to render the choice of the single-phase system of doubtful expediency. Little has transpired of the Norwegian attitude towards the subject, for the natural extension of the Riksgräns line from the Swedish border to the ice-free port of Narvik cannot be assumed to indicate a general policy. Switzerland is apparently committed to the single-phase system. The British authorities, through the medium of an Advisory Committee of the Ministry of Transport, which studied the matter in all its aspects, have decided in favour of the continuous current system, with a preferred line pressure of 1,500 volts, permitting, however, a multiple or sub-multiple of this figure where local conditions demand it. As regards extra-European countries, other than the States, it is of interest to note that both Brazil and Chile have adopted that high voltage continuous current system for extensive electrification schemes. The South African Railways have also adopted this system.

* 1,500 volts.

CHAPTER II

THE LOCOMOTIVE

The motive apparatus for electric trains is in some cases preferably distributed through the train, employing the coach axles as driving axles, and in other cases more advantageously collected in locomotives designed for the sole purpose of accommodating it. In suburban passenger service, if the full advantage of electrical working is to be realized, the multiple unit operation of motor coaches is essential, and this disposition of the motive apparatus can often be employed with advantage in other classes of passenger service; although the extent to which it is economical to use it depends largely on the system of operation employed, being greater in the continuous current system than in the single-phase system, and least of all in the polyphase system. For goods traffic and for high-speed long-distance passenger traffic, on the other hand, the independent locomotive is the preferable and in fact the only practicable means of applying tractive force to the trains. From the present point of view, however, which is concerned rather with the mechanical features of the drive than with economy of operation, it is unnecessary to distinguish between the locomotive and the motor coach, for although the latter is burdened with certain restrictions which the former escapes, these restrictions are of so little consequence that locomotives are frequently designed to employ the same type and arrangement of drive as is used on the motor coach. In the present work, therefore, the term "locomotive" may be taken as including "motor coach" wherever the matter is applicable to this form.

Like most human contrivances, the electric locomotive is composed of elements which have usually to effect a compromise between more or less conflicting ideals, and the extent to which it is advisable to allow the various ideals to influence

the design and construction depends on the circumstances and particularly on the system of operation and the class of service for which the locomotive is to be used. Much is to be learned in this regard from experience with the steam locomotive, although, as will readily be realized, the electric locomotive presents other problems and has its limitations in other directions. Much is also to be learned from tramway experience, and in fact some of the most successful electric locomotives have been developed directly from such experience. An uncritical description of existing electric locomotives would fail to furnish a reliable guide to the most desirable practice; for some are known to be unsatisfactory, and comparatively few have passed the test of having been duplicated at later date. Such a condition of affairs is of course to be expected in the early stages of development of a difficult art.

CLASSIFICATION OF LOCOMOTIVES

The number of types of electric locomotives that have been developed is large, and it is a matter of some difficulty to classify them clearly. They, however, admit of a primary division into two categories, namely, those in which the driving axles are actuated each by a separate motor, and those in which these axles are grouped and driven collectively, through the medium of side coupling rods, by one or more motors.

Individual Drives.—SINGLE REDUCTION GEARING.—In locomotives having independently driven axles, the commonest form of drive employs single reduction gearing, with the motor suspended between axle and transom. This method—a development from tramway practice—is in universal use for the motor coaches of multiple unit trains, and is still the commonest for locomotives of moderate speed and capacity. Fig. 1 shows the usual arrangement of the motor in which the gearing is entirely at one end, whilst fig. 2 shows an arrangement now frequently employed with powerful motors, in which twin gears are used. The unsymmetrical drive of fig. 1 tends to wear the journal and linings at the pinion end of the motor more than, and on the opposite side to, those at the commutator end, thus throwing the armature shaft slightly out of parallelism

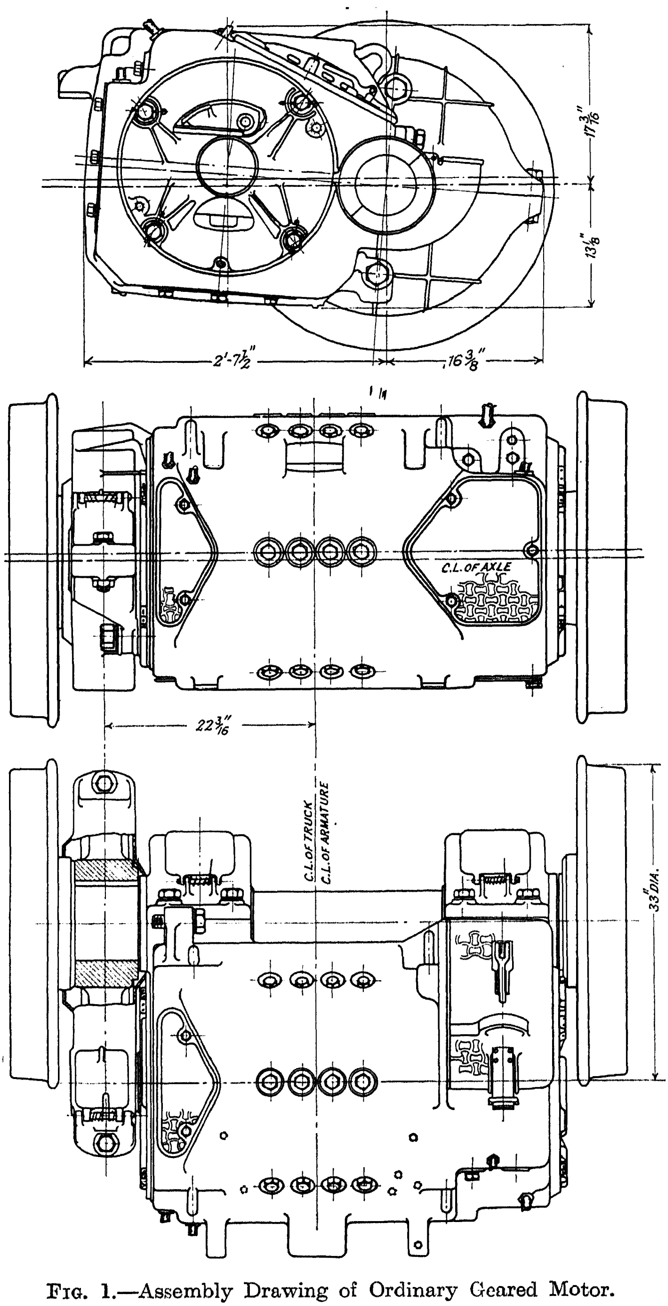


FIG. 1.—Assembly Drawing of Ordinary Geared Motor.

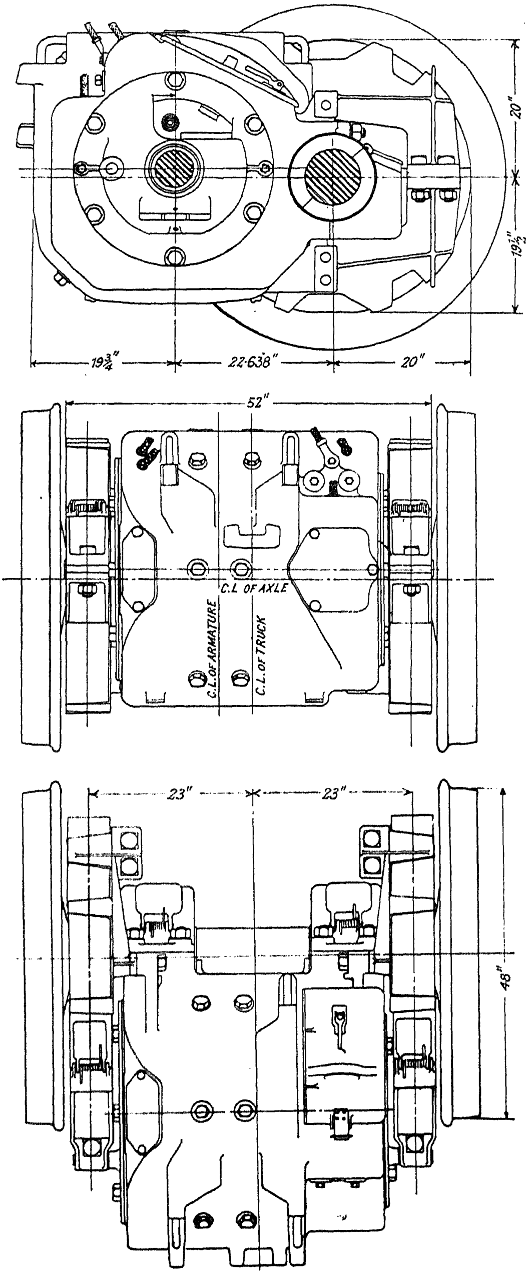


FIG. 2.—Assembly Drawing of Twin-Geared Motor.

with the axle and increasing the stresses at the inside end of the gear teeth. The gear face with large motors is usually made some 5 inches to $5\frac{1}{2}$ inches wide, and increase in width beyond this does not, in practice, increase the effective tooth strength appreciably. The arrangement of fig. 2 makes better use of the material of the teeth, but with rigid gears results in indeterminate tooth stresses, and accordingly requires very accurate

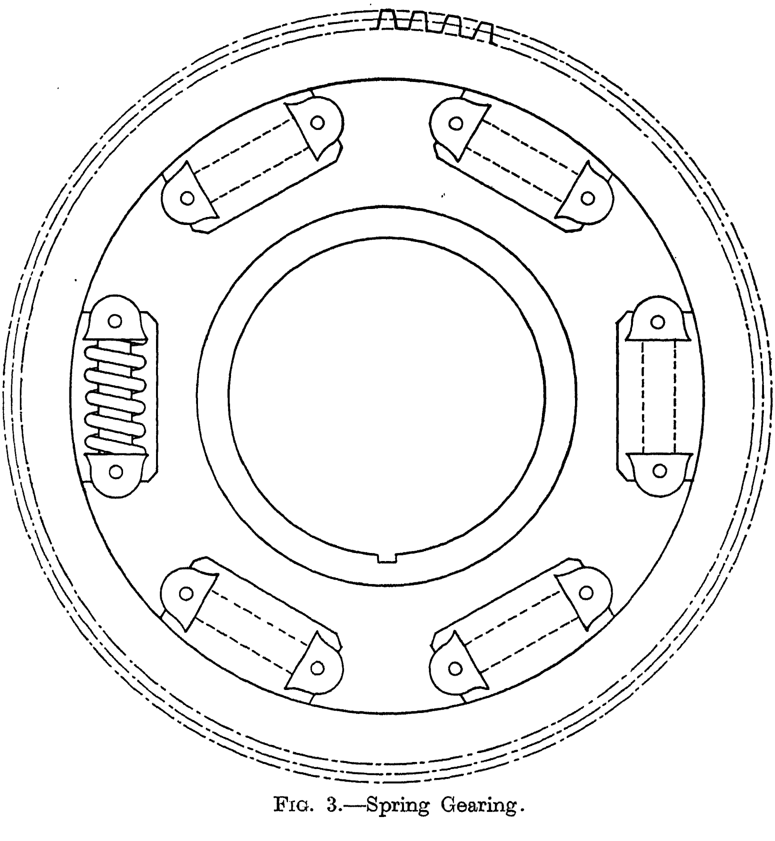


FIG. 3.—Spring Gearing.

fitting in order to make the two sets of gearing engage with approximately equal stresses. The need of such extreme accuracy may be avoided by the use of right and left-handed helical gears, between which the armature floats. Another means employed with the same object is to transmit the force from gears to axle through springs which by permitting appreciable strain tend to equalize the stresses (see fig. 3). Examples

of the use of twin gears are to be found in the Chicago Milwaukee and St. Paul locomotives, the Butte, Anaconda and Pacific locomotives, the Detroit River tunnel locomotives, the St. Clair tunnel locomotives, the Hoosac tunnel locomotives, and the Shildon-Newport locomotives.

Quill Drive.—In some cases the axle-gears are mounted on sleeves or quills which surround the axles with adequate clearance and drive the wheels through springs (fig. 4). With this construction, since the axle bearings are carried by the quill, it is practicable to mount the motor above the axle, as is done in the Hoosac tunnel locomotives, and in many of the New York, New Haven and Hartford locomotives. Quill mounting

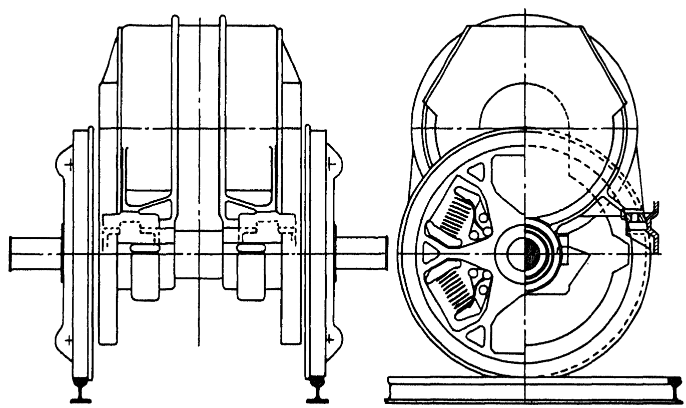


Fig. 4.—Geared Quill Drive.

is particularly useful in connection with single-phase motors, the elastic transmission serving to cushion the impulsive stresses of the driving forces which fall to zero or reverse twice in each period of alternation. By the use of this construction substantially uniform tractive effort is maintained at the wheels, whilst the heavy motor is supported elastically and the gear teeth relieved of shocks; and on this account the construction is frequently employed even when the single-phase motors are carried horizontally, as in the New Haven motor coaches.

Gearless Drive.—Another method extensively used for driving locomotive axles independently, employs gearless motors having their armatures mounted concentrically with the axles. In some of the older locomotives, such as those

originally used on the Central London Railway, the whole motor was carried directly on the axle without the intervention of springs. This construction would not now be considered good practice, and in fact, the locomotives in question were early superseded on account of the excessive pounding effect of the heavy uncushioned masses. In the Grand Central Terminal locomotives of the New York Central and Hudson River Railroad, and in some of the C.M. and St. P. locomotives, the armature of the driving motor is built directly on the axle, whilst the field structure is carried on the locomotive frame, the motor being bipolar and so constructed that exact adjustment between armature and field in a vertical direction is unnecessary. Fig. 5 is a drawing of one of the New York Central locomotives in part section, and shows how the motors are mounted; figs. 58 and 59 show longitudinal and transverse sections of such bipolar motors. In the New York, New Haven and Hartford Railroad passenger locomotives of the first type, the motor is carried on the locomotive frame, whilst its armature is

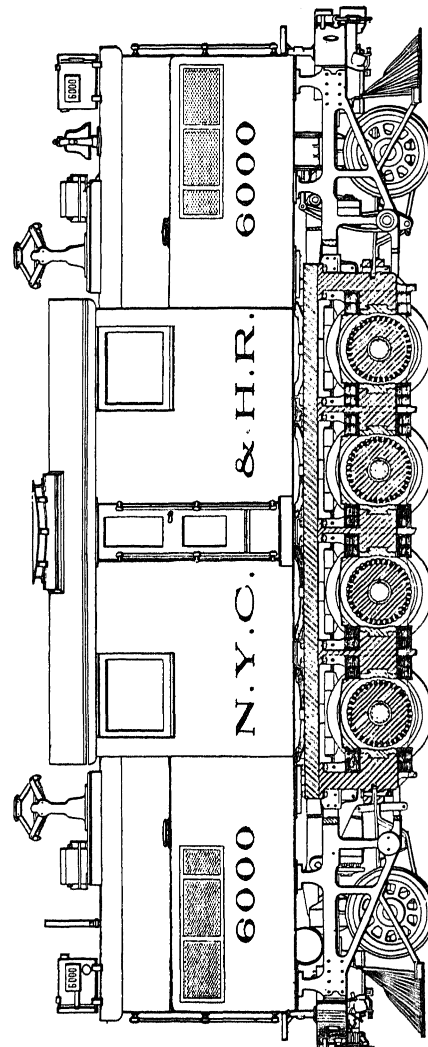


FIG. 5.—New York Central Locomotive in Part Section.

built up on a quill surrounding the axle, and driving it through springs. The whole motor is therefore elastically supported; but this construction has not been repeated. The gearless method of independent driving is particularly applicable to high-speed locomotives operating on continuous current systems, for which motors can readily be designed to make effective use of the materials of construction.

Collective Drives.—Locomotives, the axles of which are driven collectively by means of side coupling rods, exhibit greater diversity in arrangement than those having independently driven axles. They admit, however, of sub-division into two main groups; namely, those in which the power is

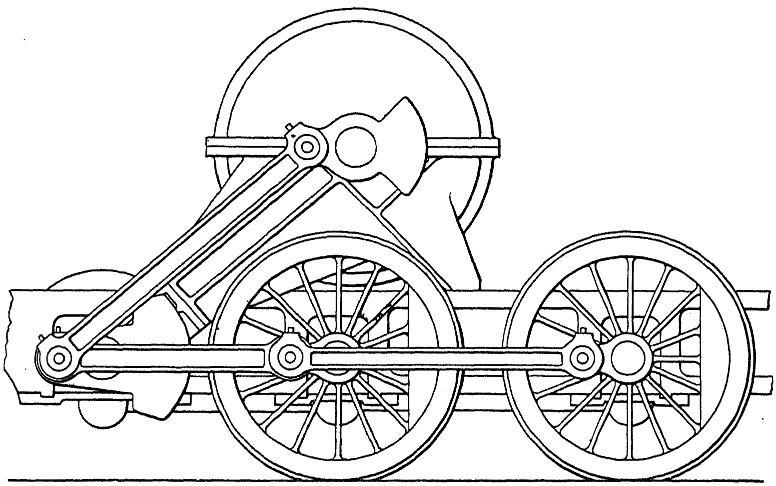


FIG. 6.—Side-Rod and Jack Shaft Drive.

transmitted, and the relative motion of the parts determined, by the aid of suitable auxiliary axles or jack shafts, carried in bearings in the main frame about on the level of the wheel axles, fig. 6, and those in which auxiliary axles are absent and the transmission to the wheels is effected directly by the aid of triangular side-members, usually called "Scotch Yokes" in this country,* fig. 7. In these locomotives, it may be noticed, the motors are carried on the main frames, thus being entirely spring-supported from the axles; and a primary requirement

* The Scotch yoke drive is generally known on the Continent as the Kando drive.

of the transmission system is that it should be arranged so as not to interfere appreciably with the freedom of the driving axles as regards displacement in a vertical direction. The side-rods, coupling the wheels with the jack shafts, are accordingly in all cases sensibly horizontal, and so jointed as to allow the necessary freedom. In the Scotch yoke drive, moreover, the crank-pin brasses of the central wheels are made free to work in a vertical slot in the yoke, with the same object. These features are shown clearly in figs. 6 and 7 and in the illustrations given in the appendix.

Where a group of axles are driven from a single motor, the transmission has hitherto usually been through quartered connecting-rods and jack shafts, and many locomotives have been constructed on these lines. The Pennsylvania and the

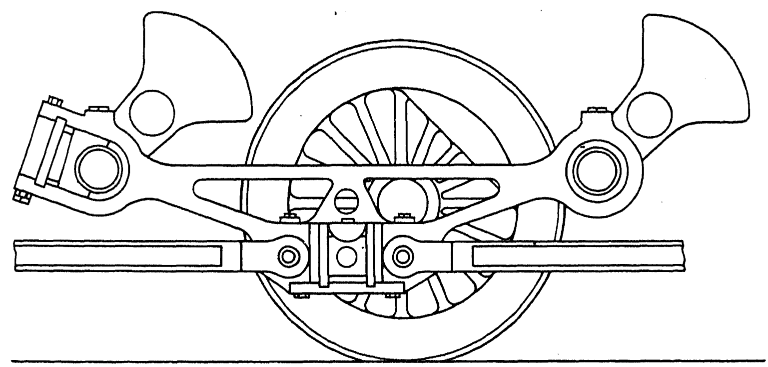


FIG. 7.—Scotch Yoke or Kando Drive.

Dessau-Bitterfield locomotives may be cited as typical examples. In the earlier Lötschberg locomotives, however, the transmission to the jack shaft takes place through gears ; and this practice appears to be reviving and spreading. Sometimes two motors are used in connection with a single jack shaft, the later Wiesental locomotives (Baden State Railways) furnishing an example in which connecting-rods are used, and the Norfolk and Western locomotives an example in which gears are used to transmit the power to the jack shaft. Sometimes, on the other hand, two motors are used to drive a group of wheels through two jack shafts, as in the earlier Wiesental locomotives of Siemens-Schuckert, and the Midi locomotives designed by the A.E.G. In the Scotch yoke drive the two motor-shafts are usually connected directly by means of the

yokes, which transmit the power to the wheels; this is the case in the Giovi and Simplon locomotives and many others; in the later Lötschberg locomotives, however, the motors are geared to yoked auxiliary shafts.

Distribution of Types.—Collective driving has found less favour among American engineers than on the European continent, for having taken the leading part in the development of the continuous current system of railway operation, they follow natural and sound lines of evolution in preferring methods which experience with this system has justified. Many continental engineers on the other hand have convinced themselves that main line electrification is essentially an alternating current problem, and have developed locomotives with particular reference to the limitations imposed by the corresponding system of operation. Possibly, however, the difference is in some measure an accident of development, for the New York, New Haven and Hartford locomotives, the first single-phase locomotives of consequence to be made in America, were under the necessity of running both on single-phase and continuous current lines. The single-phase motor, on account of limitations of design, has an armature wound for about 300 volts, and in order to use it on a 600-volt continuous current system at least four motors are required as a control unit; this favours independent driving of axles. The motors of the New Haven locomotives are, as above mentioned, for the most part carried on the locomotive frame directly above the axles, and are geared each to a quill surrounding the axle with due clearance and driving it through springs. A side rod locomotive driving through a quill, an inclined connecting-rod, and a jack shaft was, however, supplied to the New Haven railway, but has apparently proved unsuccessful, and this experience, together with the troubles that have arisen in connection with European side-rod locomotives, has probably tended to deflect American development from the type. It is worthy of mention, however, that the chief example among powerful locomotives of the use of side-rod drive with continuous current motors is American, being that of the Pennsylvania Railroad locomotives, of which thirty-three are employed for passenger traffic in the New York tunnel and terminal service.

Other Differences.—As a whole, the locomotive may be a

single unit, or may consist of two units arranged to be employed together. With respect to the wheel system, the whole weight may be carried on driving wheels or part of it may be on running wheels. The driving wheel base of any unit may itself be a rigid unit, or it may be divided into sections corresponding to as many trucks, and capable of swivelling with respect to each other. The trucks may be connected together through their centre pins and the underframe of the cab, as in the ordinary motor coach; or in the Metropolitan Railway locomotives; and in this case the draw and buffing gear is usually carried on the underframe; or they may be connected by means of a draw bar as in the North-Eastern Railway, Shildon & Newport locos, or by a mallet hinge, as in the Detroit River Tunnel locos, and in these cases the draw and buffing gear is carried on the truck frames.

Although by no means exhausting the possibility of variation in the electric locomotive, the above remarks show how great the variety is, and although some types are doubtless ephemeral, there are nevertheless a number of types which exhibit signs of permanence. With a few exceptions it may be said that locomotives having collectively driven axles pertain to alternating-current systems, that of these, those which use jack-shaft drive pertain to single-phase systems, and those which use the Scotch yoke drive pertain to polyphase systems. The ultimate reasons for the development and distribution of the types must be sought partly in the properties of the driving motors as affecting their essential design under the limitations imposed by locomotive service, partly in the class of service, partly in customary methods of handling traffic, partly in the influence of steam locomotive engineers, and partly in some cases in bureaucratic prejudices.

Classification by Wheel-arrangement.—It will perhaps assist intelligent discussion of the subject if a system of classification according to wheel-arrangement is here explained. The system usually employed in England and America has been adopted from steam-locomotive practice; but it does not furnish the same information as in the case of the steam locomotive, particularly when the axles are driven by independent motors. In the first place, the distinction between driving-wheels and guiding-wheels is frequently lost, since the same wheels may serve both purposes; in the second, the distinction

between a main locomotive-frame and a truck-frame is sometimes indefinite. It is convenient, however, to consider the locomotive or locomotive unit as possessed in general of three groups of wheels: namely, a front guiding group whose axles are capable of radiating with reference to the main locomotive frame, an intermediate group whose axles are transverse to the main locomotive frame, and a rear group whose axles are capable of radiating. The middle group is sometimes divided into sections corresponding to articulated sections of the locomotive frame. The wheel symbol consists of three or more figures, of which the first and last give respectively the number of wheels in the front and rear radiating groups, and the intermediate figures the numbers in the groups whose axles are transverse to the main frames. It should be noted that, whereas in the steam locomotive, the centre figure gives the number of driving wheels, in the electric locomotive the symbol is not to be interpreted in this manner. Thus the Metropolitan Railway locomotives,* like ordinary motor coaches, have the symbol 4–0–4, having leading and trailing bogies, but no wheels carried directly on the main frame. In the articulated truck locomotive, the truck frame forms the main frame of the locomotive; the Detroit River Tunnel locomotive,* for example, has symbol 0–4–4–0. One type presents some difficulty, and if it were not exceptional would compel reconsideration of the system of classification. This is the original New Haven passenger locomotive,* which was designed as a 4–0–4 type, having the draw gear on the underframe of the cab and transmitting the tractive effort through the centre pins. Afterwards an independent radial axle was added to each truck, and the locomotive is now usually classed as of the 2–4–4–2 type. Where the locomotive consists of two units, each of which forms a complete locomotive in itself, it will be convenient to connect the symbols of a + sign. Thus the complete B & O locomotives * of 1903 may be represented by the symbol 0–8–0 + 0–8–0, and the Norfolk and Western * by 2–4–4–2 + 2–4–4–2. The Pennsylvania locomotive,* on the other hand, is usually represented by the symbol 4–4–4–4 rather than by 4–4–0 + 0–4–4, since the units although capable of being separated are not themselves complete locomotives. There is indeed no distinction in wheel arrangement between the

* See Appendix.

4-4-4-4 Pennsylvania locomotive of fig. 204 and the 4-4-4-4 New York Central locomotives of fig. 201, for the fact that in the latter, the two units of the frame carry a single cab between them has no special significance from the present point of view.

Continental System of Classification.—Continental engineers use another system of classification, devised particularly to suit the types of locomotive with which they have usually to deal. The symbol consists of a figure to give the number of radiating leading axles, one or more letters to represent the number of coupled driving axles—A standing for single drivers, B for two-coupled, C for three-coupled, and so on—and a figure to give the number of radiating trailing axles. Thus the Pennsylvania locomotive is represented by the symbol 2-B-B-2 and the Norfolk & Western by 1-B-B-1 + 1-B-B-1. The system does not suit locomotives in which the guiding trucks carry driving motors as in the later New York Central locomotives. Neither system indeed necessarily gives information as to the function of the wheels; or takes account of all the variations to which the electric locomotive is susceptible.

GENERAL DESCRIPTION OF LOCOMOTIVES

The electric locomotive consists in general of one or more main frames, each supported from axle-boxes through springs. The axle-boxes slide vertically between machined guides in the main frames, but except for the small sliding clearances have usually no other freedom. Pivotally connected with the main truck may be one or two auxiliary trucks, spring-supported from secondary axle-boxes and arranged to carry part of the weight of the main truck. The auxiliary trucks are usually permitted a certain amount of freedom to move laterally against elastic centering forces, in order to allow the locomotive to pass round curves. Sometimes, however, as in the motor-coach and locomotives of similar construction, the whole weight is carried on a pair of swivelling trucks, and lateral displacement is then unnecessary. In other cases, the secondary axle-boxes are hung from the main frames, with a certain freedom for lateral and radial displacement against elastic constraint. The locomotive frame, or frames, carry a superstructure containing the driver's cab, with the controlling and other auxiliary gear, the superstructure being in some

cases rigidly connected with the main truck, and in others pivotally connected to two trucks by centre-plates or pins.

Degrees of Freedom.—The locomotive is substantially rigid with regard to the main wheel-base, for three independent modes of displacement, namely, for longitudinal, and transverse linear displacements, and for rotation about a vertical axis. For three other modes of displacement, namely, for translational displacement in a vertical direction, and for rotational displacement about longitudinal and transverse axes, the frame has a certain freedom against elastic constraint, and is accordingly subject to three kinds of bodily oscillation, namely, tossing, rolling and pitching. To put the matter more concisely, the locomotive structure is rigid with its wheel-base for displacements parallel to the plane of the wheel-base, but is carried flexibly as regards other displacements. This is also true of the auxiliary trucks, when considered in relation to their own wheel-bases. In relation to the main wheel-base, however, the auxiliary truck frames are capable of rotation about a vertical axis, and usually of independent lateral displacement against the opposition of elastic or gravitational forces.

Main Frame.—It is necessary to make the main frame of the locomotive of great strength and rigidity in order that it may retain its truth under the stresses which service imposes. Forms of construction which may conceal internal stresses should be avoided, for it is difficult in such to prevent warping under the severe percussion of service; and for this reason frames built up of rolled parts have generally been found to give the most permanent satisfaction. The same is true to a somewhat smaller extent of the frames of auxiliary trucks, particularly when these are used as motor trucks. Accurate workmanship is very necessary, particularly as regards the location of axle-box guides, for by parallelism of axles only is smooth running attained, and the maintenance costs, both for locomotive and track, reduced to a minimum.

Superstructure.—The superstructure of the locomotive consists of a stiff underframe, or platform, largely built up of channel sections, surmounted by a framework of tees and angles covered in for the most part with steel plates. The

chamber so formed contains contactors, resistances, and other controlling apparatus, besides providing accommodation for the driver. In some locomotives, as in figs. 190 and 193, the whole cab consists of a single compartment in which the driver's gear is located at the two ends. In others, as in figs. 189 and 192, the driver occupies a central compartment, which looks out over lower end compartments containing the auxiliary gear. In fig. 198 there are two drivers' compartments, separated by a central compartment containing an oil-fired boiler for providing steam for heating the train. In some locomotives, as already explained, the underframe bears the drafting stresses and is designed accordingly, the structure being carried on centre plates and side bearing blocks attached to rigid top bolsters, and in such the tractive effort of the wheels is transmitted to the underframe through the centre plates and king pins. In other locomotives the drafting stresses are taken through the truck frames and through a draw-bar or hinge pin, the underframe being then designed simply for the support and carriage of the cab and its contents; and in such cases the superstructure is located with reference to two centre plates, one of which is allowed a certain amount of freedom in a longitudinal direction. In still other locomotives the superstructure is built on the truck frame, its underframe being only designed for the support of the floor: the B. and O. locomotive * of 1903 is an example of this.

Bogie Trucks.—The four-wheeled bogies commonly used for carrying passenger stock, and almost exclusively for motor coaches and locomotives having similar drive, are either unequalized or only partially equalized. In British practice the bogies are usually unequalized, the frames being supported directly from axle-box springs, the flexibility of which results in approximately uniform distribution of the centrally-carried load, having regard to the accuracy of workmanship and good roadbed which characterizes British railway practice (see fig. 14). In American practice it is more usual to adopt some form of partial equalization. In the M.C.B. type of truck, for instance (fig. 15), the frame is supported by nests of springs from a pair of longitudinal equalizing beams, the ends of which rest on the axle boxes. It is clear that if the springs were

* See Appendix.

located at the centre of the beams the longitudinal equalization would be perfect; but the frame would be unstable, tilting under the stresses of braking or acceleration; whilst if the springs were over the axle-boxes there would be no equalization. The springs are located near to, and between, the axle-boxes, an arrangement which gives the necessary stability and provides also a certain measure of equalization. In the guiding trucks of the New York Central locomotives of fig. 201, longitudinal equalization is provided, stability being secured by giving the locomotive frame a long bearing base of support from truck; thus any tendency of the frame to tilt is accompanied by a redistribution of weight opposing the tilting.

The motor bogie does not differ in essentials from the ordinary trailing bogie; but the presence of the motors, and the stresses which they impose, necessitate extensive modification in the arrangement and strength of the parts. The use of longitudinal and cross bracing being rendered impracticable by the presence of the motors, the transoms are greatly increased in size; and in fact the whole frame is made heavier and is very stiffly gussetted. Central brake rods, and brake beams between the axles, being inadmissible for the same reason, and outside-hung brakes exercising a considerable tilting action on the frame, it is usual to apply the braking forces by means of side rods pulling in the plane of the brake shoes and actuated in unison from a radius beam carried in guides attached to the inner headstock. The brake-blocks are preferably applied to both sides of each wheel. In the built-up bogie of fig. 14, the frames are composed of 12 inch by 4 inch by 0·6 inch channel for solebars and transoms, with $\frac{7}{8}$ inch plates for horns; the bolster rests on two sets of four helical springs and is built up, in box-girder form, of 8 inch by 3 inch channel and $\frac{1}{2}$ inch plates, and has a lateral movement of $1\frac{1}{2}$ inches each way. The laminated bearing springs support the frame through auxiliary helical and rubber springs at the ends.* The combination of the two kinds of spring in the truck is found to result in improved riding qualities and freedom from excessive oscillation; and with the same object the

* See "The Electrical and Mechanical Equipment of the All-Metal Cars of the Manchester-Bury Section, Lancashire and Yorkshire Railway," by George Hughes, M.Inst.C.E.—*Minutes of Proc. Inst. C.E.*, vol. 208, p. 201.

M.C.B. truck frame of fig. 15 is supported on compound helical equalizer springs, whilst the bolster is carried on triple elliptic laminated springs.

In locomotives of the swivelling truck type, designed for low speed work, the bolster of the truck is in many cases attached rigidly to its side frames, so that there is only one set of springs in series between the rails and locomotive body. For somewhat higher speeds, however, the bolster is floated on springs carried on a spring plank which is attached at its ends to the truck frame. In locomotives intended for still higher speeds, and invariably in coaching stock, the bolster is floated from the spring plank, and this is itself swung from the transoms by means of links directed outwards at their lower ends. The clearance provided allows a certain amount of lateral movement to the body, with gravity centering. Sometimes lateral motion springs are fitted to assist the centering and cushion the blows of the bolster on the side frames.

Motor Coaches.—In the motor-coach, the main power-controlling equipment, comprising contactors, reversers, air compressor, governor, air reservoirs, etc., is frequently carried below the underframe, where it does not occupy revenue-earning space. The master-controlling equipment only, including cut-off and certain safety devices, is located in the driver's cab. This arrangement is practicable only where the train runs in the open or in large tunnels, so that in case of need the apparatus can be got at from the road. In the London tube lines, the tunnel clearances are insufficient to permit of its use, and it becomes necessary to carry all auxiliary apparatus within the body of the coach. Even without this limitation, however, many operators prefer to carry the controlling equipment in a compartment of the coach body, where it is instantly handy for inspection or adjustment. Motor coaches are of two general kinds, those having two-motor equipments and those having four-motor equipments. Both are used in suburban service, the former naturally in the less exacting kinds.

Equalization.—A very important problem in the design of a locomotive is that of equalization, whereby the effects of imperfect construction or warping of side frames are reduced in so far as they result in inequality of loading of the several axle-journals. Without equalization, the locomotive is in

much the same indeterminate condition with regard to the weight on wheels, as is a table standing on four or more feet. On even track and with all parts true and properly adjusted, the weight carried by each wheel may be the same, but on uneven track or with distorted frame, the several weights may

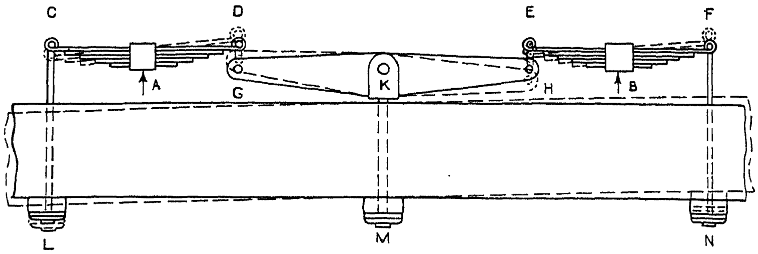


FIG. 8.—Equalizing Bar.

be far from equal, depending principally on the amount of strain in the corresponding riding springs. The purpose of equalization, as its name implies, is to cause the weight of the locomotive to be distributed equally between wheels of the same kind, and its effect is to reduce the number of points by

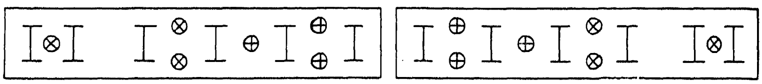


FIG. 9.—Equalization. Chicago Milwaukee and St. Paul Locomotive.

which the frame is supported. The designer of the locomotive generally aims at reducing this number to three, located at the angles of some triangle within which the vertical from the centre of gravity always falls. If he succeeds in attaining this object, not only will the support be stable, but the dis-

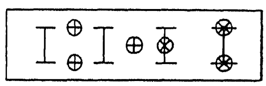


FIG. 10.—Equalization. Butte Anaconda and Pacific Locomotive.

tribution of weight on the several wheels will be determinate and will not depend to an appreciable extent on small imperfections, whether in the track or in the locomotive structure. Equalization is therefore of great importance in all locomotives, and particularly so in those designed for operation at high speeds, contributing largely to smooth riding over the irregularities of rail surface and alignment.

EQUALIZING BAR.—The system of equalization usually employed on locomotives is made up of elements of the type represented diagrammatically in fig. 8. Here A and B mark the positions of the axle-boxes; CD and EF are axle-box springs which support the side frame directly at L and N, and indirectly, through the equalizing beam GH, at M. It will readily be seen by regarding the arrangement as a system of levers that the weight carried is equally divided between the axle-boxes A and B, the springs taking the strains appropriate to this condition. Furthermore with the axle-boxes fixed, the frame can be tilted by a small amount about the centre K, as shown by dotted lines, without affecting the strain of the springs or disturbing the static equilibrium of the system. Thus the frame is in effect supported from the axle-box system at the point K only. If, as is usual, the wheels on the opposite ends of the axles are equalized in a similar manner, a second point of effective support for the frame is provided. Many examples of this form of equalization might be cited, as in the trucks of the Chicago Milwaukee and St. Paul locos of fig. 197, in which the third point of support for the first main frame is at the centre plate of the auxiliary truck, and for the second main frame at the joint between the frames.

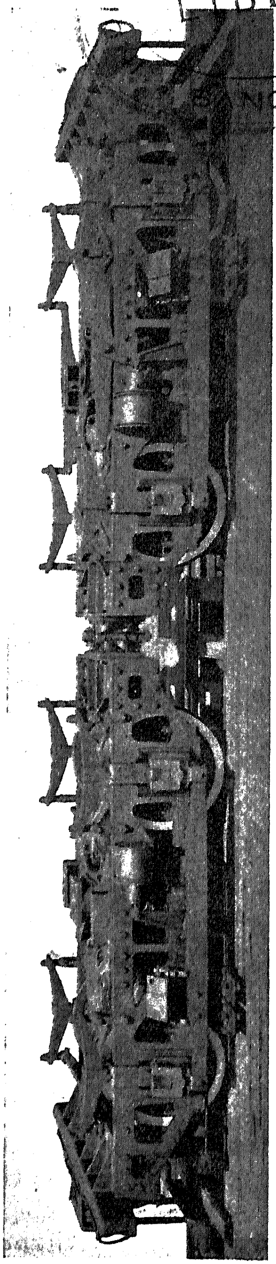


FIG. 11.—Trucks and Motors. Detroit River Tunnel Locomotive.

This joint is of the Mallet-hinge form and therefore capable

of taking vertical stresses, which are, however, in this case merely residual out-of-balance stresses. Fig. 9 shows diagrammatically in plan, the effective points of support of the frames in these locomotives. In some cases the two wheels on the same axle are equalized together, by means of a transverse equalizing bar, which results in an effective point of support on the longitudinal axis of the locomotive. Examples of this practice are to be found in one of the articulated trucks of the Butte Anaconda and Pacific locomotives, or of the Detroit River Tunnel locomotives, the second truck being longitudinally equalized in each case. Fig. 10 shows diagrammatically the arrangement of effective points of support in the former of these locomotives. Fig. 11 gives a view of the trucks of the latter.

As an example of a more complicated system of equalization, that presented by the first N.Y.C. locomotives (see fig. 5),

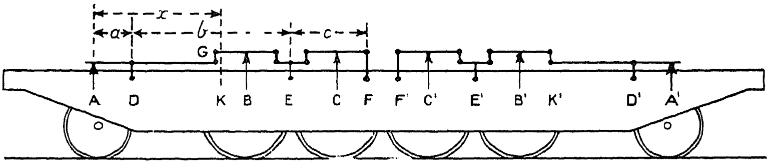


FIG. 12.

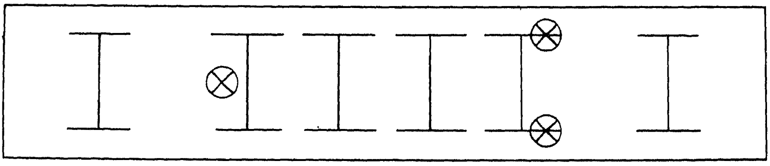


FIG. 13.
Equalization, New York Central Locomotive.

and shown diagrammatically in figs. 12 and 13, may be considered. Here ABC A′B′C′ mark the positions of axle boxes of which AA′ are those of pony wheels, which are equalized with the main driving wheels, the lengths of the lever arms being chosen to give suitable pressure on the two types of wheel. DEF D′E′F′ are points of support of the frame and of these D is really on the longitudinal axis of the locomotive, a transverse lever being interposed between the lever system shown and that on the opposite side. Let KK′ be the effective points of support for the two lever systems, for tilting displacements in the plane of the figure, and let a b c x be the

dimensions indicated in the figure. If the frame is tilted through a small angle, θ, about K, F is raised $(a + b + c - x)\,\theta$, E is raised $(a + b - x)\,\theta$, D is lowered $(x - a)\,\theta$: G is lowered by the amount that F is raised added to twice the amount that E is raised, or $[a + b + c - x + 2\,(a + b - x)]\,\theta$, whilst D is lowered $a\,/\,a + b - c$ of this amount, accordingly :

$$x - a = \frac{a}{a + b - c}\left\{3(a + b) + c - 3x\right\}$$

$$\text{or } x = \frac{4a(a + b)}{4a + b - c}$$

The effective points of support of the structure are accordingly at K′, at the corresponding point on the opposite side of the locomotive, and on the longitudinal axis opposite to K. Fig. 13 shows the location of these points in plan.

Influence of Springs.—The greater part of the mass of the locomotive, being spring-borne, does not contribute to the impulsive shocks arising from vertical and transverse irregularity in the track. The springs minimize the effect of vertical impulses directly, and where the weight is carried at some height above the plane of the spring seats, permit of a rocking motion about the longitudinal centre line in this plane, and thus in a measure relieve the transverse impulses of the great mass of the locomotive. The smaller the transverse spring base and the greater the height of the centre of gravity of the locomotive above it, the less will be the strain of the springs for a given transverse displacement of the wheels ; thus forms of locomotive having the axle journals outside the wheels and the motors placed low, require greater flexibility in the riding springs than is necessary in locomotives having the axle journals between the wheels and the motors in the superstructure.

RIDING OF LOCOMOTIVES

Pressure on Rails.—When running on curved track the weight of the locomotive is no longer distributed equally between the two rails. If w is the weight of the locomotive, considered as a rigid body, h the height of its centre of gravity above track level, and v its velocity, s the superelevation of the outer rail, b the gauge of the track, k the curvature of the track (i.e. the reciprocal of its radius of curvature), and g the

acceleration due to gravity, the normal pressure on the outer rail is given approximately by :—

$$P_o = w \left\{ \frac{1}{2} \left(1 + \frac{kv^2}{g} \cdot \frac{s}{b} \right) + \frac{h}{b} \left(\frac{kv^2}{g} - \frac{s}{b} \right) \right\} \quad . \quad (1)$$

that on the inner rail is given by :—

$$P_i = w \left\{ \frac{1}{2} \left(1 + \frac{kv^2}{g} \cdot \frac{s}{b} \right) - \frac{h}{b} \left(\frac{kv^2}{g} - \frac{s}{b} \right) \right\} \quad . \quad (2)$$

The transverse force required to cause the locomotive to travel in the curve is given by :—

$$P_r = w \left\{ \frac{kv^2}{g} - \frac{s}{b} \right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This, which, it will be noted, is independent of h, is supplied by pressure on the flanges of the wheels, and acts on the locomotive structure either directly through its axle-box guides, or indirectly through the centering springs of a guiding truck. If the speed and conditions are such that P_i (equation 2) becomes negative the locomotive commences to overturn, and it will be noted that for given track conditions the tendency to overturn is greater not only with higher speed but with greater height of centre of gravity. The removal of the restriction which treats the locomotive as a rigid body makes the tendency to overturn somewhat greater than is indicated above.

Effect on Riding Qualities of the Height of Centre of Gravity.—There is a general impression, derived from experience with steam locomotives, that for easy riding, the centre of gravity of the locomotive should be carried high. In dealing, however, with locomotives in which the natural distribution of masses is so different as in the steam and electric locomotive, it behoves the engineer to examine the basis of the impression, particularly as experience with electric locomotives does not appear to lend support to the popular notion. A defect of alignment in the track, peculiarity of the locomotive or other circumstances which results in a wheel-flange striking the rail, may be regarded as giving a blow to the wheel which impresses on it a certain transverse velocity, v, depending on the nature of the irregularity and on the velocity of running. The wheels and axle affected, with such other parts as move rigidly with them, take the velocity v and impress a certain blow on the spring-borne structure, through the horn-blocks: it is the

effect of this blow that determines the riding quality of the locomotive in so far as it depends on the distribution of masses. If h is the height of the centre of gravity of the spring-borne structure above the axles, M the mass of this portion of the locomotive and k its radius of gyration about a longitudinal axis through the centre of gravity, if further u is the transverse velocity of the centre of gravity and ω the angular velocity about the above axis, consequent on the blow on the wheel, the velocity v is given by:

$$v = u + h\omega$$

The blow on the horn block is:

$$\mathrm{M}u = \mathrm{M}k^2\omega/h$$

$$\therefore \frac{u}{k^2} = \frac{\omega}{h} = \frac{v}{k^2 + h^2}$$

thus:

$$u = k^2v/(k^2 + h^2) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the energy of the impulse is:

$$\tfrac{1}{2}(\mathrm{M}u^2 + \mathrm{M}k^2\omega^2) = \tfrac{1}{2}\mathrm{M}v^2k^2/(k^2 + h^2) \quad . \quad . \quad (4\mathrm{A})$$

The impulse is accordingly the smaller, and the riding quality the better, as h/k is the larger. It is the height of the centre of gravity of the spring-borne structure, in its relation to the radius of gyration of this structure about the longitudinal axis, that it is desirable to make as large as is practicable. In particular, it may be remarked that the carriage of the motors on the axles is not incompatible with easy riding; nor is their carriage by the superstructure of necessary advantage from the present point of view. It may also be remarked incidentally that it is generally desirable to distribute the apparatus carried in the superstructure longitudinally rather than transversely, in order to keep down the value of k.

The original New York Central locomotives carried their centre of gravity about 44 inches above rail level; and that of the superstructure about 29 inches above the axles. The first New Haven passenger locomotives, in their original 4–0–4 form, carried theirs at a height of about 51 inches above rail level; and that of the superstructure approximately 24 inches above the axles. American steam locomotives sometimes carry their centre of gravity as high as 73 inches above the rails.* With regard to English rolling stock, for which it must be remembered the height of limiting structures is smaller

* Sprague, *Transactions A.I.E.E.*, vol. 26, p. 756.

than in America, the following figures for the height of the centre of gravity above rail level have been given by Mr. Aspinall * : for a 10-wheeled bogie passenger engine, 66 inches; for an 8-wheeled radial tank loco, 58 inches; for a Liverpool-Southport trailer car, 56 inches; motor car, 45 inches; and motor bogie, 22 inches.

Discussion of Running Qualities.—The first requisite for the satisfactory running of a locomotive is that the wheel arrangement should be geometrically and kinematically correct. By this is implied that, if the track were perfectly true to gauge, the locomotive would be able to pass over it without encroaching on the clearances, and with the wheels simply rolling; or if the clearances were annulled the locomotive should be able to run everywhere without binding, or tending to mount the rails. It is not practicable to satisfy this condition fully. For instance, it is necessary to have the wheels in pairs of the same size, rigidly connected through their axle, hence it is impossible to pass round a curve with pure rolling, as the outer rail is longer than the inner. It is practically necessary also that some at least of the axles should be parallel, whereas to satisfy the ideal conditions, the axles should everywhere radiate from the centre of curvature of the track. The nearer the axles are to being at right angles to the direction of motion, the less the tendency of the wheels to mount the rails; hence the rigid wheel base should be short compared with the radius of the curves in order that the axles may approximate to the condition. If more than two axles are within a rigid wheel base, it is obvious that the wheels cannot lie on a curve without encroaching on the clearances, and on this account intermediate axles are often allowed additional clearance or side play.

Radiation of Trucks.—Treating the set of wheels within a rigid wheel base as a unit, if two such units are included in one locomotive, it is necessary either that both should radiate with reference to the frame, as in ordinary bogie coaches and locomotives constructed on similar lines, or, if one is rigidly attached to the frame of the locomotive, that the other should be capable both of radiating and of moving transversely, as in many steam and electric locomotives having guiding wheels

* Presidential Address, I.M.E., 1909.

additional to the driving wheels. In the latter case, the transverse movement of the guiding wheels is opposed by centering springs or their equivalent, which by their reaction on the structure turn it in accordance with the guidance of the truck. Sometimes, as in the ordinary leading or trailing pony axle, the amount of transverse movement is geometrically connected with the amount of swivelling, being effected by a rotation about a pin at some distance from the pony axle. With this arrangement radiation is imperfect, unless additional end play is allowed in the axle. Where two trucks are connected by means of a hinge joint, considerations of geometry require somewhat greater flexibility than is provided by this type of joint; inasmuch as the longitudinal centre line of each truck should be in the direction of motion of the truck, and should touch the mean line of the track at the centre of its wheel base. This implies that the tangents from the hinge pin to the track line should be equal, which, whilst possible when the curvature is uniform, is not in general possible in transition from straight to curved track. With the curvature usual on railway tracks, however, the deviation from accuracy so introduced is not of importance; but it would be otherwise if two long bogie coaches had their underframes articulated by means of a simple hinge. In this case the four bogie centres could not generally be placed on a curve with the two underframes meeting at the hinge pin, unless the curve were one of uniform curvature, and considerable stresses might be set up in transition from straight to curved track, if the hinge connection were used between coaches, in place of the draw bar.

Forces at Wheel-treads.—It is accordingly impracticable to satisfy, under all circumstances, the geometrical and kinematical conditions necessary to secure that the locomotive may always make progress by simple rolling of the wheels, and with departure from correct conditions slipping and creeping occurs at the wheel treads, accompanied by frictional and elastic forces and flange pressures. At high speeds further large stresses appear between wheel and rails, due to certain inertia effects which will be discussed later. In addition, however, to these forces, which would arise equally with perfectly true and rigid trackwork as with such track as can practically be laid, others of an impulsive nature are up set,

due to irregularity of surface and alignment. These, which must be made the subject of separate study, involve generally and principally the portion of the mass which is not carried on springs, the wheels, the axles, the axle-boxes, the side-rods, etc. For steady motion on curves, which will be considered first, the locomotive may be assumed to move as a rigid body. Its whole mass accordingly comes into question, the clearances and the deviation of the springs being exhausted at the inception of the motion.

Motion in Curve.—A rigid body, moving with velocity v in a plane curve of curvature k, has an acceleration compounded of $\frac{dv}{dt}$ in the tangential direction and kv^2 in the normal direction. The angular velocity of the body is kv and its angular acceleration $k\frac{dv}{dt} + v^2\frac{dk}{dl}$, l being the distance measured along the curve from a fixed point. Thus if M is the mass of the body and I its moment of inertia about an axis through its centre gravity at right angles to the plane of the curve, the outside forces acting on the body have resultants $\mathrm{M}\frac{dv}{dt}$ and $\mathrm{M}kv^2$ along the tangent and inwardly drawn normal respectively, and moment $\mathrm{I}\left(k\frac{dv}{dt} + v^2\frac{dk}{dl}\right)$ about the above-mentioned axis through the centre of gravity.

In the case of a locomotive the external forces acting on it may for convenience be divided into two classes. The component of gravity due to gradient or to superelevation of the rails, the head resistance, the draw bar pull of the train, the centering forces of auxiliary trucks, and the like, comprise the first class, and these forces must be assumed known in the circumstances of any particular problem. The interactions between the several wheels and rails form the second class, and these will be made the subject of discussion. Distinguishing the known forces by dashed letters, and writing T for the tangential, N for the normal force, and G for the moment of the forces, the equations of motion may be written :

$$\mathrm{M}\frac{dv}{dt} = \mathrm{T} + \mathrm{T}' \qquad \ldots\ldots \qquad (5)$$

$$Mkv^2 = N + N' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$I\left(k\frac{dv}{dt} + v^2\frac{dk}{dl}\right) = G + G' \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In discussing the question of running on curves, the acceleration in the tangential direction has no particular interest and it will generally be sufficient to assume that the axles are running idly, so that $T = O$, not only for the whole locomotive, but for each pair of wheels. Moreover, if the track is laid with due regard to easy transition, the term involving $\frac{dk}{dl}$ is small, and for most purposes equation 7 may be written $G + G' = 0$. It will be convenient in general to divide the transverse force N' into two parts, namely, N_1 due to centering forces of auxiliary trucks and the like, and Mgs/b due to superelevation of the track, s being the value of the superelevation and b the track gauge, as in equation 3. If v_o is the speed for which the track is superelevated, or that which makes $P_r = 0$ in equation 3 :

$$kv_0{}^2 = \frac{s}{b}g$$

thus equation 6 can be written in the form

$$Mk(v^2 - v_0{}^2) = N + N_1 \quad . \quad . \quad . \quad . \quad . \quad (7\text{A})$$

Interaction between Wheel and Rail.—There are three general modes of progress of a wheel on a rail, which may be designated respectively, rolling, slipping and creeping. The first is the natural function of the wheel and requires no explanation. The second also is readily apprehended as a form in which there is relative motion between wheel and rail at their point of contact. The third, however, is somewhat more recondite and requires explanation. The interaction between wheel and rail is really a very complicated one—the material in the region of the area of contact being strongly stressed. When the wheel simply rolls on the rail, the state of stress is symmetrical about the axial plane bisecting the area of contact and the resultant interaction is a simple pressure. If, however, as the wheel rolls, a force is applied to it in any direction tangential to the area of contact, the material in the region of this area is deformed, extended elements of one member being

successively brought into contact with compressed elements of the other, with the visible result that the wheel, besides rolling, creeps slowly in the general direction of the force. Thus a driving wheel makes a somewhat greater number of revolutions in a given distance than an idly running wheel of the same size, and the difference, considered in relation to the tractive effort, measures the creepage. As the tangential force is increased, a value is ultimately reached at which the surfaces in contact can no longer grip with sufficient intensity to transmit the force, and slipping then commences, continuing as long as the force is maintained. Conversely when the motion and the geometrical connections are such as to compel creeping as an accompaniment to the rolling of the wheels, a tangential force is set up at the contact face. The value of this force depends upon the rate of creeping as compared with that of rolling, until a certain limiting rate is attained, at which slipping commences, and the force becomes sensibly independent of the rate. Under ideally uniform conditions it is possible to conceive an abrupt transition from creeping to slipping, but with the unevenness of actual surfaces and the vibration of running the boundary between the two states is naturally indistinct.

CREEPAGE.—It is convenient to define creepage as the ratio of the rate of displacement by creeping to the rate of displacement by rolling, and to adopt the word "gliding" to include either slipping or creeping. With the above definition, the creepage, within limits set by the yielding of the material, is clearly proportional to the elastic deformation in the region of the interface and therefore to the tangential force between wheel and rail, at any rate for displacements in principal directions. If accordingly α and α' are components of creepage in the longitudinal and transverse directions respectively, the corresponding components of tangential force on the wheel may be written $-f\alpha$ and $-f'\alpha'$. No data are available from which to determine values of f and f', and although conjecture may set limits, there is difficulty even in obtaining a plausible estimate. It may, however, be assumed as the result of experience that the stresses involved in creeping are not in general sufficiently great to cause continuous yielding in the materials of the rail or wheel-rims. This limitation appears at first sight to reduce the possibility of creeping to a very small range, for

from tests made on standard test pieces of the material, a permanent set is usually obtained at stresses of the order of 35,000 lbs. per sq. inch (2,500 kg. per sq. cm.), corresponding to a maximum recoverable strain of the order of 0·15 per cent. for each member, or a maximum creepage of 0·3 per cent. As is well known, however, the stresses in a well-ironed surface layer may greatly exceed those possible in a machined test piece, without fracture of the material; and a steel which would commence to yield at 35,000 lbs. per sq. inch in the mass, may be found to stand 150,000 lbs. per sq. inch without yielding if drawn into a wire of $\frac{1}{10}$ inch diameter, whilst the amount of flexure that such a wire will stand without permanent set indicates an even greater limit of allowable stress in the surface layers. It would appear accordingly that there is nothing in the nature of steel to prevent creepage, even up to 1 per cent., when the surfaces have been well-ironed by use, and this is the order of most of the gliding motions incidental to progress along curves. Whilst therefore it may be necessary, in the interests of definiteness and in order to obtain limiting forces, to assume that the gliding motions on curves are of the nature of slipping, it appears not unlikely that creeping is the more normal condition and actual slipping somewhat exceptional.

Motion of Locomotive on Curved Track.—When a locomotive is moving steadily and smoothly in a curve it may be treated as revolving about an axis at right angles to the plane of the curve and passing through the centre of curvature, this being the instantaneous axis of rotation for the locomotive. The rigid wheel base will not set itself symmetrically on the curve, but will turn a little outwards at the forward end, where the wheel flange is guided by the rail (fig. 16). A perpendicular from the centre of curvature, O, to the line of the rigid wheel-base will accordingly not strike this at its centre, but at some point to the rear thereof; and the same is true of each independently swivelling truck. In general, all wheels both roll and glide, but there is a point, H, on the perpendicular to the wheel-base, such that if a wheel of equal diameter to the actual wheels were located there, having its axis in the same direction as the others, it would simply roll at the same revolution rate as the actual wheels. The instantaneous centre of rotation for the locomotive being

at O, the velocity of H is at right angles to OH and may be represented by the line OH; to this scale the velocity of the wheel A is represented by the line OA, and is at right angles to this line. It is therefore compounded of OH at right angles to the direction OH, and HA at right angles to the

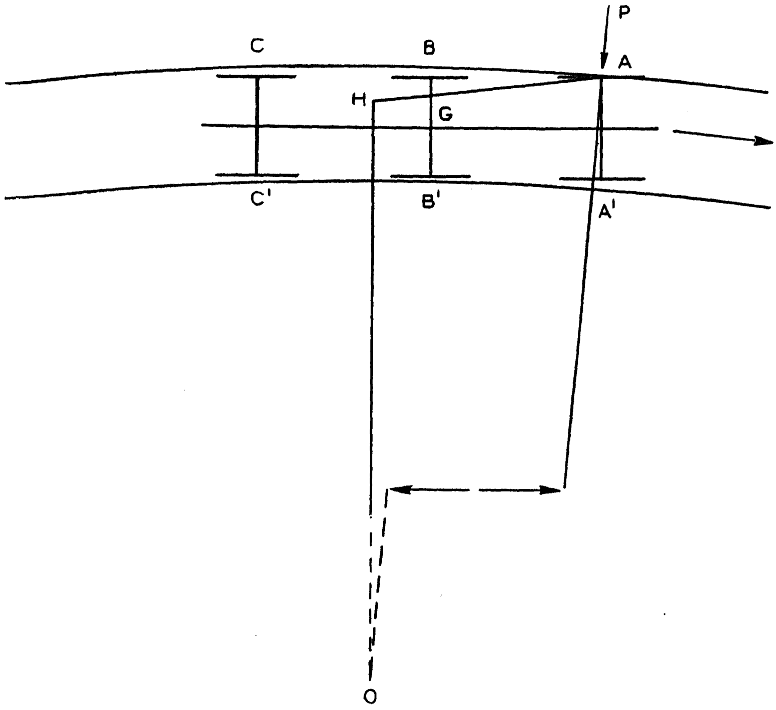


FIG. 16.—Rigid Wheel-Base on Curve.

direction HA. The first of these components represents the pure rolling, and the second, the gliding. The gliding, expressed as a fraction of the rolling, is accordingly HA/OH or $k \times$ HA, where k is the curvature of the track.

EQUATIONS FOR RIGID WHEEL-BASE.—Let the origin of coordinates be taken at the centre of the wheel-base and let axes be taken in the longitudinal and transverse directions, which directions are also, for practical purposes, tangential and normal respectively to the curve described by the centre of gravity of the locomotive; let the coordinates of the wheels A, A′, B, B′, etc., be: $(\alpha\ \eta)$, $(\alpha\ -\eta)$, $(\beta\ \eta)$, $(\beta\ -\eta)$, etc.;

let $(x\ y)$ be the coordinates of the pure rolling point H, and $(x'\ y')$ those of the centre of gravity G′; let P be the transverse force on the flange at A. Assuming, for the present, that the wheels creep, the longitudinal forces on the wheel treads at A, B, etc., all have values $-fk(\eta - y)$, whilst those on the wheel-treads at A′ B′, etc., have values $fk(\eta + y)$; hence if n is the number of axles, the tangential force (see equation 5) is approximately:

$$T = 2nfky \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The transverse forces on the wheel-treads at A and A′ are each

$f'k(\alpha - x)$, those at B and B′, $f'k(\beta - x)$, etc.

Hence the normal force (directed inwards), including the flange pressure P at A, is approximately (see equations 6 and 7A):

$$N = 2nf'kx + P \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The moment of the wheel forces about the centre of gravity is (see equation 7):

$$G = +2nfk\eta^2 + 2f'k(\alpha^2 + \beta^2 + \ldots) - P\alpha + Ty' + Nx' \quad (10)$$

Equations 8, 9 and 10, taken in connexion with equations 5, 6 and 7, determine the three unknown quantities x, y and P.

Four-wheeled Bogies.—In the case of a four-wheeled bogie truck, loaded over its centre, and exerting no tractive effort; if $2c$ is the length and $2b$ the width of the wheel base, $\alpha = -\beta = c$, $\eta = b$, $x' = y' = 0$.

Hence, taking account of the remarks that follow equation 7:—

$$y = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$P = \frac{4k}{c}(fb^2 + f'c^2) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$Mk(v^2 - v_0^2) = 4f'kx + P$$

$$\text{or } M(v^2 - v_0^2) = \frac{4}{c}(fb' + f'c^2 + f'cx) \quad . \quad . \quad (13)$$

Thus at the speed for which the track is superelevated:

$$x = -c - \frac{f}{f'}\frac{b^2}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The rolling point H is accordingly in this case behind the

rear axle of the truck, and recedes further as the speed falls below that for which the track is superelevated. Equation 12 shows that, under the conditions assumed, the flange pressure varies directly as the curvature of the track, but is independent of speed. With variation of wheel-base, it passes through a minimum when $fb^2 = f'c^2$. Equation 13 shows that the position of the rolling centre H is independent of the curvature of the track.

Example. For the sake of obtaining a clearer impression, assume $f = f' = 300{,}000$ lbs.; then on a curve of 1,000-feet radius, if $2c = 6$ feet, $2b = 5$ feet, equations 12 and 14 give:

$$P = 6{,}100 \text{ lbs.}$$
$$x = -3 - 21/12 = -5 \text{ feet } 1 \text{ inch.}$$

ARTICULATED LOCOMOTIVES.—The articulated locomotive having its trucks connected by mallet hinges, presents a special problem; for the trucks are able to influence each other; and, when they have been forced to set themselves at an angle appropriate to the curve, no flange pressure is necessary, in many cases, to maintain the motion, the frictional forces at the wheels being balanced by the reaction at the hinges. Thus in the case of a locomotive having two similar articulated four-wheeled trucks (fig. 17) running steadily at the speed for which the track is superelevated, the point H, the foot

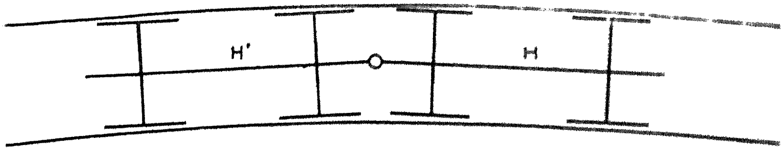


FIG. 17.—Articulated Wheel-Base on Curve

of the perpendicular from the centre of curvature to the wheel-base, is ahead of the centre of the wheel-base for the first truck and behind it for the second, the actual distance being the same in each case and being obtained by taking moments for either truck about the hinge. If $2d$ is the distance between the truck centres, the equation for x for the first truck is obtained by putting $P = 0$, $x' = -d$, $y' = 0$, in equation 10, giving:

$$x = \frac{fb^2 + f'c^2}{f'd} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The angle between the trucks while so running is, in circular measure:

$$\theta = 2k(d + x)$$

$$= 2\frac{k}{d}\left(\frac{f}{f'}.b^2 + c^2 + d^2\right) \quad . \quad . \quad . \quad . \quad (16)$$

Equation 16 suggests a possible method of determining the ratio f/f' under working conditions. The absolute value of f might be obtained as indicated above by observation of creepage on straight track when the wheels are exerting tractive effort; this would probably be best effected by choosing a long uniform gradient and hauling a load against it, including in the load a similar locomotive running light, with which the revolutions of the wheels could be exactly compared, tests being made with the locomotives driven alternately.

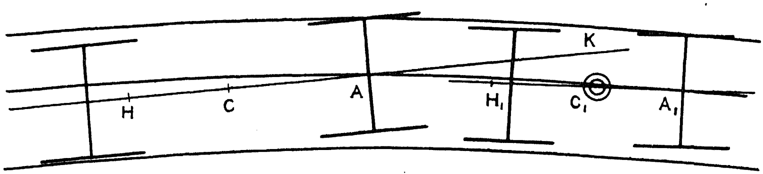


FIG. 18.—Rigid Wheel-Base with Guiding Bogie on Curve.

GUIDING BOGIE.—Reverting to the general case, if in fig. 18 HCA is the longitudinal centre line of the wheel-base, C being the centre and H the foot of the perpendicular from the centre of curvature of the track, and if the curve of the track be drawn through A, the distance of any point K on the centre line from this curve is:

$$\delta = \frac{k}{2}\,\mathrm{AK}(\mathrm{HA} + \mathrm{HK})$$

or, writing CK $= h$, and with CA $= c$, CH $= x$, as before:—

$$\delta = \frac{k}{2}(h - c)(h + c - 2x). \quad . \quad . \quad . \quad (17)$$

If K is taken at C:

$$\delta = -\frac{k}{2}c(c - 2x). \quad . \quad . \quad . \quad . \quad (17\text{A})$$

If now a locomotive is fitted with a guiding bogie having its centre at distance h from the centre of the main wheel-base,

the transverse displacement of the bogie centre when on the curve is:

$$\delta - \delta_1 = \frac{k}{2}\{(h - c)(h + c - 2x) + c_1(c_1 - 2x_1)\} \quad . \quad . \quad (18)$$

where the suffix 1 refers to the bogie. If the centering force is $Q(\delta - \delta_1)$, its effect on the main truck is to add components to N' and G' (equations 6 and 7) of values:

$$N' = Q(\delta - \delta_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$G' = - Q(\delta - \delta_1)(h - x') \quad . \quad . \quad . \quad . \quad (20)$$

whilst for the bogie truck the corresponding components are:

$$N'_1 = - Q(\delta - \delta_1) \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$G'_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Thus from linear equations of the forms of 6 and 7 the four quantities P, P_1, x, x_1, are determined. It may be noted that the above calculation applies whether the bogie leads or trails, h being negative in the latter case.

As an example of the use of these equations consider a locomotive having four main wheels in a 10 feet wheel-base and with a leading bogie of 6 feet wheel-base. Let the distance h between truck centres be 13 feet; let $Q = 18{,}000$ lbs. per foot displacement; let $k = \cdot 001$ and $f = f' = 300{,}000$ lbs., as before, then:

$$Q(\delta - \delta_1) = 1{,}377 - 144x - 54x_1.$$

At the speed for which the track is superelevated, from equations 7A, 9 and 19:

$$1{,}200x + P + 1{,}377 - 144x - 54x_1 = 0$$

from equations 7A, 9 and 21:

$$1{,}200x_1 + P_1 - 1{,}377 + 144x + 54x_1 = 0$$

from equations 7, 10 and 20:

$$37{,}500 - 5P - 13[1{,}377 - 144x - 54x_1] = 0$$

from equations 7, 10 and 22:

$$18{,}300 - 3P_1 = 0$$

Whence:

$$P = 2{,}130 \text{ lbs.} \qquad P_1 = 6{,}100 \text{ lbs.}$$

$$x = - 3\cdot 50 \text{ feet.} \qquad x_1 = - 3\cdot 36 \text{ feet.}$$

Slipping Wheels.—If, for the sake of obtaining limiting figures, it be assumed that the wheels slip on the rails, the

calculation becomes somewhat more complicated. If w is the weight on a wheel and μ the coefficient of friction the equation corresponding to equation 8 becomes:

$$T = -\Sigma\left\{\frac{\mu w(\eta - y)}{\sqrt{[(a-x)^2+(\eta-y)^2]}} - \frac{\mu w(\eta + y)}{\sqrt{[(a-x)^2+(\eta+y)^2]}}\right\} \quad . \quad . \quad (23)$$

the equation corresponding to equation 9 becomes:

$$N = -\Sigma\left\{\frac{\mu w(a - x)}{\sqrt{[(a-x)^2+(\eta-y)^2]}} + \frac{\mu w(a - x)}{\sqrt{[(a-x)^2+(\eta+y)^2]}}\right\} + P \quad (24)$$

whilst the equation corresponding to equation 10 becomes:

$$G = \Sigma\{\mu w\sqrt{[(a-x)^2+(\eta-y)^2]} + \mu w\sqrt{[(a-x)^2+(\eta+y)^2]}\}$$
$$- P(a-x) + T(y'-y) + N(x'-x) \quad . \quad . \quad . \quad . \quad (25)$$

If the wheels exert no tractive effort, $y = 0$ as before, or H is on the longitudinal centre line; in this case the above equations are somewhat simplified.

Considering as an example the four-wheeled bogie, loaded over its centre and running without tractive effort, at the speed for which the track is superelevated, if, as before, $2c$ is the length and $2b$ the width of the wheel-base: $a = -\beta = c$, $\eta = b$, $x' = y' = 0$, $y = 0$,
whilst P and x are given by:

$$-\frac{2\mu w(c-x)}{\sqrt{[(c-x)^2+b^2]}} + \frac{2\mu w(c+x)}{\sqrt{[(c+x)^2+b^2]}} + P = 0\ . \quad (26)$$

$$2\mu w\sqrt{[(c-x)^2+b^2]} + 2\mu w\sqrt{[(c+x)^2+b^2]} - P(c-x) = 0\ . \quad (27)$$

From these equations, P is readily eliminated, yielding the quartic:

$$b^2\sqrt{[(c+x)^2+b^2]} + [b^2+2c^2+2cx]\sqrt{[(c-x)^2+b^2]} = 0 \quad (28)$$

Thus if $2c = 6'$, $2b = 5'$, $\mu = 0{\cdot}3$, $w = 12{,}000$ lbs., equations 26 and 27 give:

$x = -4{\cdot}425$ feet. $\qquad$ $P = 2{\cdot}89\mu w$
$\qquad\qquad\qquad\qquad\qquad = 10{,}400$ lbs.

Since the assumption of slipping at all wheels is not justified, for it is probable in any case that such slipping is confined to the pair of fore wheels, the use of correct dynamical reasoning furnishes no guarantee of results in accordance with physical facts. As it hardly is worth while to discuss the case of some wheels creeping and others slipping, it appears expedient, in

the interests of facility of calculation, to deduce the value of x on the assumption of creeping wheels and use it in the equation of moment (equation 25) to determine a limiting value for P. Thus in the case of the four-wheeled bogie considered above, from equations 14 and 27, assuming $f = f'$:

$$P\left(c + \frac{b^2}{2c}\right) = 2\mu w \sqrt{(b^2 + c^2)}\left\{\sqrt{\left(1 + \frac{b^2}{4c^2}\right)} + \frac{b}{2c}\right\} \quad . \quad (29)$$

For the numerical example discussed, this equation yields $P = 2{\cdot}9\mu w$, which is practically the same value as is given by the more complicated method.

Natural Alignment of Locomotive.—An elementary condition necessary to secure satisfactory running of a locomotive on straight track, particularly at high speed, is that the several groups of axles may tend to set themselves parallel to one another. Consider a four-wheeled vehicle such as represented in fig. 19, one pair of wheels having their axle transverse to the main frame, and the other pair being capable of swivelling about a point O on the longitudinal centre line, COA. Let OB be drawn perpendicular to the

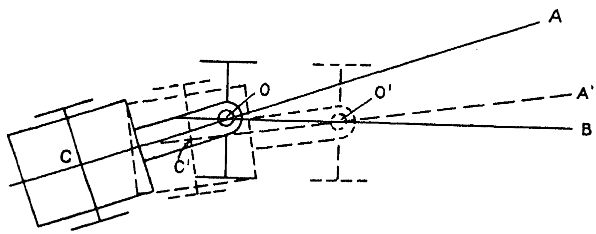


FIG. 19.—Diagram Illustrating Tendency to Align or Deviate.

swivelling axle, so that the angle AOB represents the angle between the two axles. If the vehicle be given a push and then allowed to run freely on a flat surface, since the main frame is unable to exert a torque on the swivelling truck, the line OB retains its initial direction. Let the broken lines represent a subsequent position of the vehicle, C′O′A′ being the new position of the longitudinal centre line. It is clear that the angle A′O′B is smaller than the angle AOB. Thus in displacement from the full-line position to the broken-line position of the vehicle, the axles approach parallelism. The motion however is obviously reversible, and if the vehicle

is propelled in the opposite direction, the angle between the axles continually increases. In a locomotive, whilst the divergence of the axles is limited by flange clearance, the same tendency is operative. The locomotive carried on a main group of wheels and a leading bogie, runs on straight track without excessive guiding stresses; but the locomotive fitted with a trailing bogie is apt to exhibit rearing proclivities and to give rise to large stresses tending to spread the rails. Other wheel arrangements show other tendencies, but before discussing these more particularly it is advisable to obtain certain general equations of the forces which act through the wheel-treads.

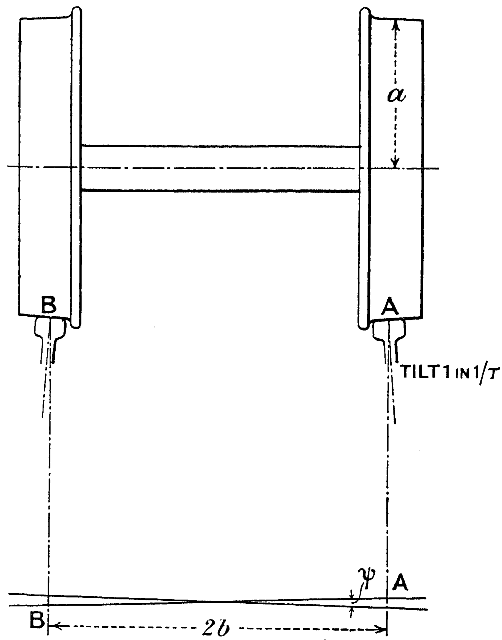


FIG. 20.—Diagram, Wheels and Axle.

FREE RUNNING OF PAIR OF WHEELS AND AXLE.*—The fact that the wheels are coned slightly, and the rails canted to correspond, causes a locomotive to progress with somewhat of a sinuous motion; and although the amplitude of this motion is restricted by the amount of flange-clearance its stability or instability has considerable influence on the performance of the locomotive, particularly with reference to the nosing and rearing tendencies, and the destructive track stresses resulting therefrom. Consider a pair of wheels and axle in simple rolling motion (fig. 20): let τ in 1 be the coning of the wheels, or the cant of the rails; let (x, y) be the coordinates of the centre of the axle, the axis of abscissae being taken along the centre line of the track, so that the effective

* See *Minutes of Proc. Inst. C.E.*, vol. 201, p. 239.

radius of wheel A is $a + \tau y$ and of wheel B, $a - \tau y$; let ψ be the angle between the axle and the transverse direction, and let $2b$ be the gauge of the track. The coordinates of the wheels are:

$$\begin{aligned} &\text{of A,} \quad \{x - b\psi, \qquad y + b\} \\ &\text{of B,} \quad \{x + b\psi, \qquad y - b\} \end{aligned} \quad . \quad . \quad . \quad . \quad (30)$$

Thus the component displacements are therefore:

$$\begin{aligned} &\text{of A,} \quad \{dx - bd\psi, \qquad dy\} \\ &\text{of B,} \quad \{dx + bd\psi, \qquad dy\} \end{aligned} \quad . \quad . \quad . \quad . \quad (31)$$

But the component displacements due to rolling are

$$\begin{aligned} &\text{of A,} \quad \{(a + \tau y)d\theta, \ a\psi d\theta\} \\ &\text{of B,} \quad \{(a - \tau y)d\theta, \ a\psi d\theta\} \end{aligned} \quad . \quad . \quad . \quad . \quad (32)$$

Hence:

$$\begin{aligned} dx - bd\psi &= (a + \tau y)d\theta \\ dx + bd\psi &= (a - \tau y)d\theta \\ dy &= - a\psi d\theta \end{aligned}$$

or:

$$\begin{aligned} dx &= ad\theta \\ dy &= a\psi d\theta \\ bd\psi &= - \tau y d\theta \end{aligned}$$

Eliminating y and θ:

$$ab\frac{d^2\psi}{dx^2} + \tau\psi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The solution of this is of the form:

$$\psi = \text{A} \sin\left[\sqrt{\left(\frac{\tau}{ab}\right)}x + a\right] \quad . \quad . \quad . \quad . \quad (34)$$

The motion therefore repeats in distance D given by:

$$\text{D} = 2\pi\sqrt{\frac{ab}{\tau}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Thus if the cant of the rails is 1 in 20, the diameter ($2a$) of the wheels 4 feet, and the track of standard gauge, the distance covered per complete oscillation is approximately 60 feet.

FORCES AT WHEEL-TREADS.*—Where axles are carried in a locomotive frame so that they have no other horizontal

* See *Minutes of Proc. Inst. C.E.*, vol. 201, p. 248.

motion relatively to it than their natural one of rotation, a simple rolling motion of the wheels is not in general possible, the constraint imposed by the frame causing the wheels to

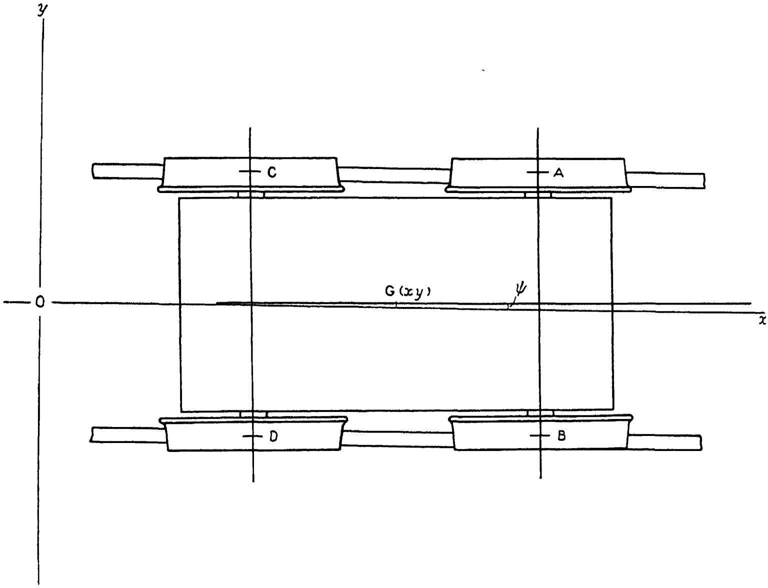


FIG. 21.—Truck on Rigid Wheel-Base.

creep as well as roll (see fig. 21). The components of creepage displacement are (from 31 and 32 above):

$$\begin{array}{ll} \text{of A,} & \{dx - ad\theta - bd\psi - \tau y d\theta, \quad dy - a\psi d\theta\} \\ \text{of B,} & \{dx - ad\theta + bd\psi + \tau y d\theta, \quad dy - a\psi d\theta\} \end{array}. \tag{36}$$

Hence the components of creepage are:

$$\begin{array}{ll} \text{of A,} & \left\{\dfrac{dx}{ad\theta} - 1 - b\dfrac{d\psi}{ad\theta} - \dfrac{\tau}{a}y, \qquad \dfrac{dy}{ad\theta} - \psi\right\} \\ \text{of B,} & \left\{\dfrac{dx}{ad\theta} - 1 + b\dfrac{d\psi}{ad\theta} + \dfrac{\tau}{a}y, \qquad \dfrac{dy}{ad\theta} - \psi\right\} \end{array}. \tag{37}$$

The component forces at the wheels, assumed proportional to the component creepages, are:

$$\begin{array}{ll} \text{at A,} & \left\{-f\left[\dfrac{dx}{ad\theta} - 1 - \dfrac{b}{a}\dfrac{d\psi}{d\theta} - \tau\dfrac{y}{a}\right] \quad -f'\left[\dfrac{dy}{ad\theta} - \psi\right]\right\} \\ \text{at B,} & \left\{-f\left[\dfrac{dx}{ad\theta} - 1 + \dfrac{b}{a}\dfrac{d\psi}{d\theta} + \tau\dfrac{y}{a}\right] \quad -f'\left[\dfrac{dy}{ad\theta} - \psi\right]\right\} \end{array} \tag{38}$$

This system of forces at the wheel-treads is equivalent to :—

1. A longitudinal force acting at the centre of the axle, of value :

$$X_1 = -2f\left(\frac{dx}{ad\theta} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (39)$$

2. A transverse force acting at the centre of the axle, of value :

$$Y_1 = -2f'\left(\frac{dy}{ad\theta} - \psi\right) \quad . \quad . \quad . \quad . \quad . \quad (40)$$

3. A couple, having a vertical axis, and moment of value :

$$G_1 = -2f\left(\frac{b^2}{a}\frac{d\psi}{d\theta} + \tau\frac{by}{a}\right) \quad . \quad . \quad . \quad . \quad . \quad (41)$$

If the system of forces, instead of being referred to the centre of the axle, is referred to some point, O, [coordinates $(x\ y)$], on the centre line of the locomotive and distant c behind the axle, it becomes :

1. A longitudinal force acting at 0, of value :

$$X = -2f\left(\frac{dx}{ad\theta} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (42)$$

2. A transverse force acting at 0, of value :

$$Y = -2f'\left(\frac{dy}{ad\theta} + c\frac{d\psi}{ad\theta} - \psi\right) \quad . \quad . \quad . \quad . \quad (43)$$

3. A couple having a vertical axis and moment of value :

$$G = -2\left[\frac{fb^2 + f'c^2}{a}\frac{d\psi}{d\theta} + \tau\frac{bc}{a}f\psi - cf'\psi + f'\frac{c}{a}\frac{dy}{d\theta} + f\tau\frac{b}{a}y\right] \quad (44)$$

Fundamental Equations of Motion.—These may be taken as the fundamental equations for the force-system at the wheel-treads. If 0 be taken at the centre of gravity of the part affected by the forces, M the mass and I the moment of inertia about a vertical through 0 ; X', Y', G', the force-system, reduced to the point 0, of any other forces acting on the part, the equations of motion are :

$$M\frac{d^2x}{dt^2} = X' + \Sigma X \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$M\frac{d^2y}{dt^2} = Y' + \Sigma Y \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$\mathrm{I}\,\frac{d^2\psi}{dt^2} = \mathrm{G}' + \Sigma\mathrm{G} \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where the summation extends over the several axles.

Equation 45 has no particular interest in the present connection, being concerned only with the normal progression of the locomotive. It should be noted, however, that f is large, so that $dx/ad\theta$ differs only infinitesimally from unity; and accordingly in equations 43 and 44, dx may be substituted for $ad\theta$. It is not proposed here to take account of the effect of acceleration on the phenomena of deviation, and accordingly, if V is the velocity of progression, it is permissible to write:

$$\frac{d^2}{dt^2} = \left(\frac{dx}{dt}\right)^2 \frac{d^2}{dx^2} = \mathrm{V}^2\frac{d^2}{dx^2}$$

thus equations 46 and 47 reduce to:

$$\mathrm{MV}^2\frac{d^2y}{dx^2} + 2\Sigma f'\left(\frac{dy}{dx} + c\,\frac{d\psi}{dx} - \psi\right) = \mathrm{Y}' \quad . \quad (48)$$

$$\mathrm{IV}^2\frac{d^2\psi}{dx^2} + 2\Sigma\left[(fb^2 + f'c^2)\frac{d\psi}{dx} + \left(f\tau\frac{b}{a} - f'\right)c\psi + f'c\frac{dy}{dx} + f\tau\frac{b}{a}y\right] = \mathrm{G}' \quad 49$$

These are the fundamental equations which determine the deviation of the motion from normal progression. The derivation from these of suitable equations to correspond with the circumstances of any given case usually presents no difficulty, and the recognized methods of solution for simultaneous linear differential equations having constant coefficients are applicable. The full solution of the equations is, however, not possible with our present knowledge as the data is lacking from which the constants f and f' can be determined. It is, therefore, expedient to discuss the matter in a number of partial aspects, introducing separately the several causes which influence the motion.

Slow Motion of Locomotive.—Consider first the case of a symmetrical locomotive or truck carried on two equal pairs of wheels in a rigid wheel-base, and moving slowly without train or other constraint. Equations 48 and 49 give:

$$\frac{dy}{dx} - \psi = 0$$

$$(fb^2 + f'c^2)\frac{d\psi}{dx} + f\frac{\tau b}{a}y = 0$$

Accordingly:

$$(fb^2 + f'c^2)\frac{d^2\psi}{dx^2} + f\frac{\tau b}{a}\psi = 0 \quad . \quad . \quad . \quad (50)$$

The motion is accordingly periodic, repeating in distance:

$$\mathrm{D} = 2\pi\sqrt{\left[\frac{ab}{\tau}\left(1 + \frac{f'}{f}\frac{c^2}{b^2}\right)\right]} \quad . \quad . \quad . \quad (51)$$

Consider next the case in which the pairs of wheels are unequal; let a be the radius of the front pair, and a' that of the rear pair. The equations of slow motion now become:

$$\frac{dy}{dx} - \psi = 0$$

$$(fb^2 + f'c^2)\frac{d\psi}{dx} + \tfrac{1}{2}f\tau b\left(\frac{1}{a} + \frac{1}{a'}\right)y + \tfrac{1}{2}f\tau bc\left(\frac{1}{a} - \frac{1}{a'}\right)\psi = 0$$

giving:

$$(fb^2 + f'c^2)\frac{d^2\psi}{dx^2} + \tfrac{1}{2}f\tau bc\left(\frac{1}{a} - \frac{1}{a'}\right)\frac{d\psi}{dx} + \tfrac{1}{2}f\tau b\left(\frac{1}{a} + \frac{1}{a'}\right)\psi = 0 \quad (52)$$

The solution of this is of the form:

$$\psi = \psi_o e^{-px}\sin(qx + a)$$

where

$$p = \tfrac{1}{4}c\left(\frac{\tau}{ab} - \frac{\tau}{a'b}\right)\Big/\left(1 + \frac{f'}{f}\frac{c^2}{b^2}\right)$$

$$q = \left\{\frac{\frac{1}{2}\left(\frac{\tau}{ab} + \frac{\tau}{a'b}\right)}{\left(1 + \frac{f'}{f'}\frac{c^2}{b^2}\right)} - \frac{c^2\left(\frac{\tau}{ab} - \frac{\tau}{a'b}\right)^2}{16\left(1 + \frac{f'}{f'}\frac{c^2}{b^2}\right)^2}\right\}^{\frac{1}{2}}$$

The period is therefore approximately that of a truck having equal wheels of radius $2aa'(a + a')$, but the amplitude of the motion now increases or diminishes according as p is negative or positive, that is, according as the front or the rear wheels are of greater radius. Thus the truck runs in one direction with less tendency to deviate than when running in the other.

More generally, when a locomotive is carried on more than one truck, each truck tends to impose on the locomotive an oscillation of period depending on its wheel-base and on the radius of its wheels. The construction of the truck, the outside constraints or other circumstances (e.g. the inertia) may

diminish or increase the tendency and with it the violence of the oscillation, and some of these effects will now be considered.

Motion of Locomotive Steadied by Train-haulage.—The general effect of hauling a train is to impose on the locomotive a counter-torque tending to diminish any deflection from the direction of the track and thus making for stability. The exact direction of the draw-bar pull is perhaps a little indefinite, but in general the counter-torque it imposes is certainly not less than would be the case if the draw-bar remained always parallel with the track. Making this assumption accordingly, if P is the draw-bar pull and e the distance of its point of application from the centre of the locomotive, the outside force system is represented by

$$Y' = 0 \qquad G' = -\mathrm{P}e\psi.$$

Thus, for slow motion, equations 48 and 49 reduce to:

$$\frac{dy}{dx} - \psi = 0$$

$$(fb^2 + f'c^2)\frac{d\psi}{dx} + f\frac{\tau b}{a}y + \tfrac{1}{4}\,\mathrm{P}e\psi = 0$$

whence: $$(fb^2 + f'c^2)\frac{d^2\psi}{dx^2} + \tfrac{1}{4}\,\mathrm{P}e\frac{d\psi}{dx} + f\frac{\tau b}{a}\psi = 0 \quad . \quad . \quad . \quad (53)$$

the solution of which is of the form:

$$\psi = \psi_0 e^{-px} \sin(qx + a)$$

where: $$p = \mathrm{P}e/8(fb^2 + f'c^2). \quad . \quad . \quad . \quad . \quad . \quad (54)$$

The oscillation accordingly diminishes as the locomotive progresses, showing that haulage tends to stabilize the motion; the period of the oscillation is not sensibly affected by the haulage.

Conditions of Natural Alignment.—When a locomotive is fitted with auxiliary trucks so arranged as to be able to affect, and be affected by, the deviation of the main truck, the combination, as has already been shown in a particular case, may possess a certain tendency for the several groups of wheels to align themselves or to run out of alignment as the case may be. This tendency has no connection with the coning of the wheels, although it may affect or determine the stability

of the oscillation resulting from the coning. The matter is accordingly most easily studied apart from the coning, assuming the wheels to be of uniform radius and treating the locomotive as if it were running on a plane surface instead of on rails. If under these conditions the groups of wheels tend naturally to align with one another, it may reasonably be concluded that the combination makes for stability; if, on the other hand, the natural tendency of any group is to run out of alignment with the others, it may be inferred that the combination possesses corresponding elements of instability.

LOCOMOTIVE WITH PONY AXLE.—Consider first the case of a locomotive having an auxiliary axle pivotally connected with the main frame, as shown diagrammatically in fig. 22. Let h and h_1 in the figure be taken as positive when

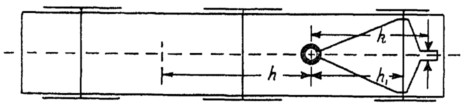


FIG. 22.—Locomotive with Pony Axle.

the auxiliary axle leads, and in general let the suffix 1 denote a quantity applying to the auxiliary member. Assume an elastic centering force, tending to parallel the axles, to act between the two members at distance k from the pivot, as shown in the figure; the value of this force on the main frame may be written $Qk\ (\psi_1 - \psi)$, where Q is a constant. The pivotal relation between the members is expressed by the equation:

$$y_1 = y + h\psi + h_1\psi_1.$$

Putting $V = 0$, $\tau = 0$, in equations 48 and 49, the following equations are deduced, the first from the transverse forces acting through the wheels on the two members jointly, and the two last by taking moments about the pivot for the two members separately:

$$f'\left(\frac{dy}{dx} - \psi\right) + f_1'\left(\frac{dy_1}{dx} - \psi_1\right) = 0$$

$$4(fb^2 + f'c^2)\frac{d\psi}{dx} - 4hf'\left(\frac{dy}{dx} - \psi\right) - Qk^2(\psi_1 - \psi) = 0$$

$$4f_1b^2\frac{d\psi_1}{dx} + 4h_1f_1'\left(\frac{dy_1}{dx} - \psi_1\right) + Qk^2(\psi_1 - \psi) = 0$$

Putting the solution as proportional to e^{mx}, the equation for m reduces to:

$$4\{(f+f_1')(fb'+f'c^2)f_1b^2+f'f_1'[(fb^2+f'c^2)h_1^2+f_1b^2h^2]\}m \\ +4f'f_1'[f_1b^2h-(fb^2+f'c^2)h_1]+ \\ Qk^2\{(f'+f_1')[(f+f_1)b^2+f'c^2]+f'f_1'(h+h_1)^2\}=0 \qquad (55)$$

The condition of natural alignment of the truck is accordingly:

$$4f'f_1'[f_1b^2h-(fb^2+f'c^2)h_1]+Qk^2\{(f'+f_1')[(f+f_1)b^2+f'c^2] \\ +f'f_1'(h+h_1)^2\}>0.$$

It is usually quite possible to satisfy this condition whether the auxiliary axle leads or trails, the value of Qk^2 necessary being generally well within the range of practice. When the auxiliary axle trails indeed, so that h and h_1 are both negative, the trucks tend to align without the centering force, provided that:

$$(-h_1)(fb^2+f'c^2)>(-h)f_1b^2,$$

a condition which is usually satisfied. If the effect of coning the wheels be taken into account, the equation for m becomes a cubic and the above condition, with a modification of small importance, is among those of stable running. Detailed discussion of the cubic is hardly justified, inasmuch as inertia, represented by the terms involving the velocity in equations 48 and 49, modifies the conditions of stability. It may be inferred, however, that a locomotive of the kind under discussion has but one period of kinematical oscillation, the pivotal connection compelling a synchronous movement of the two members, although their individual periods may be very different. This can hardly fail to be severe on both track and locomotive, and to militate against smooth running.

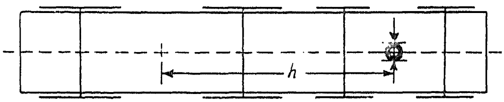


FIG. 23.—Locomotive with Auxiliary Bogie.

LOCOMOTIVE WITH AUXILIARY BOGIE.—Consider next the case of a locomotive having an elastically centered guiding bogie (fig. 23), the distance between the centres of the two groups of wheels being h. Adopting suffix 1 to distinguish

quantities applying to the auxiliary truck, the outside forces acting are:

$$Y' = -Q(y - y_1 + h\psi), \qquad Y_1' = Q(y - y_1 + h\psi)$$
$$G = -Qh(y - y_1 + h\psi), \qquad G_1 = 0$$

Putting $\tau = 0$ and $V = 0$ in equations 44 and 45, there results:

$$4f'\left(\frac{dy}{dx} - \psi\right) + Q(y - y_1 + h\psi) = 0$$

$$4f_1'\left(\frac{dy_1}{dx} - \psi_1\right) - Q(y - y_1 + h\psi) = 0$$

$$4(fb^2 + f'c^2)\frac{d\psi}{dx} + Qh(y - y_1 + h\psi) = 0$$

$$\frac{d\psi_1}{dx} = 0$$

Thus ψ_1 is constant and putting $\psi - \psi_1$, as proportional to e^{mx}, the equation for m is:

$$4f'f_1'(fb^2 + f'c^2)m^2 + Q[(f' + f_1')(fb^2 + f'c^2) + f'f_1'h^2]m + f'f'_1Qh = 0 \quad . \quad . \quad . \quad . \quad (56)$$

The trucks therefore tend to align when h is positive, that is, when the bogie leads, and to diverge when the bogie trails—a conclusion already reached for the simpler case in which the elastic centering feature is absent ($Q = \propto$). The locomotive accordingly runs smoothly with the bogie leading but tends to rear when the bogie trails. If the coning of the wheels be taken into account, the resulting oscillation has two components, the periods of which are sensibly those of the individual trucks. The oscillation tends to increase when the bogie trails and to decrease when it leads.

Elastically Aligned Bogie.—Consider now the case in which the bogie is in the rear, and is not only elastically centered but also subject to an elastic aligning couple. Let the aligning couple have the moment G times the angular deviation from alignment. Adopting the same notation as in last paragraph, equations 48 and 49 give, putting $\tau = 0$ and $V = 0$ as before:

$$4f'\left(\frac{dy}{dx} - \psi\right) + Q(y - y_1 + h\psi) = 0$$

$$4f_1'\left(\frac{dy_1}{dx} - \psi_1\right) - Q(y - y_1 + h\psi) = 0$$

$$4(fb^2+f'c^2)\frac{d\psi}{dx}+Qh(y-y_1+h\psi)+G(\psi-\psi_1)=0$$

$$4(f_1b^2+f_1'c_1^2)\frac{d\psi_1}{dx}-G(\psi-\psi_1)=0.$$

Writing $f'e^2=fb^2+f'c^2$ for brevity, with a similar expression for $f_1'e_1^2$, if the solution varies as e^{mx}, m is given by:

$$\begin{vmatrix} 4f'm+Q & -Q & -(4f'-Qh) & 0 \\ -Q & 4f_1'm+Q & -Qh & -4f_1' \\ Qh & -Qh & 4f'e^2m+G+Qh^2 & -G \\ 0 & 0 & -G & 4f_1'e_1^2m+G \end{vmatrix}=0$$

This determinant can be reduced to the simpler form:

$$\begin{vmatrix} 4f'm & f'+f_1' & -4f' & -f'h \\ -Q & f_1' & -Qh & 0 \\ 0 & f_1'h & 4f'e^2m & f'e^2+f_1'e_1^2 \\ 0 & 0 & -G & f'_1e^2_1 \end{vmatrix}=0$$

This yields:

$$16f'^2f_1'^2e^2e_1^2m^2+4f'f_1'\{(f'e^2+f_1'e_1^2)G+[(f'+f_1')e^2e_1^2+f_1'h^2c^2]Q\}m$$
$$+\{[(f'+f_1')(f'e^2+f_1'e_1^2)+f'f_1'h^2]G+4f'f_1'^2he_1^2\}Q=0 \quad . \quad (57)$$

Hence when h is positive the trucks tend to run in alignment for all positive values of G, but when h is negative the trucks tend to align only if:

$$G>\frac{4f'f_1'^2e_1^2(-h)}{(f'+f_1')(f'e^2+f_1'e_1^2)+f'f_1'h^2}$$

Thus a rear bogie should for smooth running be acted on by an aligning couple whose moment per unit angular deflection exceeds the amount given in the above inequality; but a front bogie requires no such alignment.

As an example (fig. 23), let $f=f_1=f'=f_1'=300{,}000$ lbs.
$e=5$ ft. $\qquad e_1=4$ ft. $\qquad h=12$ ft.

$$G>\frac{4\times300{,}000\times12}{2\left(1+\frac{25}{16}\right)+9}>\frac{14{,}400{,}000}{14\cdot1}$$

$$>10^6$$

If the couple is produced by springs at each end of the bogie

at a distance of 6 feet from its centre, the stiffness of the springs per inch displacement is given by:

$$12\text{P} = \frac{1}{72}\text{G}$$

$$\text{P} > \frac{10^6}{864} > 1{,}200 \text{ lbs.}$$

Influence of Inertia.—The effect of the inertia terms is readily seen in the case of a locomotive on rigid wheel-base. Where no train is hauled, equations 48 and 49 reduce to:

$$\text{MV}^2\frac{d^2y}{dx^2} + 4f'\left(\frac{dy}{dx} - \psi\right) = 0$$

$$\text{IV}^2\frac{d^2\psi}{dx^2} + 4\left[(fb^2 + f'c^2)\frac{d\psi}{dx} + f\frac{\tau b}{a}y\right] = 0$$

and the equation for m becomes:

$$\text{IMV}^4m^4 + 4[f'\text{I} + \text{M}(fb^2 + f'c^2)]\text{V}^2m^3 + 16f'(fb^2 + f'c^2)m^2 + 16ff'\frac{\tau b}{a} = 0 \quad . \quad . \quad . \quad . \quad (58)$$

The absence of a first power of m in equation 58 indicates that a pure motion of progression is unstable. In order to obtain an approximate solution of the equation, put $m = \xi + i\eta$, where:

$$(fb^2 + f'c^2)\eta^2 = f\frac{\tau b}{a}$$

ξ is then given by:

$$32f'(fb^2 + f'c^2)\xi = 4[f'\text{I} + \text{M}(fb^2 + f'c^2)]\text{V}^2\eta^2$$

or

$$\xi = \frac{f'\text{I} + \text{M}(fb^2 + f'c^2)}{8f'(fb^2 + f'c^2)^2}f\frac{\tau b}{a}\text{V}^2 \quad . \quad . \quad . \quad (59)$$

Since ξ is positive, the oscillation tends continually to increase in amplitude, and, if appropriate values are given to the quantities involved, it will be found that the increment factor becomes significant, and in fact, important at speeds well within the range of ordinary operation of railway trains. The inference that a locomotive carried on a rigid wheel-base is unsuited for high-speed service is of course in accordance with experience. When the locomotive is hauling a train, the

increment factor of the oscillation may be deduced approximately from equations 54 and 59, viz.:

$$\xi = \frac{f'\mathrm{I} + \mathrm{M}(fb^2 + f'c^2)}{8f'(fb^2 + f'c^2)^2} f\frac{\tau b}{a}\mathrm{V}^2 - \frac{\mathrm{P}e}{8(fb2 + f'c^2)} \quad . \quad (60)$$

This, although negative at low speed, becomes positive as the speed rises, the region of instability being generally reached within limits of speed of railway trains.

Bogie and Articulated Truck Locomotives.—Conclusions similar to the above hold in regard to locomotives carried on independently swivelling trucks, incapable of influencing one another in the matter of deflection, and also in regard to articulated truck locomotives. Such locomotives stress the track and their own parts if run at high speed, although the smaller masses involved in the kinematical oscillation reduce the destructiveness of the stresses, as compared with those due to locomotives carried on a single rigid wheel-base. Coaches also oscillate on straight track as far as the wheel flanges will permit, and the violent swaying that frequently accompanies running at certain speeds is doubtless to be attributed to resonance between the forced oscillation arising from the coning of the wheels and the natural rolling oscillation under the control of the suspension springs.

The friction of bearing plates, although reducing to some extent the influence making for instability, can hardly be said to exert a true dampening influence on the oscillation. Such friction has the effect of introducing in G in equation 49 a term which may be taken as proportional to $-\frac{d\psi}{dx}$, and this merely increases the term of like denomination, so diminishing the increment factor of equation 59, without tending to change its sign. It in fact diminishes the rate of increment of the oscillation, but has no tendency to convert it into a decrement. An active stabilizing influence is provided by a restoring force or couple proportional to the displacement from true alignment, such as results in a measure from the draw-bar pull, or from the centering action of a guiding truck. In this important respect, as well as in the fact that it is not isochronous, the kinematical oscillation arising from the coning of the wheels differs from natural oscillations, which are actively promoted

by restoring forces proportional to the displacement and invariably dampened by frictional forces.

The actual motion of a locomotive is of course less simple than the above discussion would indicate. The clearances between journals and bearing brasses, for instance, and between axle-boxes and horn-blocks, allow the axles to have a certain amount of individual freedom, an effect of which is probably to render the period of oscillation somewhat indefinite. If the oscillation is sufficient to cause the wheel-flanges to strike the rails, the constraint thereby imposed undoubtedly deflects the locomotive more quickly than is natural to its proper motion on coned wheels; an effect of such impact is therefore to reduce the distance in which the motion repeats and to render it dependent on the degree of instability of the oscillatory motion. It is in fact the use of the wheel-flanges for limiting the divergence resulting from instability that gives rise to the principal rough riding phenomena, and the pernicious track-spreading effects which often accompany high-speed operation is evidence of the magnitude of the flange forces required to deflect the locomotive with the necessary vigour.

Symmetrical Locomotives.—The facility with which an electric locomotive can be controlled from any desired point has led to the extended use of mechanically symmetrical machines, devised for hauling trains irrespective of the end by which they are attached. Most electric locomotives have, in fact, symmetrical wheel-bases, although some are made up of two unsymmetrical units. The advantages of a symmetrical locomotive, from the point of view of train operation, are doubtless great, particularly when used in terminal service; but such a locomotive has the incidental disadvantage of restricting the leading and trailing wheel-groups to similar arrangements, and since their functions are different, of making an aggregate which is not usually the most desirable from the point of view of smoothness of running. The matter is generally of little consequence in the case of goods locomotives, but many of the high-speed passenger locomotives that have been built reveal a tendency to instability to which the symmetrical wheel arrangement is doubtless a contributory cause. From the above discussion, it appears that elastic alignment of certain of the wheel-groups gives promise of a solution which

fulfils the conditions for smooth running in spite of symmetry of the wheel-base. Before investigating this possibility, however, it may be advisable to summarize the results already established, in order to show what arrangements are to be preferred.

LEADING AND TRAILING AXLES. Fig. 24 shows wheels and swivelling centres for a locomotive having a main wheel-base and leading and trailing pony axles. Fig. 25 is similar in principle, but has four-wheeled trucks, articulated with the main frame, in place of the single-axle trucks. It has been shown that whereas the rear truck trails naturally, the front truck requires an aligning force to prevent it tending always to run against the flanges. The auxiliary trucks are more

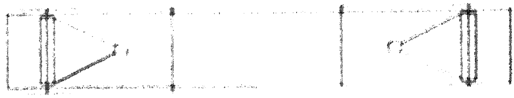


FIG. 24. Locomotive with Symmetrical Wheel Base, Pony Axles.

FIG. 25. Locomotive with Symmetrical Wheel Base, Articulated Trucks.

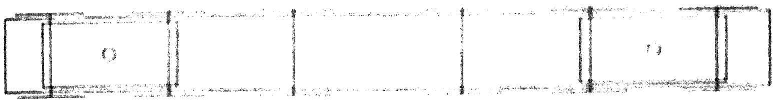


FIG. 26. Locomotive with Symmetrical Wheel Base, Guiding Bogies.

over geometrically connected with the main frame, so that their free deflection is prevented, with the result that considerable stresses are passed back and forth between main wheels and auxiliary wheels, through the rails on the one hand and the frames and swivelling centre-pins on the other. The main wheel group and auxiliary wheel groups indeed do not follow each its natural rolling course of sinuous motion; but each is constrained by the others, and all are compelled by rail forces to take up a compromise motion. The principal source of trouble is the leading truck, for the trailing truck tends naturally to align itself, so that its deflection is never very great. If the leading truck is impulsively deflected by reason of the wheel-flanges striking the rail, the blow is greatly

increased by the pivotal connection of the truck, which brings the mass of the main frame to oppose the deflection. The auxiliary truck so arranged is therefore quite satisfactory when used trailing, but cannot be recommended as a leading truck for a high-speed locomotive.

LEADING AND TRAILING BOGIES.—Fig. 26 shows the wheel arrangement of a locomotive having a main wheel base with leading and trailing bogies centred in the usual manner. In this case it has been shown that the main wheels tend naturally to follow the leading bogie ; and the combination, as far as concerns these two elements, is a satisfactory one, which makes for smooth running. Any force tending to deflect the bogie from its path is resisted only by the inertia of the bogie itself, which is accordingly able to ride over the irregularities of the track without stressing it unduly. The main wheels follow, and, on straight track, without appreciable guidance by their flanges. The rear bogie, however, does not behave so satisfactorily, tending continually to run across the track, and so causing the locomotive to rear.

Author's Reversible Locomotive.—The conclusion to be drawn from the foregoing discussion is that for high-speed locomotives the wheel-arrangement preferably comprises a leading bogie, of no greater mass than need be, and swivelling freely about its centre, with the remainder of the locomotive in trail. The locomotive is therefore not directly reversible end for end. It is, however, not difficult to devise a locomotive in which the character of the auxiliary trucks is determined by the position of the reverse-handle, in such manner that suitable conditions for high-speed running are fulfilled for either direction of motion. Figs. 27 and 28 show in diagrammatic form the principle of a possible arrangement.* Here the direction of motion is as shown by the arrow. The leading truck is able to swivel freely about its centre (A), which is constrained towards the centre plane of the locomotive by suitable devices not represented in the diagrams. The rear truck is pivotally connected to a block held at a point (B) approximately midway between the centre of the truck and the centre of the main wheel-base, so that it can deflect freely in passing about a curve, but is nevertheless always subject to an aligning torque. By the motion of the rod carrying the jaws, which may be

* See British Patent 163185.

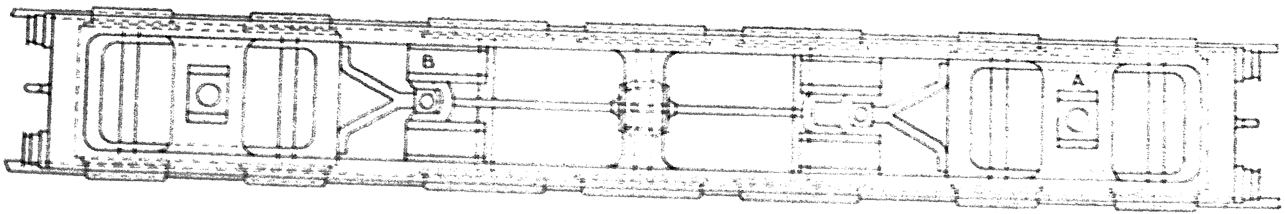


Fig. 27.—Reversible Locomotive with [illegible] Wheel Base.

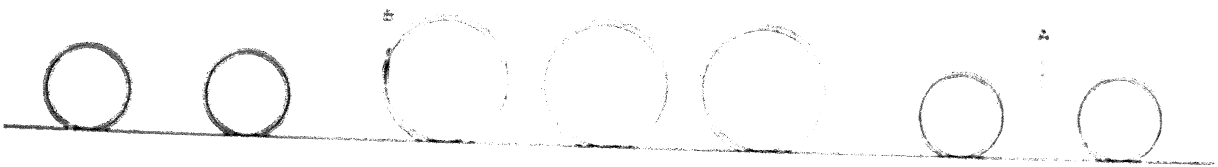


Fig. 28.—Diagram showing [illegible] of Fig. 27.

regarded as possibly representative of a more complicated mechanism, either truck may be left free, as befits a leading truck, and the other then has the characteristics of a trailing truck. The rod is shown as passing through an air-cylinder, pressure being applied to one side or the other of the piston, according to the position of the reversing handle, but this feature also is intended to be regarded as representative rather than necessarily actual.

General Conclusions as to Riding Qualities.—The above discussion shows, or at least suggests, that bad riding qualities result from one or both of two general causes, the constraint of the wheels to follow other courses than those towards which they naturally tend, and the setting up of resonant oscillations under the control of the springs, and usually as a consequence of the coning of the wheels. The former cause may be operative at all speeds, as when the wheel-base is so composed that its units tend naturally to diverge, or may increase in effect as the speed rises, as is the result of inertia of the parts carried by each rigid wheel-base, but in either case the manifestations become more violent the higher the speed. The latter produces effects which become noticeable on approaching certain speeds and pass away as these speeds are surpassed. The phenomena of "nosing" and "rearing" may be ascribed generally to the former cause and those of "rolling" and "pitching" to the latter.

Nosing.—A certain measure of nosing is a natural tendency of a locomotive running on coned wheels, although certain guiding-wheel arrangements tend to suppress the oscillation, which accordingly never becomes noticeable within the range of the working speed. Conditions favourable to nosing are short rigid main wheel-base, large flange clearance, large moment of inertia about a vertical axis and high speed. Excessive overhang of the locomotive body may also be mentioned, but this, apart from the greater moment of inertia involved, is a condition which shows off the nosing rather than causes it. It is probable that in some instances nosing is a phenomenon of resonant oscillation, the sinusoidal motion of the wheels synchronizing with the natural period of oscillation of the locomotive under the action of the centering springs of the guiding truck; and this form of nosing may be recognized

by its occurrence at about a definite speed. It is minimized by making the centering devices lock somewhat at the centre position, so that small motions are taken by deflecting the riding springs. This destroys the isochronous character of the centering devices. It is possible also that equality between periods of oscillation of the main and auxiliary trucks results in an enhancement of the nosing amplitude, and arrangements which lead to this condition are perhaps best avoided. It is for constructional reasons that steam locomotive bogies are furnished with wheels of much smaller diameter than the driving wheels, but the practice nevertheless probably makes for smooth running.

Rearing.—Rearing is indicated above as resulting from the use of a trailing bogie, capable of turning the main frame, but incapable of being itself turned by the reaction of this member. The arrangement and the fault are fairly common in locomotives designed for running equally in either direction. Thus the Pennsylvania Railroad locomotive of fig. 204 consists of two units connected by a draw-bar and having wheel arrangement 4-4-4-4; of these it has been written: * "The rear half of the locomotive does not seem to articulate well with the front half. Some action tends to lift the rear half from the tracks." The New York Central 4-4-4-4 locomotive of fig. 201, which consists of two units connected by a hinge joint, also gave trouble of similar nature, necessitating the introduction of a friction device to restrict the relative movement of the trucks, and thus give each unit the approximate characteristics of a rigid wheel-base; the arm shown joining the trucks in fig. 201 is part of the friction device in question.

Rolling.—Rolling has been mentioned above in connection with coaches and explained as due to synchronism between the periodic sideway motion of the wheels and the natural rolling oscillation on the riding springs. Violent rolling is apt to take place at the appropriate speed with an unstably running locomotive, and is more likely to occur within the range of working speeds when the axle journals are between the wheels and the centre of gravity is carried high, than when opposite conditions hold. The employment of laminated riding springs, in which the friction between the plates absorbs energy and

* *Electric Traction for Railway Trains*, by E. R. Burch, p. 328.

thus dampens the oscillation, mitigates rolling, as well as the other modes of oscillation involving these springs.

Pitching.—Pitching, being a motion in a longitudinal direction, might at first sight appear to be unaffected by sideway oscillations produced by the coning of the wheels; in a properly balanced locomotive this conclusion is justified, but where the masses are so distributed that the longitudinal axis of the locomotive is not in the direction of a principal axis of inertia it is possible for a periodic sideway motion to set up a pitching oscillation, and since the period of pitching is comparatively large, the oscillation is likely to occur within the working range of speed. The records of the Berlin-Zossen high-speed tests furnish an example of oscillation which, as far as can be judged from the imperfect information given, was of the nature in question. Of the two locomotive coaches

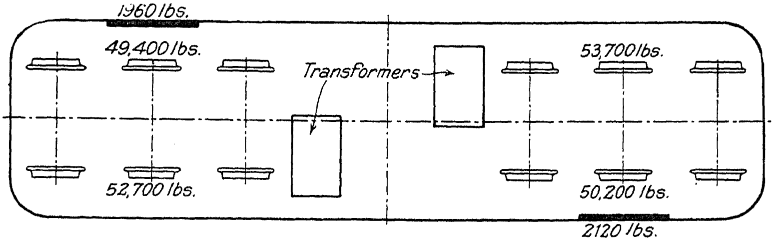


FIG. 29.—Loading of Berlin-Zossen Coach.

employed, one was symmetrically loaded, and this ran to the highest speeds obtainable without oscillation. The other, whilst substantially balanced about the longitudinal and transverse axes, had the transformers set somewhat off the centre line of the coach, as shown in fig. 29, and this coach commenced to oscillate violently at about 90 m.p.h. so that higher speeds could not be reached with safety. The pressure on the wheels on each side of each bogie was measured, and found to be 52,700 and 49,400 lbs. respectively for one truck, and 50,200 and 53,700 lbs. respectively for the other, the greater reading corresponding to the transformer side of the bogie. Weights of 1,960 and 2,120 lbs. respectively were put at the positions shown in fig. 29 at a distance of 53 inches from the centre line, and on the lighter side, these weights having approximately the same moment about the centre line as the excess of pressure on the wheels, thus equalizing the pressures in pairs. When

this had been done, the coach was able to run to the highest speeds (125 m.p.h.) without oscillation. The addition of the compensating masses, besides equalizing the pressures on the wheels, has the effect of bringing the axis of inertia into parallelism with the longitudinal axis of the locomotive.

CONCLUSION.—The foregoing discussion of the performance of locomotives as affected by the interaction between wheel and rail, shows it to be a complex phenomenon, depending in no simple manner on a number of factors, possessing also secondary features of consequence, and altogether involving much complication in its correct mathematical treatment. It is, moreover, a subject of great importance fully repaying the labour of any investigation that may be needed to secure stable operation at all working speeds. The conclusions drawn from the discussion appear in general accord with experience, testifying to the soundness of the theory advanced and justifying the discussion itself. It will readily be appreciated, however, that a particular locomotive presents less of difficulty than a more general case, and if reliable values were obtainable for certain constants it would not be difficult to detect radical defects in design likely to result in trouble from unstable running, and to indicate suitable means of amelioration.

Blows on Rail.—The blow which a wheel receives in passing over a high rail joint or other irregularity in the track depends

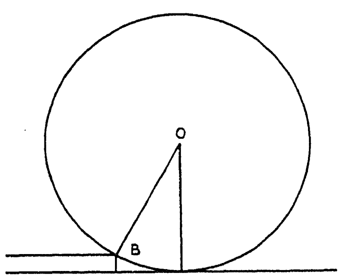


FIG. 30.—Diagram. Wheel and Obstacle.

on the uncushioned mass (m) carried by the wheel, on the speed (v) of the train, on the diameter (d) of the wheel and on the height (h) of the obstruction above normal track level. In fig. 30, if BC represent the obstacle to be surmounted, the motion, originally at right angles to AO, is changed by the

blow to one at right angles to BO ; the impressed component is directed along BO and is represented in magnitude by the length CA, as compared with the original motion, represented by AO. Thus if v' is the impressed velocity:

$$v' = \frac{\mathrm{CA}}{\mathrm{AO}} v$$

$$= 2\frac{\sqrt{\mathrm{BC.2.AO}}}{2\mathrm{AO}} v$$

$$= 2\sqrt{\frac{h}{d}}v \quad . \quad . \quad . \quad . \quad (61)$$

The impulse of the blow is:

$$\mathrm{P} = 2mv\sqrt{\frac{h}{d}} \quad . \quad . \quad . \quad . \quad (62)$$

It therefore varies as the uncushioned mass and inversely as the square root of the wheel diameter. The permissible value of $m/\sqrt{d}$ depends on the speed of operation (v), as well as on the quality of the track, as represented by h. In the original Central London Railway locomotives the total uncushioned mass amounted to about 8 tons per axle, the wheels being 42 inches in diameter; but although the speed was comparatively low, rarely exceeding 30 miles per hour, the results were unsatisfactory and the type was shortly superseded. In the bipolar gearless drive used on the locomotives of the New York Central and Hudson River Railroad, the whole armature is carried inelastically on the axle. In the case of the first types of these locomotives the total uncushioned mass amounted to about 5½ tons per axle, the wheels being 44 inches in diameter. Later types have been lighter in this respect, one of the latest having an uncushioned mass of less than 3 tons per axle, carried on 36-inch wheels. These locomotives were designed for high speed, the latter type sometimes attaining to 70 miles per hour in service.

Uncushioned Masses.*—The approximate determination of the uncushioned masses presents no difficulty in so far as they consist of wheels and axles and parts carried entirely by these members without the intervention of springs, e.g. axle-boxes, shoe beams, coupling rods and counterweights. Where, however, a motor is supported partly by means of axle bearings

* See *Minutes of Proc. Inst. C.E.*, vol. 201, p. 228.

and partly from the transom of a truck, as in fig. 31, the equivalent uncushioned mass is less obvious. A blow given to the wheel is transmitted to the motor partly through the axle bearings, and, unless flexible gears are used, partly through the gear teeth ; whilst if the nose of the motor is attached rigidly to the transom, a further portion is passed on to the truck frame through this. The last of the three portions enumerated, when it exists, is usually inconsiderable, but since it depends on the mass and arrangement of the truck frame, adds considerably to the complication of the problem. A superior limit can, however, be set to its value by assuming the mass

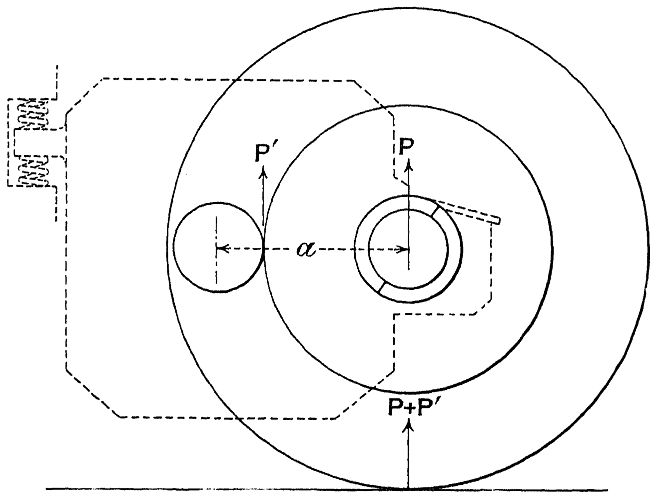


FIG. 31.—Diagram. Nose-suspended Motor.

of the truck to be infinite compared with that of the motor, so that the nose acts as the pivot about which the motor turns, and in this case the several impulses are readily determined. However, it is more in accordance with reality to assume the nose of the motor free, particularly since it is now common practice with powerful motors to support the nose between elastic buffers, as represented diagrammatically in fig. 31, an arrangement which leaves it free as regards the impulses.

Let M be the mass of the complete motor and m the equivalent uncushioned mass, I the moment of inertia of the motor-frame and field-structure about the centre line of the shaft, assumed to pass through the centre of gravity of the motor,

and I′ the moment of inertia of the armature and pinion about the same line; a the distance between axle and armature shaft; γ the ratio of gear reduction, so that the pitch radius of the pinion is $a/(\gamma + 1)$. Let P be the impulse transmitted through the axle bearings and P′ that transmitted through the gear teeth; the velocity impressed on the centre of gravity is then $(\mathrm{P} + \mathrm{P}')/\mathrm{M}$; the angular velocity of the frame is $\mathrm{P}a/\mathrm{I}$; the angular velocity of the armature is $\mathrm{P}'a/\mathrm{I}'(\gamma + 1)$. The additional velocity of the axle, or of the gear tooth, caused by the blow, is accordingly:

$$v' = \frac{\mathrm{P} + \mathrm{P}'}{m} = \frac{\mathrm{P} + \mathrm{P}'}{\mathrm{M}} + \frac{\mathrm{P}a^2}{\mathrm{I}} = \frac{\mathrm{P} + \mathrm{P}'}{\mathrm{M}} + \frac{\mathrm{P}'a^2}{\mathrm{I}'(\gamma + 1)^2}$$

hence:

$$\frac{\mathrm{P}}{\mathrm{I}} = \frac{\mathrm{P}'}{\mathrm{I}'(\gamma + 1)^2} = \frac{\mathrm{P} + \mathrm{P}'}{\mathrm{I} + \mathrm{I}'(\gamma + 1)^2}$$

and

$$m = \mathrm{M}\frac{\mathrm{I} + \mathrm{I}'(\gamma + 1)^2}{\mathrm{M}a^2 + \mathrm{I} + \mathrm{I}'(\gamma + 1)^2} \quad . \quad . \quad (63)$$

This is the equivalent uncushioned mass of the motor.

A point at horizontal distance x from the armature shaft commences to move with upward velocity:

$$\frac{\mathrm{P} + \mathrm{P}'}{\mathrm{M}} + \frac{\mathrm{P}ax}{\mathrm{I}} = (\mathrm{P} + \mathrm{P}')\left[\frac{1}{\mathrm{M}} + \frac{ax}{\mathrm{I} + \mathrm{I}'(\gamma + 1)^2}\right]$$

hence the centre of rotation for the imposed motion is given by:

$$x = -\frac{\mathrm{I} + \mathrm{I}'(\gamma + 1)^2}{\mathrm{M}a} \quad . \quad . \quad . \quad (64)$$

The centre of rotation is therefore on the side of the shaft remote from the axle, and is in fact frequently outside the motor altogether. In the latter event the whole motor starts with an upward velocity, so that the provision of an elastic buffer below the nose, with a rigid strap above it, is ineffective to reduce the shocks to the minimum possible.

The above calculation assumes that the action takes place in a plane, or that both wheels on an axle are raised simultaneously and equally by irregularity in the track, whereas in general one wheel only would be lifted at a time. The appropriate three-dimensional calculation is not difficult to perform,

but it will suffice here to state the result. If v_1 v_2 are upward velocities simultaneously impressed on the two wheels, c_1 c_2 the transverse distances of the centre of gravity of the motor from the corresponding track rails, the sum of the blows on the two rails due to the presence of the motor is given by :

$$Q = M\frac{I + I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2} \cdot \frac{c_1v_2 + c_2v_1}{c_1 + c_2} \quad . \quad . \quad (65)$$

Thus equation 63 still gives the uncushioned mass correctly provided that it is associated with velocity $(c_1v_2 + c_2v_1)/(c_1 + c_2)$ which is that of the point of the axle opposite the centre of gravity of the motor. In most cases c_1 and c_2 are not very different, and if one wheel only is raised, with velocity v', the sum of the blows on the two rails is approximately the same as if both wheels were raised with equal velocity $v'/2$. The same is true, however, of the wheels themselves, the axle boxes and other elements of uncushioned mass, and accordingly the mass determined by equation 63 can appropriately be regarded as the uncushioned mass of the motor. The blow is not equally divided between the two rails, but is for the most part taken by that on which the obstacle occurs. Thus for practical purposes, if h is the height of the representative obstacle, the actual blow on the rail on which it occurs may be taken as a half that given by equation 55, or

$$P_1 = mv\sqrt{\frac{h}{d}} \quad . \quad . \quad . \quad (66)$$

It should be noted that there is no distinction between the pinion and commutator ends of the motor as regards the effect of an obstacle on the rail, except in so far as c_1 and c_2 may differ ; this difference usually makes the blow a little heavier when the obstacle on the rail is at the commutator end of the motor, since the centre of gravity is generally somewhat nearer to this rail (see equation 65).

Flexible Gearing.—When a motor is driven through flexible gearing, impulsive shocks are not transmitted through the gear-teeth ; it is readily seen that this is equivalent to putting $I' = 0$ in equation 63, making the effective uncushioned mass of the motor :

$$m = M\frac{I}{Ma^2 + I} \quad . \quad . \quad . \quad (67)$$

The uncushioned mass is of course much smaller in this case than when rigid gearing is employed, and the use of flexible gearing is quite justified from the present point of view where heavy motors are used for high-speed work.

Blows on Gear-teeth.—When the gears are rigid the impulse transmitted through the teeth assumes importance from its possibly destructive effects. Using the same notation as above the amount of this impulse is given by the equation:

$$P' = M \frac{I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2} \cdot \frac{c_1 v_2 + c_2 v_1}{c_1 + c_2} \quad . \quad (68)$$

If, as is usual, $c_1 = c_2$ approximately, and one of the wheels encounters an obstacle of height h, the corresponding impulsive shock on the gear-tooth is (see equation 61):

$$P' = M \frac{I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2} v \sqrt{\frac{h}{d}} \quad . \quad . \quad (69)$$

If N is the maximum allowable speed of revolution of the armature, the corresponding train speed, v, is $\pi d N/\gamma$. The blow on the gear-tooth may therefore be written:

$$P' = M \frac{I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2} \cdot \frac{N}{\gamma} \cdot \pi\sqrt{dh} \quad . \quad (70)$$

Thus as far as the motor is concerned, the quantity,

$$MI'(\gamma + 1)^2 N\sqrt{d} \Big/ \gamma[Ma^2 + I + I'(\gamma + 1)^2]$$

may be taken as representing the intensity of the blow inflicted on the gear-tooth.

STRESS IMPOSED ON GEAR-TEETH.—It is, however, of greater value to the engineer to discover the stress imposed on the teeth by the impulses. This can hardly be computed absolutely; but in so far as it depends on the motor an expression is readily obtained for it, whilst the appropriate constant of proportionality corresponding to any particular design of tooth is to be determined from experience in normal service. The energy transmitted by the elastic forces is proportional jointly to the impulse P′ and the imposed velocity v'; it is, however, also proportional to the square of the maximum stress F; hence:

$$F^2 \propto P'v'$$

or from equations 61 and 70:

$$F \propto N\frac{\gamma+1}{\gamma}\sqrt{\left\{\frac{MI'd}{Ma^2+I+I'(\gamma+1)^2}\right\}} \quad . \quad (71)$$

The quantity on the right-hand side of this proportionality is accordingly a figure of merit, subject to a superior limit for a particular design of gear-tooth. With twin gears of the rigid type the tooth stresses are indeterminate, but prudence dictates that either gear should be capable of standing the whole shock, due allowance being made for the better alignment of the teeth of gear and pinion as compared with the more usual form.

In computing the uncushioned mass of a motor, the assumption was made that the centre of gravity of the frame structure was somewhere on the armature shaft, whereas it is usually somewhat nearer to the axle. In order to take some account of this it is advisable to consider the axle bearing caps and brasses as being entirely uncushioned, which approximately adjusts the balance. The gear wheel is of course also uncushioned.

Example.—As an example of the use of the foregoing formulae, the Central Argentine Railway Motor (GE. 235), which was designed for suburban service, may be considered. Its weight complete is 8,650 lbs., of which 400 lbs. is in axle-bearing caps with brasses and 690 lbs. in the gear, leaving $M = 7{,}560$ lbs.; the weight of the armature is 2,430 lbs., or 32 per cent. of M, the frame being therefore 68 per cent. of M; the radius of gyration of the armature is estimated at 7 inches and of the frame at 16 inches; the gear reduction γ is 70/22 or 3·18, and, the pitch being $2\frac{1}{4}$ teeth per inch diameter, $a = \frac{1}{2} \times 92/2\frac{1}{4} = 20{\cdot}4$ inches. From equation 63:

$$m = 7{,}560\,\frac{\cdot 68 \times 16^2 + \cdot 32 \times 7^2 \times 4{\cdot}18^2}{20{\cdot}4^2 + \cdot 68 \times 16^2 + \cdot 32 \times 7^2 \times 4{\cdot}18^2}$$

$$= 3{,}920 \text{ lbs.}$$

To this figure should be added 1,090 lbs. for the gear, axle-bearing caps and brasses, making the total uncushioned mass for the motor, 5,010 lbs.; the wheels, axle, axle-boxes, etc., add approximately 4,500 lbs. to this figure, making a total of about 9,500 lbs. for each axle, the wheels being 42 inches in diameter. The distance of the centre of rotation of the

motor from the armature shaft as given by equation 57 is:

$$x = -\frac{\cdot 68 \times 16^2 + \cdot 32 \times 7^2 \times 4\cdot 18^2}{20\cdot 4}$$
$$= -21\cdot 9 \text{ inches.}$$

The centre is therefore outside the motor. With masses expressed in pounds and lengths in inches, the figure given by the proportionality (71) as representing the stress in the gear-tooth, is 100 N, and the maximum working value of N is about 1,650 r.p.m., making a figure of merit of 165,000.

EFFECT OF SIZE OF WHEEL.—If the wheel diameter and the ratio of gear reduction be varied in the same proportion the speed and power characteristics of the locomotive are unaffected. With the increase of wheel diameter, however, both the mass of the wheel and the equivalent uncushioned mass of the motor are increased, whilst the blow on the rail per unit mass is diminished. The total effect of the change is in this respect usually in favour of the smaller wheel. With regard to the gears, whilst increase in wheel diameter generally increases the blow on the teeth, the stresses set up in them are usually but little affected by the change.

Blows on Armature-Shaft Journals.—The impulses transmitted through the armature-shaft journals are of interest in connection with the design of the bearings. If M′ is the mass of the armature, the impulse in question is:

$$M' \frac{P + P'}{M} = \frac{M'}{M} mv'$$
$$= M' \frac{I + I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2 v'} \quad . \quad . \quad (72)$$

it is accordingly the same as that of a mass, carried directly by the axle, of amount:

$$m' = M' \frac{I + I'(\gamma + 1)^2}{Ma^2 + I + I'(\gamma + 1)^2} \quad . \quad . \quad (73)$$

and this mass may be compared with that of the axle-boxes, and their rigid appurtenances. In the case of the Central Argentine motors, for instance:

$$m' = \frac{2,430}{7,560} \times 3,920$$
$$= 1,260 \text{ lbs.}$$

This is some five times the uncushioned mass carried by the axle journals. Thus although the total weight of parts supported on the latter journals is many times greater than that of the armature, the blows to which the uncushioned masses subject them have only about one-fifth of the intensity of those borne by the armature-shaft journals. To such considerations must be attributed the general failure of ball and roller bearings for the armature-shaft journals of heavy railway motors, although experience has approved such bearings for axle journals. The shocks of service are reduced by the use of flexible gearing and practically annulled if the axle be driven through a flexibly supported quill; the engineer employing these devices may accordingly consider the use of ball or roller bearings for the armature-shaft journals.

VIBRATIONS IN SIDE-ROD LOCOMOTIVES

The motors usually employed in the continuous current system of electric traction are capable of developing large power in a confined space, whilst the relation between economical motor speed and axle speed lies within the limits imposed by considerations of satisfactory design. There is, moreover, no sacrifice of efficiency or other desirable features involved in using motors of comparatively small size, whilst extended experience has developed a motor which it is quite unnecessary to keep under constant surveillance. With such machines available it is practicable to drive the several axles by independent motors, and the excellent mechanical features of the transmission system involved in the drive appeal to the engineer as giving promise of successful operation and low maintenance cost, a promise which experience constantly supports. The alternating-current motor, on the other hand, and particularly the single-phase commutator motor, is in many respects closer to the limits of satisfactory design, and the imposition of space restrictions is accordingly felt as much more onerous. The single-phase commutator motor, moreover, with its large number of brushes and generally indifferent commutation, is the better for being readily accessible for the purposes of inspection and adjustment, whilst the larger the unit the higher is the practicable efficiency, and the more satisfactory the operation. It is principally for the reasons that such assiduous efforts have been made to develop a satis-

factory side-rod locomotive, in spite of certain grave mechanical demerits of the transmission system, in which circular motion has to be converted into reciprocating motion, only to be reconverted into circular motion at the axles. It is true that the type has certain advantages, but these are incidental and of a minor nature when weighed against its inherent defects. It is easily practicable, for instance, to arrange that the locomotive has a high centre of gravity; whilst, the motors being entirely spring-borne, the uncushioned mass carried by the axle is generally smaller than where the axles are driven by independent motors.

Mechanical Defects attending Collective Driving.—Collective driving of locomotive axles through the medium of side-rods appears to appeal to certain steam-locomotive engineers, doubtless as possessing familiar features and as promising security on account of the ready accessibility of the motors. Steam and electrical locomotives are, however, by no means similar in mechanical arrangement, and side-rod driving presents a much more difficult mechanical problem in the electrical locomotive than in the steam locomotive. In the latter the primary forces are reciprocating and parallel to longitudinal members of the locomotive. The requirements of driving being of a similar character and in the same direction, the stresses imposed on the frame are for the most part local in their incidence and unable to produce much distortion. Moreover, the pistons form cushioned ends to the driving system, so that any distortion of the parts and any inaccuracy in fitting is taken up in the clearance thus provided. In the electric side-rod locomotive, on the other hand, the original drive is in the form of a uniform torque, and this cannot be converted to reciprocating drive without the introduction of large couples having transverse arms. The forces tending to produce distortion in the structure are considerable, but the permissible distortion, together with any inaccuracy of fitting, must nevertheless be no more than can be provided in the clearances between running surfaces; and these must be kept within fine limits if knocking is to be avoided. The fitting and adjustment required is, therefore, much closer than is usual in locomotive work, whilst constant skilled attention is necessary to keep the machine in satisfactory running order.

Calculation of Forces on Locomotive Frame.*—If T is the torque of a shaft transmitting its power through quartered connecting rods and a the radius of the crank circles, then, neglecting for a moment the indeterminateness of their inci-

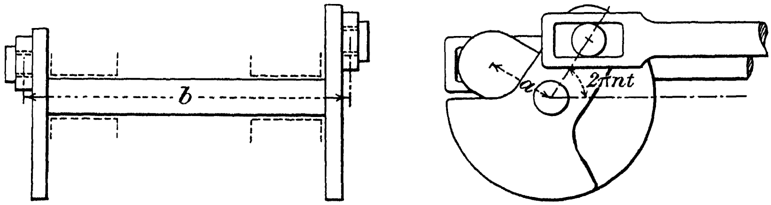


FIG. 32.—Diagram. Crank Action.

dence, the forces in the two connecting rods may be written (see fig. 32):

$$\frac{T}{a}\sin 2\pi nt = \frac{T}{a\sqrt{2}}\left\{\sin\left(2\pi nt + \frac{\pi}{4}\right) + \sin\left(2\pi nt - \frac{\pi}{4}\right)\right\} \quad . \quad (74)$$

$$\frac{T}{a}\cos 2\pi nt = \frac{T}{a\sqrt{2}}\left\{\sin\left(2\pi nt + \frac{\pi}{4}\right) - \sin\left(2\pi nt - \frac{\pi}{4}\right)\right\} \quad . \quad (75)$$

where n is the speed in revolutions per second and $2\pi nt$ is measured from the line of centres of driving and driven shafts. These forces act on each shaft at its crank-pins, and if b is the transverse distance between them, equations (74) and (75) show that besides balancing the torque of the shaft, the connecting-rods impose on the locomotive structure, through the shaft-bearings, forces resolvable into (1) a central alternating force, in the direction of the connecting rods and of maximum amount $T\sqrt{2}/a$; (2) an alternating couple in the plane of the two shafts, and of maximum moment $Tb/a\sqrt{2}$. The torque transmitted by a side-rod at any instant is:

$$T' = T\sin^2 2\pi nt = \tfrac{1}{2}T(1 - \cos 4\pi nt) \quad . \quad . \quad (76)$$

FORCES DUE TO JACK SHAFT.—A jack shaft is subject to a double set of such forces, one set delivering power and the other receiving it. In fig. 32, if a is the angle between connecting and wheel-coupling rods, the forces acting on the locomotive frame may be resolved into (1) a force

$$T\sqrt{2}\sin\left(2\pi nt + \frac{\pi}{4}\right)\Big/a$$

* See *Minutes of Proc. Inst. C.E.*, vol. 201, p. 231.

acting at the centre of the shaft, in the direction of the coupling rods; (2) a force

$$-\mathrm{T}\sqrt{2}\sin\left(2\pi nt+\frac{\pi}{4}-\alpha\right)\Big/a$$

acting at the same point in the direction of the connecting rods; (3) a couple of moment

$$\mathrm{T}b\sin\left(2\pi nt-\frac{\pi}{4}\right)\Big/a\sqrt{2},$$

having its axis at right angles to the shaft and to the coupling rods; (4) a couple of moment

$$-\mathrm{T}b\sin\left(2\pi nt-\frac{\pi}{4}-\alpha\right)\Big/a\sqrt{2},$$

having its axis at right angles to the shaft and to the connecting rods. The two forces (1) and (2) may be expressed as a horizontal force:

$$\begin{aligned}\mathrm{H} &= \frac{\mathrm{T}\sqrt{2}}{a}\left\{\sin\left(2\pi nt+\frac{\pi}{4}\right)-\cos\alpha\sin\left(2\pi nt+\frac{\pi}{4}-\alpha\right)\right\}\\ &= \frac{\mathrm{T}\sqrt{2}}{a}\sin\alpha\cos\left(2\pi nt+\frac{\pi}{4}-\alpha\right)\\ &= \frac{\mathrm{T}\sqrt{2}}{a}\sin\alpha\cos\left(\alpha-\frac{\pi}{4}-2\pi nt\right) \quad . \quad . \quad . \quad (77)\end{aligned}$$

together with a vertical force:

$$\begin{aligned}\mathrm{V} &= -\frac{\mathrm{T}\sqrt{2}}{a}\sin\alpha\sin\left(2\pi nt+\frac{\pi}{4}-\alpha\right)\\ &= \frac{\mathrm{T}\sqrt{2}}{a}\sin\alpha\sin\left(\alpha-\frac{\pi}{4}-2\pi nt\right). \quad . \quad . \quad . \quad (78)\end{aligned}$$

These forces accordingly have a constant resultant $\mathrm{T}\sqrt{2}\sin\alpha/a$, which rotates about the shaft in the opposite direction to the cranks, with the same speed n. In a similar manner, the two couples (3) and (4) may be expressed as a couple about a horizontal axis, of moment:

$$\begin{aligned}\mathrm{H}_1 &= -\frac{\mathrm{T}b}{a\sqrt{2}}\sin\alpha\sin\left(2\pi nt-\alpha-\frac{\pi}{4}\right)\\ &= \frac{\mathrm{T}b}{a\sqrt{2}}\sin\alpha\cos\left(\alpha-\frac{\pi}{4}-2\pi nt\right) \quad . \quad . \quad . \quad (79)\end{aligned}$$

with a couple about a vertical axis, of moment:

$$V_1 = \frac{Tb}{a\sqrt{2}}\left\{\sin\left(2\pi nt - \frac{\pi}{4}\right) - \cos a \sin\left(2\pi nt - \frac{\pi}{4} - a\right)\right\}$$

$$= \frac{Tb}{a\sqrt{2}} \sin a \sin\left(a - \frac{\pi}{4} - 2\pi nt\right). \quad . \quad . \quad . \quad (80)$$

The couples accordingly have a resultant of constant magnitude $Tb \sin a / a\sqrt{2}$, whose axis lies in the line of the resultant force, thus revolving in the opposite direction to the cranks and at the same rate. It is evident that the jack shaft bearings have to be held stiffly against forces in all radial directions. The most unfavourable value for a is clearly 90°, and in this case the whole torque of the motor is transmitted from one side of the locomotive to the other through the jack shaft four times per revolution.

Example.—The order of magnitude of the forces imposed on the locomotive frame in side-rod driving, will be best appreciated by the consideration of an example. Take for this purpose the Dessau-Bitterfield express locomotives (see page 406; these have vertical connecting rods so that $a = 90°$; the weight of the locomotive is 157,000 lbs., of which 68,500 lbs. is on driving wheels; taking the maximum tractive effort as a third of the weight on driving wheels (and being a single-phase locomotive with rigid transmission of power, this implies an effective tractive effort of something over a sixth of the weight on drivers), its value is 22,800 lbs. The driving wheels have a diameter of 63 inches, whilst the crank-circle has a diameter ($2a$) of 23·6 inches; the horizontal distance, b, between the centres of crank-pins is approximately 80 inches. Thus the revolving force found above has magnitude:

$$\frac{22{,}800 \times 63 \times \sqrt{2}}{23{\cdot}6} = 86{,}000 \text{ lbs.}$$

whilst the moment of the revolving couple is:

$$\frac{86{,}000 \times 80}{2 \times 12} = 287{,}000 \text{ lb.-feet.}$$

this is $b/a\sqrt{2}$, or 4·8 times the maximum torque of the motor.

The above discussion is based on the assumption that the load is divided between the rods on the two sides of the loco-

motive in a certain ideal manner, which is quite unattainable in practice. A cranked shaft of the kind involved possesses four principal regions at which it reacts against outside constraint, namely, its two journals and its two crank-pins (fig. 32). Three of these determine the direction and angular position of the shaft at any instant, the fourth being for the time redundant. Except for certain transition periods, during which the parts are being relieved of elastic strain, the shaft reacts at three of its bearing surfaces only, whilst the fourth is passing through its clearance. Each bearing surface may therefore be said to be active on an average for three-fourths of the time. The exact manner in which the driving is effected accordingly depends on the clearances in the several bearing brasses, on the accuracy of adjustment, on the inertia of the parts, and on the elastic deformation consequent on the forces.

More Detailed Calculation.—It is not practicable to take account of all the influences affecting the problem, but useful information can be gained if the parts are assumed to be rigid and without inertia, whilst the centres of pressure of

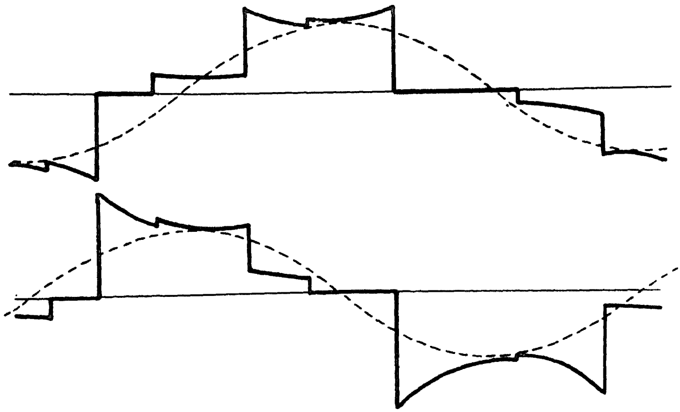


Fig. 33.—Force in Side Rods.

the several bearing surfaces are assumed invariable. With these assumptions, and with any particular clearances and adjustments, it is a simple matter to determine which three of the four bearing surfaces of the shaft are in use at any particular instant, and what forces act on the several parts with a given torque on the shaft. Fig. 33 gives curves of the variation of

force in the side rods determined in this manner for a particular and quite normal case of uniform total torque, whilst fig. 34 shows the variation in the torque impressed by the side rods. In these figures the chain lines show the corresponding curves traced from equations 74, 75 and 76. The elastic yielding of the parts would have a general tendency to mitigate the sharpness of the peaks and the abruptness of the changes and slipping

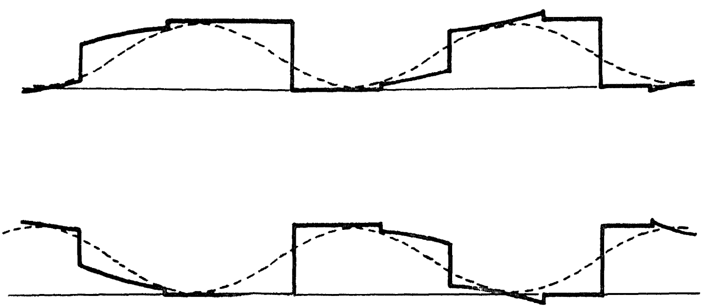


FIG. 34.—Torque produced by Side Rods.

of the wheels would also tend to relieve the structure of the worst shocks. Inertia of parts on the other hand would tend generally to accentuate local stresses. In spite of ameliorating circumstances, however, it may fairly be concluded that the forces acting on the structure, and the stresses in the side rods and crank-pins, have values at times greatly in excess of those indicated by the simpler theory, and that the forces may come upon the parts very suddenly.

Stresses due to Short Rods.—The clearances assumed in figs. 33 and 34 are reasonable and such that actual binding does not occur; but a small change in adjustment might have resulted in binding, with great increase in the peaks of force. In the case of rods between jack shaft and wheels, the rise and fall of the axle-boxes in their ways has the effect of changing the adjustment slightly as the locomotive sways, and in some cases results in very severe stresses on the parts; the cause of this is readily seen from fig. 35, in which A is the centre of the jack shaft, and BC the travel of the axle-box, very much exaggerated by comparison with the centre-spacing. AD represents the length of the rods in the central position of the axle-box, and AB or AC the length in an extreme position, thus DE is the amount by which the length of the rods must

vary in order that they may accommodate themselves to the rise and fall of the axle-boxes. This trouble is clearly the greater the shorter the rods, thus showing that long rods are

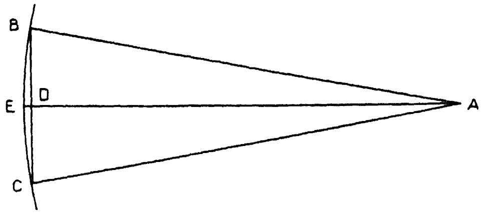


FIG. 35.—Effect of Obliquity of Side Rods.

desirable for this connection; the steam locomotive is free from the defect, the variation in question being taken up by piston clearance.

Fine Clearances Necessary.—It might be supposed that some of the troubles referred to would be avoided by the provision of larger clearances in axle-boxes, bearings and crank-pin brasses. Such a course, however, results in excessive pounding vibration, and, furthermore, by permitting the rods to get out of parallelism, renders locking possible in passing the dead centres, a condition likely to result in fracture or distortion of the side rods and crank-pins, and in any case only relieved by the agency of great stresses. Experience, in fact, shows that a large factor of safety must be allowed in designing the gear for the mechanical transmission of power to the wheels, that fine clearances are necessary to satisfactory operation, and that consequently the wearing parts must be frequently and closely adjusted, and the whole gear kept continually under expert supervision.*

Vibration.—Besides possessing the undesirable characteristics discussed above, the side-rod locomotive is apt, when running at certain speeds, to be set into a state of vibration, which may become violent enough to impair the efficiency of the locomotive and even compass the destruction of parts of the transmission gear. The character of the phenomenon suggests that it is due to synchronism of periodic impulses with some natural period of vibration of the structure, and the cause can easily be surmised. The matter has been discussed,

* See *Electric Railway Journal*, vol. xli, p. 452.

with particular reference to the second Lötschberg locomotives, by J. Buchli,* who attributes the trouble to the indeterminateness discussed above. Buchli makes the significant remarks :

"The Lötschberg locomotives which have been in regular operation since the middle of last year develop at certain speeds very serious vibrations whose cause has for a long time been unexplained. While some of these locomotives run perfectly steadily at all speeds up to 75 km. hr., in others there appear at speeds between 35 and 42 km. hr. vibrations which in several cases have caused serious defects of the driving gear. By careful adjustment of the bearings one is able to remove the trouble. The adjustment requires labour and time, and shortly after the locomotive is put in normal service the trouble occurs again. Similar vibrations to those seen on the Lötschberg locomotives I have also observed on other jack-shaft-driven locomotives. They appear under some circumstances so serious that the locomotive in question cannot be kept in service. The locomotives of the Pennsylvania Railroad appear to possess this same fault, and I do not think I am going too far when I make the statement that every main line electric locomotive built up to this time with side rods and jack shafts shows mechanical vibrations of this character in greater or less degree. It is, therefore, not surprising that the representatives of main line railroad companies are to-day looking at the side-rod drive with some mistrust and that the American construction of drive, the axle motor or the quill-drive motor, is again coming into prominence."

General Calculation of Vibrations.—The force exerted by a side rod, shown generally in fig. 33, may be expressed by a Fouriers series of the form :

$$F' = \frac{T}{a}\Sigma A_k \sin(2\pi knt + a_k) \qquad . \qquad . \qquad (81)$$

where k takes successive integral values from unity upwards. The torque imposed by the side rod takes the form :

$$\begin{aligned} T' &= T \sin 2\pi nt \Sigma A_k \sin(2\pi knt + a_k) \\ &= \tfrac{1}{2}T\Sigma A_k \left\{\cos[2\pi(k-1)nt + a_k] - \cos[2\pi(k+1)nt + a_k]\right\} \qquad (82) \end{aligned}$$

Where the motor is one which exerts a continuous torque, like

* *E.T.Z.*, vol. 35, p. 613.

the continuous current, or the polyphase induction motor, T is constant, and the force exerted by a side rod is made up of periodic components having frequencies n, $2n$, $3n$, etc., represented by the general formula kn, whilst the torque imposed on a crank-shaft is made up of components having frequencies given by the general formula $(k \pm 1)n$. The several components of force and torque cause corresponding vibrations in the structure, and should any of them chance to approximate in frequency with a natural mode of vibration, the amplitude is liable to become excessive. Thus as the speed (n) is increased, the locomotive usually passes through successive states of vibration more or less marked, intervening with quiescence. In this connection it may be noted that the polyphase locomotive is less likely to exhibit susceptibility to vibration than either the continuous current or single-phase locomotive. In the former type of machine the characteristic of the motor is such that the running speed is practically definite whatever the load or gradient; and if precautions are taken that this running speed produce no resonant vibration, any vibratory effect can only be of a transient nature, as the speed passes through a value corresponding to resonance, and this would in all probability not have time to attain destructive amplitude. In the latter types, however, the running speed varies largely with outside conditions, and is accordingly much more likely to include critical values.

Single-phase Motors.—The value of n usually varies from zero to some 6 or 7 revolutions per second, the upper limit depending on the class of locomotive; in most cases, however, the natural frequencies with which resonance is possible are of an order not less than 40 periods per second; thus in the case of locomotives driven by motors of continuous torque, the higher harmonics only of the effort curves are competent to give trouble, and the vibration produced, whilst noticeable, and even at times objectionable, is rarely destructive, since the amplitude of these harmonics is not usually very great. The case is otherwise, however, in locomotives driven by single-phase commutator motors, in which the primary effort is periodic, and may be written in the form (neglecting harmonics), $T = T_1 + T_2 \sin 4\pi ft$, the frequency being twice that of the supply (f). In this case the force in the side rod,

besides components of the form already discussed, arising from T_1, includes components of the form :

$$F_2' = \frac{T_2}{a} \sin 4\pi f t \Sigma A_k \sin (2\pi k n t + a_k)$$

$$= \frac{T_2}{a} \Sigma A_k \{\cos [(4\pi f - 2\pi k n)t - a_k]$$

$$- \cos [(4\pi f + 2\pi k n)t + a_k]\} \quad . \quad . \quad (83)$$

The frequencies of these components may be expressed by the general formula $2f \pm kn$; in a similar manner it can be shown that the torque imposed by the side rod, besides the components already discussed, includes components of frequencies given by the general formula $2f \pm (k \pm 1)n$. The value of f ranges generally from 15 to $16\frac{2}{3}$ cycles per second, and the components whose frequencies are given by the above general formula often come within the region of resonance for quite small values of k and normal running values of n ; such components may easily be comparable in magnitude with the main driving forces, and the vibrations set up by them are apt to be not merely violent, but so destructive as to necessitate the imposition of drastic limits on the speed of the locomotives. Thus, although side-rod transmission offers very decided advantages in connection with the single-phase commutator motor, these are accompanied by such grave disadvantages as render the use of such transmission of doubtful expediency in many cases. A region of excessive vibration may sometimes be avoided by the insertion of an elastic link in the transmission, which reduces the resonant frequency and removes it from the range of ordinary running speeds, that is, in so far as the vibration may be set up by lower harmonics of the effort curve. As clearances change, however, other of the harmonics become effective and vibration more or less violent is sure to be produced at times. Thus in the original form of the second Lötschberg locomotives the vibrations set up were sufficiently violent to fracture the yoke, but the trouble was mitigated and the locomotives rendered workable by the insertion of springs between the gears and auxiliary shafts. The results, however, are not considered entirely satisfactory and apparently no more of the type are to be installed. The single side-rod locomotive made for the New Haven line, in which the motor

armature is mounted on a quill and drives through springs, appears also to have proved unsatisfactory.

The foregoing deals with forces between parts of a locomotive which are connected together in a substantially rigid manner, resulting in vibrations of comparatively high frequency. The locomotive is so designed that the reciprocating driving forces have no components deflecting the spring system, for these could hardly fail to produce violent oscillation of the structure. Vibration under the control of the suspension springs may, however, arise from want of balance in the rotating and reciprocating parts; but, since all driving parts of an electric locomotive move in circles, approximate dynamical balance is generally practicable. In the case of locomotives which have their crank-pin brasses fixed in the rods, perfect balance may be attained. In the Scotch yoke drive, however, the balance is not perfect, since the crank-pin brass of the central wheel works in a vertical slot in the yoke, so that inertia forces due to the yoke are thrown, in the course of the revolution, between this crank-pin and those of the yoked shafts. Dynamical balance could be obtained in this latter case by attaching to the yoke a weight, which should bring its centre of gravity (including with it a half of the attached coupling rods) into the lines of the yoked pin centres and counterbalancing it on the yoked shafts. Vibration due to want of balance is less likely to affect polyphase locomotives than others, since these run at approximately constant speed, and it is principally on such locomotives that the Scotch yoke drive has been employed.

Efficiency of Side-rod Drive.—An effect of the large and irregular forces incidental to side-rod drive, and of the vibrations which they set up, is that the loss of energy in the transmission gear is greater than would be expected from considerations of its simplicity and directness. It is exceedingly difficult to separate the loss in question from the total energy involved, but such tests as have been made certainly indicate greater loss than in direct-geared drive. It has even been claimed, as a result of continental tests, that the efficiency of side-rod drive is at best no more than 90 per cent.; but it is probably expedient to take this figure with reserve, as the tests are necessarily very difficult and it is not easy to see

where so much wasted energy could be dissipated without immediate detriment to the parts.

SELECTION OF LOCOMOTIVES

The foregoing discussion shows that the nature of the service and the system of operation exercise chief influence in determining the general type of locomotive most suitable for use on a given railway. The question of the provision of auxiliary trucks is in some cases determined by the system of working, as affecting the weight of the parts. In the continuous-current system the natural weight of the equipped locomotive is seldom greater than can be conveniently carried on driving axles, and the additional weight needed for auxiliary trucks has to be provided in the form of ballast. In the single-phase system, on the other hand, there is usually an excess of weight to be provided for, and locomotives operating on this system are accordingly more frequently fitted with auxiliary trucks.

WEIGHT ON AXLES AND TRACTIVE EFFORT.—A limit is always imposed on the designer in respect of the total weight borne by each axle, and this frequently affects his design by imposing a limit on the attainable tractive effort per axle. On a bad rail, the maximum value of this tractive effort may not exceed 16 or 18 per cent. of the pressure on the driving wheels; and in the case of multiple unit trains, in which it is not convenient to sand the rails ahead of every driving wheel, it is usual to arrange, if possible, that the maximum tractive effort required does not exceed the lower of the above figures. In locomotives, on the other hand, where sanding gear can readily be used, it is considered conservative to provide for a maximum tractive effort of only 20 per cent. of the weight on driving wheels, and it is quite common to allow 25 per cent. as a limiting service condition. Actually the tractive effort required to slip the wheels on a good rail may amount to as much as 35 per cent. of the weight carried, and although this cannot be reckoned upon as a service condition it is necessary in general to provide that no damage may ensue if the wheels are slipped under these abnormal conditions. The limit of satisfactory operation for the motors should therefore be beyond this slipping point, whilst the mechanical strength of all parts involved in driving should be computed to provide for the stresses thus set up.

Slipping Wheels.—In locomotives driven by impulsive torque, which transmit the forces entirely through rigid links, the limiting value of the average tractive effort is naturally smaller than in machines driven by continuous torque, since the maximum permissible, as determined by the slipping, is no greater. Where the draw-bar pull may approach a limiting value, therefore, locomotives of the former class require greater weight on the driving axles than those of the latter class. The slipping, when it is first started under impulsive torque, is apt to be markedly different in character from that due to continuous torque, for whereas in the latter the wheels, having once lost grip of the rails, revolve rapidly, in the former they may, as it were, renew grip every half period and thus crawl round. It may be mentioned that the tractive effort when the wheels are slipping rapidly is only some 8 or 10 per cent. of the weight carried, so that when the grip of the rails has been lost, considerable reduction in motor torque must take place before it can be restored.

Capacity of Locomotive.—The capacity of a locomotive, as measured by the train it can haul, depends not only on the weight of the locomotive and the dynamical characteristics of its motors, but also on the nature of the service and the character of the route, particularly as regards disposition and steepness of gradients. It is the business of the designer of the locomotive to examine all the conditions of its operation, in so far as they can be foreseen, with a view to determining which are likely to constitute the greatest tax on its capacity. He may, as the result of his survey, be able to indicate more suitable methods of handling the traffic, assimilating the requirements of the several classes of service or sections of the route; for just as economical methods of operating steam traffic are ultimately governed by the limitations of the steam locomotive, so the methods of electrical operation should conform naturally with the means employed.

Many locomotives, particularly those intended for goods traffic, are required on emergency to exert all the tractive effort that their adhesion permits, being therefore limited only by the track condition and weight on driving wheels. The maximum train hauled under such circumstances is determined by the ruling gradient, it being presupposed that the limiting draw-bar pull allows a satisfactory factor of safety for the

draw-gear. The couplings used on British goods wagons, composed as they are of $1\frac{1}{2}$ inch wrought-iron chain, have an ultimate strength of the order of 60 or 70 tons. When they are broken it is usually by the surging of the train, but since the consequence of breakage may be serious, it is not advisable to allow the steady pull to exceed some 15 tons, even under the worst conditions, when starting a heavy train on a steep gradient. A locomotive having weight on driving wheels of the order of 70 tons, capable of exerting an adhesive tractive effort in service of $17\frac{1}{2}$ tons or more, is quite as heavy as it is advisable to employ at the head of a train in this country, and as the permissible weight per axle is generally at least 18 tons, the British locomotive needs no more than four driving axles. Much stronger draw-gear is used in some countries ; in America, for instance. The Chicago Milwaukee and St. Paul Railway is able to employ locomotives at the head of trains having eight driving axles, bearing between them a weight of 180 tons, whilst the locomotives used for the Elkhorn gradient electrification of the Norfolk and Western Railway have eight driving axles carrying a total weight of 200 tons.

Where the haul is long and continuous the heating of the motors rather than the ruling gradient is likely to set the limit of the capacity of a locomotive. The limit in this case depends on the average gradient, and where the route is undulating particular gradients usually have but little effect. Where, however, long continuous gradients have to be surmounted, the additional heating due to this cause should be taken into account. In most modern locomotives intended for this class of work the motors are cooled by forced draught, and the heating is thus to some extent under control. It is, however, neither usual nor necessary to provide a motor equipment capable of giving the maximum tractive effort continuously. The capability of a locomotive, measured by the maximum weight of train that it can haul, is therefore dependent both on the nature of the service and on the character of the route, being in some sections limited by the ruling gradient, and in others by the average gradient. With the former limitation the weight on driving wheels governs the capacity, whilst with the latter, the length of haul and time of layover enter into the question, as well as the nature of the motor equipment.

PASSENGER LOCOMOTIVES.—In passenger service the maxi-

mum tractive effort is exerted during the initial acceleration, and this is usually several times as great as would suffice to carry the train against the ruling gradient. The weight on driving wheels is accordingly determined so as to provide a suitable starting effort for normal trains, but the limit of capacity of a given locomotive, although affected somewhat by the character and disposition of gradients, is in the main governed by the nature of the service as regards schedule speed, frequency of stations, and duration of station stops and layovers, these being the chief factors which affect the heating of the motors. The estimation of the heating and the determination of the most suitable driving equipment to fulfil the conditions imposed by the nature of the road and traffic will be discussed in later chapters.

Shunting Locomotives.—Locomotives intended solely for shunting work are of comparatively small power, having regard to their weight, since the speed is low. In such locomotives the capacity is limited only by the adhesive weight, it being presupposed that the motor equipment is dynamically capable of furnishing the maximum tractive effort which the weight permits. The heating of the motors is rarely of much consequence in these locomotives, since the power is small, the work intermittent and the dissipating surfaces of the motors large.

CHAPTER III

RAILWAY MOTORS

The design of railway motors does not differ in principle from that of stationary power motors ; but their construction is so largely affected by the limitations of space, by the heavy intermittent duty usually required of them, by the continual vibration to which they are subject in service, by the difficulties of getting rid of the heat developed without exposing them to deleterious action of the elements, and by the necessity they are under of working for long periods without adjustment or attention, that they furnish a problem of quite a special nature, often taxing the designer's skill to the utmost. It is, however, not the author's intention to enter in detail into the subject of designing railway motors, but rather to discuss it in general, with particular reference to the peculiar conditions under which the motors have to operate.

The Continuous Current Series Motor.—The railway motor takes a number of forms according to circumstances, but the commonest and most highly developed of these is the continuous current series motor, adapted to be carried between an axle and transom of the locomotive truck, and driving the axle through single reduction gearing. A description of a typical motor of this form follows, as illustrative of the manner in which good modern practice meets the requirements of service. Figs. 36 to 43 give views of motors of the type in question. Fig. 36 is an exploded view of a motor, with lettering for identification, to which reference is made in the following description. Figs. 37 and 38 give longitudinal and transverse sections of a motor of the totally enclosed or grid ventilated type, having radial ventilating ducts. Figs. 39 and 40 give longitudinal and transverse sections of a fan-

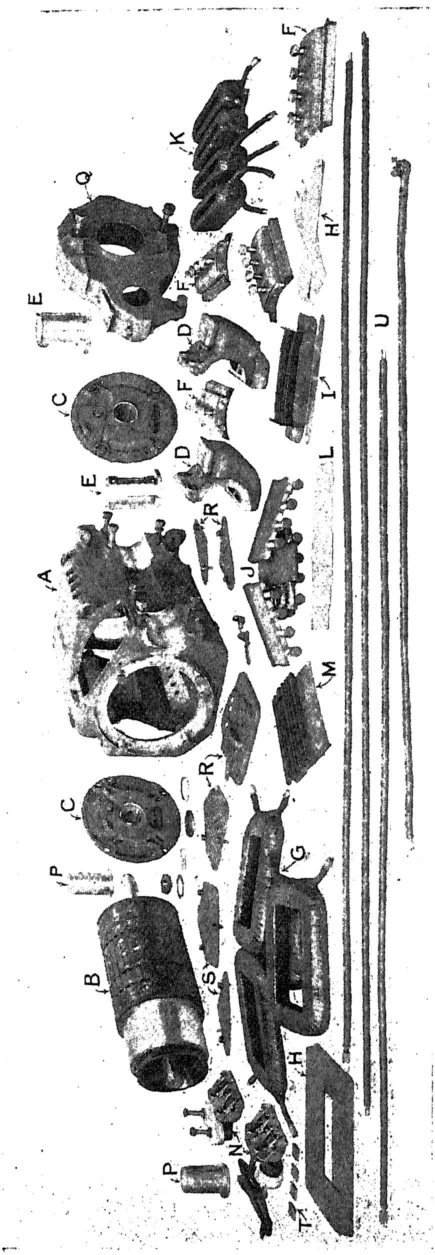


FIG. 36.—Exploded View of Railway Motor.

A. Frame; B. Armature; C. Frame Heads; D. Axle-Bearing Caps; E. Axle-Bearing Linings; F. Exciting Pole Pieces; G. Exciting Field Coils; H. Insulating Pads; I. Spring Flanges; J. Commutating Pole Pieces; K. Commutating Field Coils; L. Insulating Pads; M. Spring Flanges; N. Brush Holders; P. Armature-Bearing Linings; Q. Gear-Case; R. Grid Covers; S. Solid Covers; T. Brushes; U. Leads.

ventilated motor in which the air is drawn in one stream by a series path through the motor. Fig. 41 is a longitudinal section of a fan-ventilated motor, in which the air is drawn in two streams by multiple paths. Figs. 42 and 43 give longitudinal and transverse sections of a motor cooled by forced draught from an outside blower.

THE MECHANICAL STRUCTURE.—The frame, A, of the motor, which also serves as the magnetic yoke, is of cast steel, shaped roughly in the form of a rectangular prism, having well rounded corners and large bored openings, one at each end, into which frame-heads, C, carrying the armature shaft bearing linings, are bolted. Through the frame head opening at the pinion end the armature, B, can be put in place, or withdrawn from the frame. The motor is supported from the axle by means of axle-bearing brackets and secured by means of axle bearing caps, D, which hold the split axle-bearing linings, E, and also contain the oil wells for these bearings. It is supported from the transom-bracket by means of a lug cast in the frame, often fitted with a wearing-plate.

THE MAGNETIC CIRCUITS. The motor is of the four pole class, for this in general proves the most economical, and in fact often the only practicable design, having regard to space limitations. The pole pieces, F and J, are bolted to suitable seats in the frame by means of through bolts. The exciting pole-pieces, F, are built up of laminations chamfered alternately at the tips, which are therefore quickly saturated and so cause a stiff field to be maintained in the commutating zone. The exciting field coils, G, are of ribbon-wound copper strip, insulated internally by means of asbestos, and filled with a heavy varnish or impregnated in vacuo with a bitumen compound. They are held in the motor against insulating pads, H, which rest on machined faces, and are kept in place by means of spring flanges, I; for, with the vibration of service, rigid clamps would eventually cut through the insulating covering of the coil and ground the winding. The exciting field coils are often tapped in one or more places in order to give higher speed by reduction of excitation. The commutating pole pieces, J, are drop forged. The commutating field coils, K, are generally of edgewise-wound copper strip insulated with asbestos, filled or impregnated like the exciting coils, and supported between insulating pads, L, and spring flanges, M.

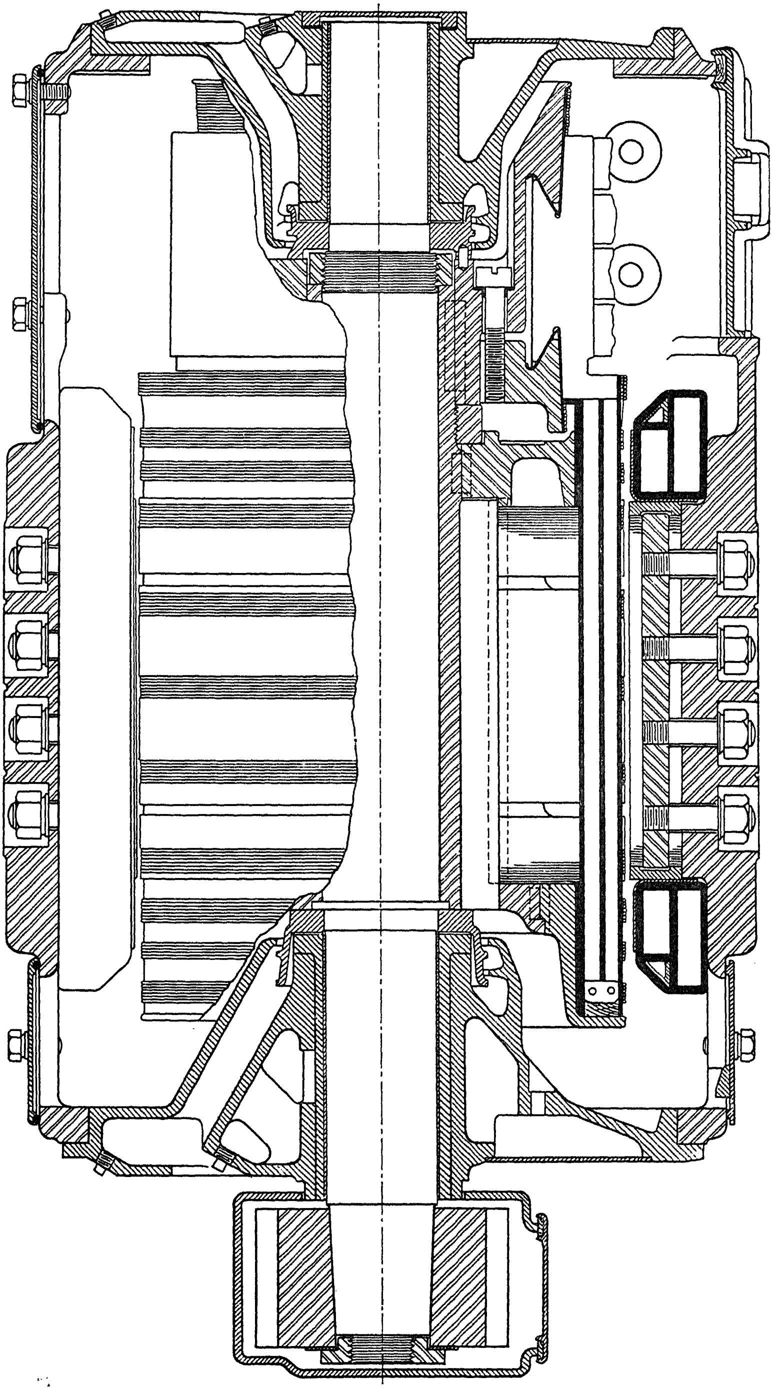


FIG. 37.—Longitudinal Section of Enclosed Railway Motor.

The exciting poles are located on the four sides and the commutating poles at the angles of the frame (see fig. 38).

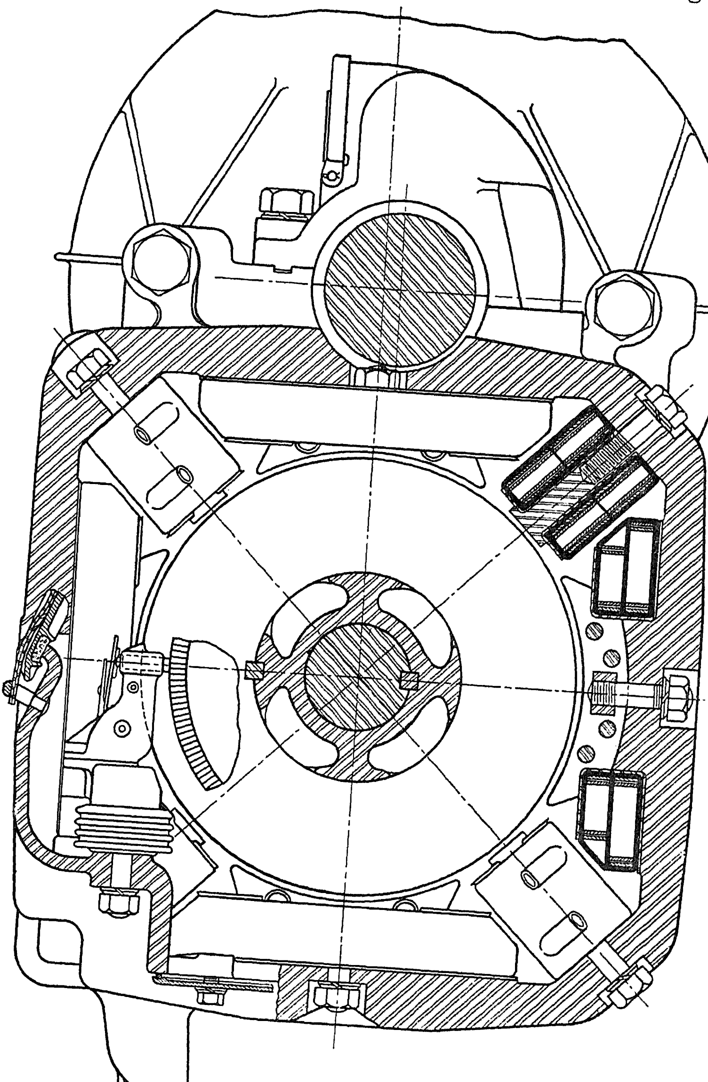


FIG. 38.—Transverse Section of Enclosed Railway Motor.

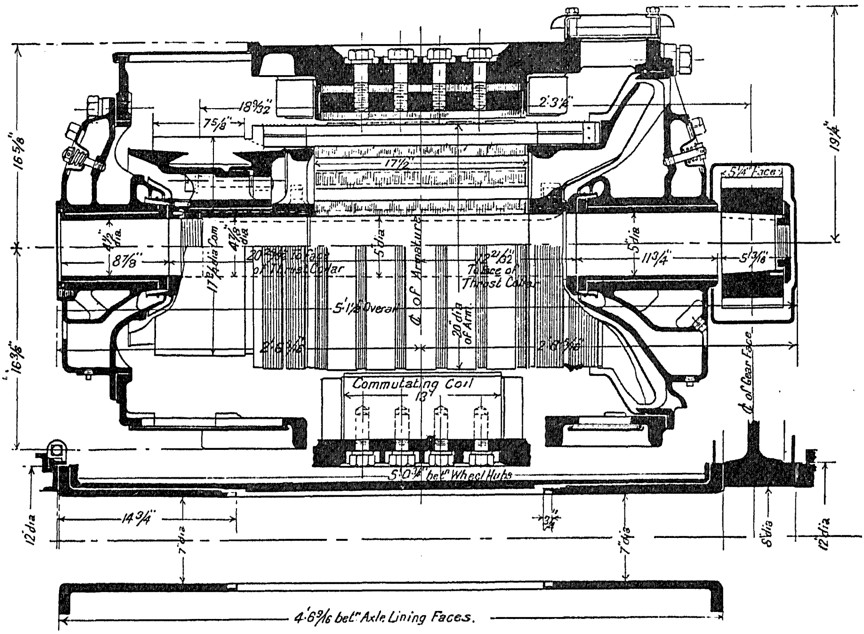
THE ARMATURE.—The armature, B, is of the series or wave-wound drum type. This type of winding makes efficient use of the slot-space, and moreover renders it practicable to lead

in the current by two sets of brushes only, which can be arranged conveniently for inspection and adjustment when the commutator cover is raised. The latter is a desirable provision when it can be attained, inasmuch as the commutator and brush gear comprise perhaps the most delicate parts of the motor; for brush holders located out of sight below the commutator may be neglected, and possibly come into contact with and damage the commutator when the bearings are worn unduly or the babbitt melted out. However, in some cases the space limitation compels the use of four brush holders (see fig. 43). In powerful motors the armature is usually bar-wound, where this design is practicable; and the pitch of the winding is then usually made fractional with respect to the slot pitch, in order to secure the most favourable conditions for commutation, although this construction necessitates the use of riveted and soldered, or welded, end-connections. The bars are split and turned over in the slot portion, in order to diminish eddy currents in them. The conductors are separately insulated with mica and assembled in sets to suit the slots. The set is further insulated with mica along the slot portion, and the whole compound bar is given a protective covering of tape filled with an impervious varnish. The pinion-end armature-head is extended to form a rigid seat for the end-connections, whilst the commutator-end armature-head fills the whole space between core and commutator, with the same object. The ends of the coils are bound down solidly to the seats so formed, over a good insulation of moulded mica. This is an important detail rendered necessary by the excessive vibration to which the motor is subjected in service. The binding bands are of tinned steel wire, some being wound in recesses in the core and others over the end-connections. The conductors are soldered directly into ears forming part of the commutator segments, thus avoiding the use of connecting leads.

THE COMMUTATOR.—The commutator is composed of hard drawn copper segments, insulated throughout with mica. The segment mica is about ·030″ thick and is kept recessed for about the same distance from the commutator face so that the brushes bear on copper only. The thick cone micas are built up and pressed solid in steam moulds. The commutator shell and cap are of cast steel and are pressed together hydraulically

before the bolts are tightened, for it is very important that the segments should be tightly clamped.

THE BRUSH GEAR.—The brush holders, N, are of cast bronze, attached rigidly to the motor frame by means of steel studs and insulated from it by a primary insulation of compressed mica and a surface insulation of porcelain. The holders have a radial adjustment only, being located so that the brushes are set at the geometrical neutral point. The brushes, T, slide in finished ways, being pressed radially against



"THE ENGINEER"

FIG. 39.—Longitudinal Section of Ventilated Railway Motor, Series Ventilation.

the commutator by independent fingers which give sensibly uniform pressure throughout the working range of the brushes. On account of the great vibration which the motor has to stand in service, the brush pressure necessary is considerably greater than is customary for stationary motors, being usually in the neighbourhood of 5 lbs. per square inch. The need of this pressure is not manifest from tests made in the shops, and satisfactory operation on the testing stand with smaller pressure should not be accepted as implying satisfactory operation in

service. The brushes, moreover, should not be accepted on the sole evidence of shop tests, for a very homogeneous, close and tough brush is required to stand the pressure and vibration of service without chipping or splitting, although an inferior brush may be found quite satisfactory in shop tests. It is not good economy to use a poor brush for this service, however low the price. The brushes, however, which have shown themselves suitable for the work have a surprisingly long life on modern commutating pole continuous-current motors, the service running into hundreds of thousands of miles. The commutator wear also is exceedingly small in this class of

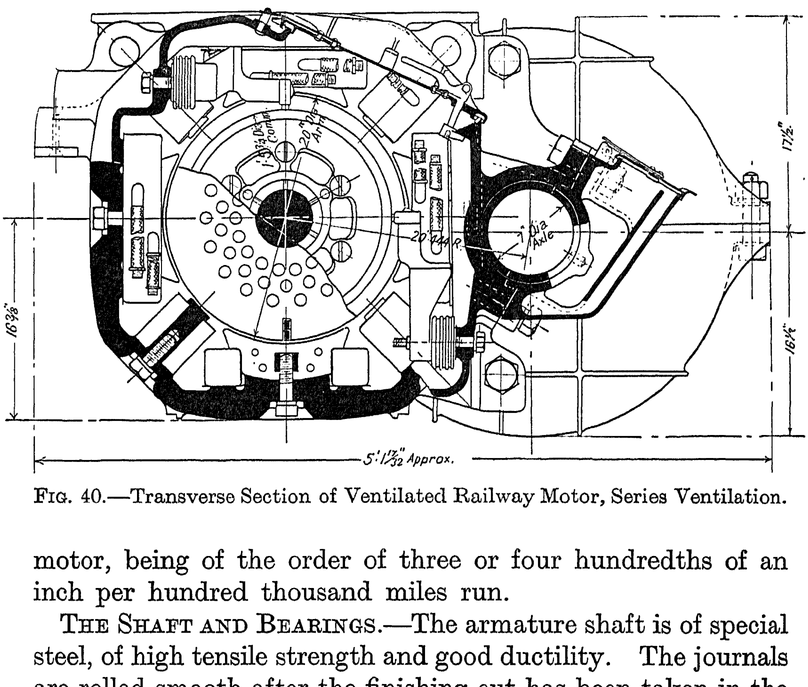


FIG. 40.—Transverse Section of Ventilated Railway Motor, Series Ventilation.

motor, being of the order of three or four hundredths of an inch per hundred thousand miles run.

THE SHAFT AND BEARINGS.—The armature shaft is of special steel, of high tensile strength and good ductility. The journals are rolled smooth after the finishing cut has been taken in the lathe. The bearing linings, P, are housed in the frame heads, C, which are of malleable iron, cast in one piece, shaped on the inside roughly in the form of truncated cones. The conical portion extends for some distance within the armature head at the pinion end of the motor, and well inside the commutator shell at the commutator end. The linings, P, are babbitted to such thickness that if the babbitt should be

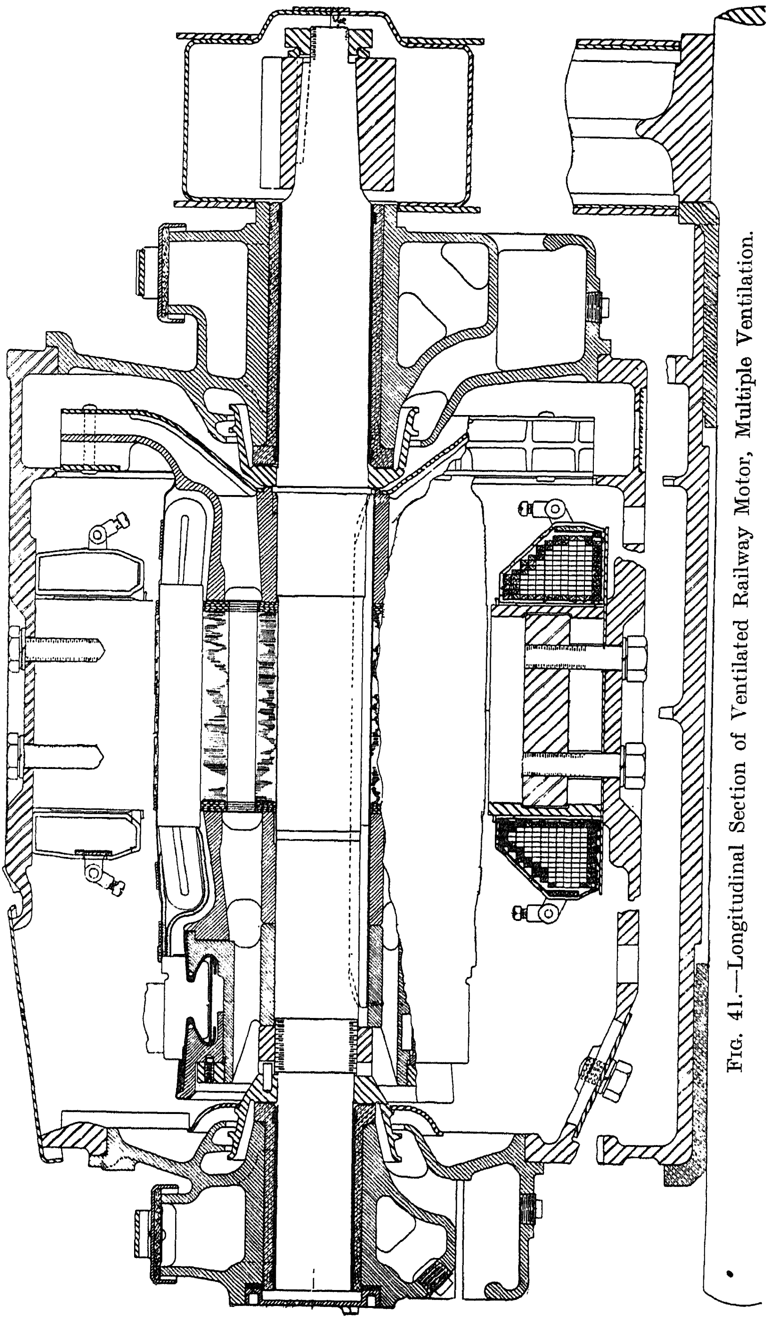


FIG. 41.—Longitudinal Section of Ventilated Railway Motor, Multiple Ventilation.

melted out, the shaft is supported on the bronze sleeves before the armature touches the poles. The pinion-end bearing, being subject to much greater pressure than that at the commutator-end, is made greater, both in length and diameter, its area usually exceeding that at the commutator-end by at least 50 per cent.

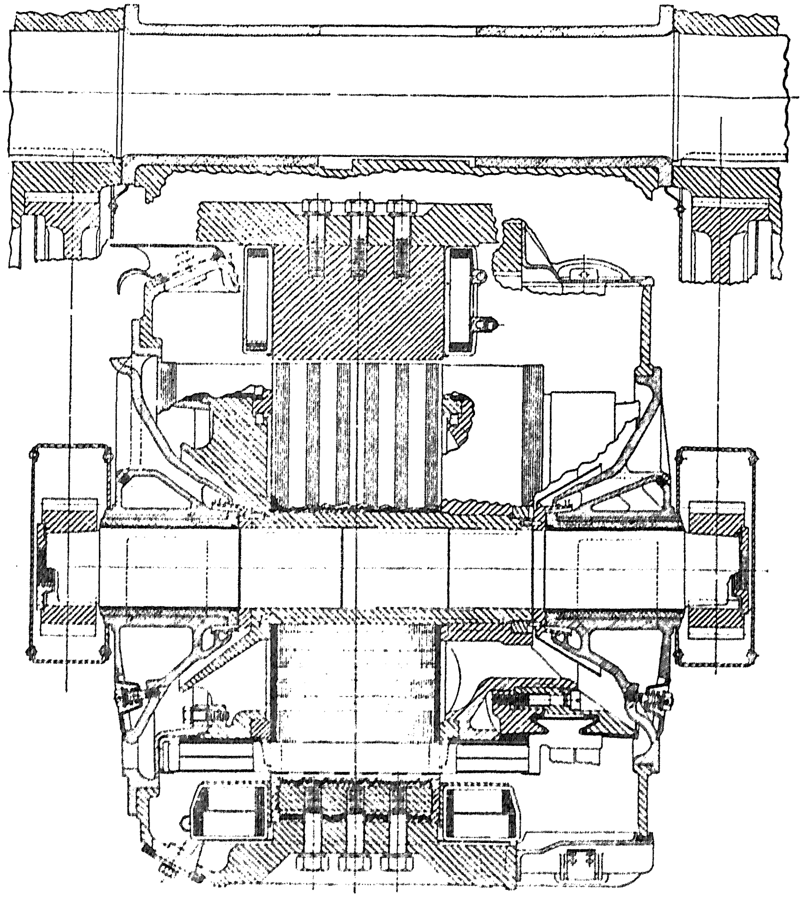


FIG. 42.—Longitudinal Section of Motor Designed for Forced Ventilation.

LUBRICATION.—The best practice favours oil and wool waste lubrication. The frame heads have large oil wells into which the waste is packed, and an opening is made in the bearing lining on the low pressure side, where the wool, which is specially selected and in long strands, is held in contact with the journal. Oil ring lubrication has been used to some extent,

and with suitable oil and frequent inspection it may be satisfactory; but it is sometimes necessary in this case to change the oil according to the temperature of the air, particularly in places where seasonal variations are great. The chief objection to oil ring lubrication, however, is that the dust and grit picked up on the road are sure to get into the oil eventually, and unless the covers of the oil wells and details of the bearing ends are carefully watched, a considerable amount may collect there. With oil ring lubrication this grit is fed on the journals, much to their detriment; but with oil and waste lubrication, which feeds by capillarity, it is strained out and the filtered oil alone reaches the journals. Modern motors are lubricated every ten or twenty days according to the service and the life of linings is from 50,000 to 200,000 miles. Improvements in material and perfection in manufacture as well as in design have contributed to these results. The axle bearing linings are necessarily split and are usually of bronze without babbitt. Oil and waste lubrication is employed for these bearings also and suitable oil wells are located in the axle bearing caps. A housing of sheet steel is now commonly used to cover up the axle between the bearings, in order to exclude dust.

GEARING.—The gearing used with railway motors has been brought to a very high state of perfection as regards the homogeneity and toughness of material, and the accuracy of workmanship. The steel used is as carefully selected as the best tool steel, which it resembles in some of its properties, and the heat treatment it receives is calculated to secure uniformity of structure with hard wearing surface and very high elastic limit. In the best quality forged gears and pinions, the metal is elastic almost up to the breaking stress of 140,000 to 150,000 lbs. per square inch. The pinions are invariably of forged steel, and are sometimes case hardened and sometimes of uniform temper, according to the nature of steel employed. They are usually applied to the taper fit on the shaft after having been heated by immersion in boiling water; for greater heating than this would, in the case of some steels, injure the temper. The gears are either forged or of case-hardened cast steel, the former being preferable for heavy service. Sometimes a forged steel rim pressed on to a cast steel centre is used. The gears are pressed on to the special seat on the driving axle before the driving wheel is fitted, split gears being practically superseded

by solid for this class of work. The best quality modern gears generally have a useful life of the order of 300,000 miles, whilst the pinions last about 100,000 miles, the wear of the members being approximately in the ratio of gear reduction. Gear-teeth should be exceedingly strong for railway service, as a broken tooth frequently leads to a bent armature shaft. In the best modern practice the diametral pitch is never made greater than $2\frac{1}{2}$ (1·257 inch circular pitch), but it is becoming common to use pitches of $2\frac{1}{4}$ or even 2 for heavy suburban service, whilst a pitch of 1·75 (1·795 inch circular pitch) is sometimes used in powerful locomotives. Teeth of involute form are used, having generally an angle of approach of $14\frac{1}{2}$ degrees, but sometimes of 20 degrees. Steel teeth are occasionally employed. The gearing is enclosed in a split gear case, Q, rigidly supported on the motor frame. Gear cases have in the past generally been made of malleable cast iron, but lately pressed steel, which has the advantage of lesser weight, has been employed to some extent.

VENTILATION.—The means adopted for getting rid of the heat developed in the motors are of the utmost importance. Before the service required of motors was as severe as is usually the case at the present time, it was considered preferable to have the motors completely enclosed in order that all road dust might be excluded; and the armature core was then provided with radial ventilating ducts so as to aid in distributing the heat (see figs. 37 and 38). Since in this case the heat is carried away through the frame, it is impossible to deal with more than a very moderate amount, a quantity usually of the order of 2 to 2·5 kilowatts. As the severity of services increased, grid covers were put over some of the openings in the frame, such as are shown in fig. 36 at R, by which a certain interchange of air between the inside and outside of the motor could take place, resulting in greater dissipation of heat. In more recent motors a definite circulation of air from outside and through the structure is provided by means of a fan, either external to the motor and separately driven or internal and attached to the pinion-end armature head. With forced ventilation there are usually no radial ducts, and the circulation of air is so arranged that any dust that may be drawn into the motor passes only over smooth surfaces, whilst the liberal passages provided for the air give little opportunity for the dust to

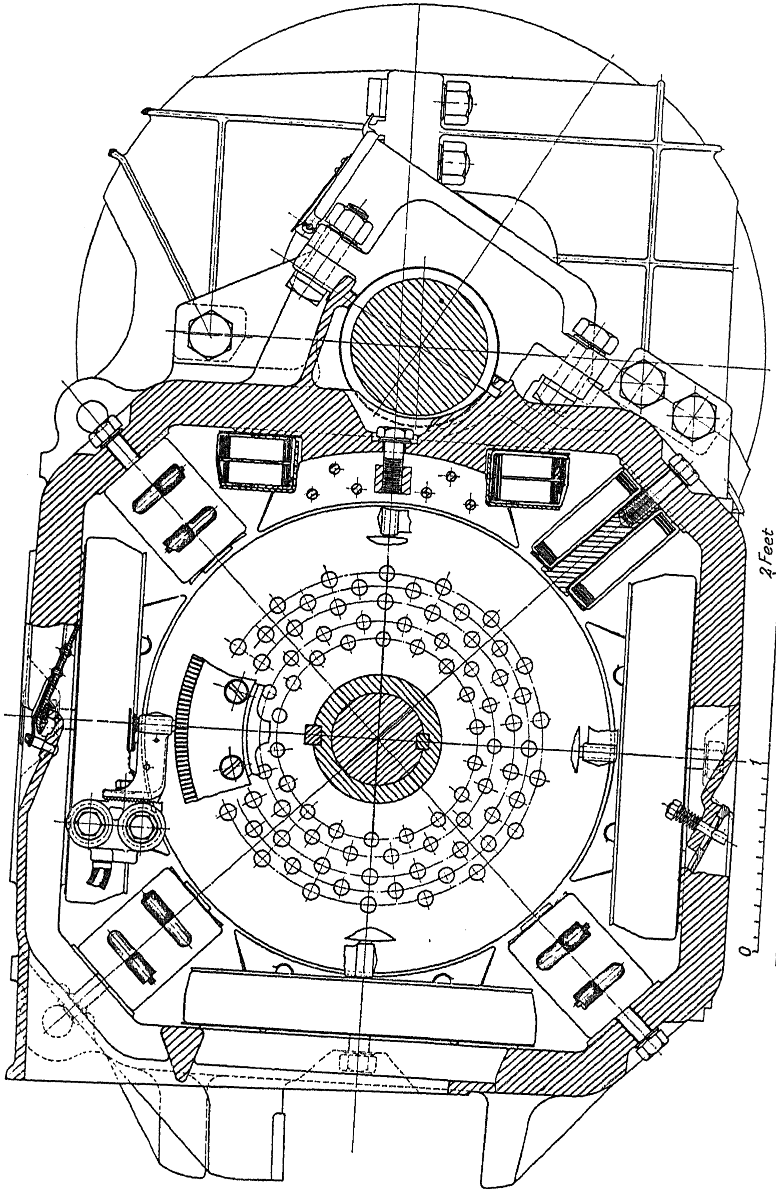


Fig. 43.—Transverse Section of Motor Designed for Forced Ventilation.

settle inside the motor. The air-conduits of the armature are longitudinal, consisting of cylindrical tunnels, arranged in concentric circles, and passing completely through core, armature heads and commutator shell. Three forms of self-ventilated motor may be distinguished. In a series-ventilated motor (figs. 39 and 40), the air is taken in at the pinion-end, passes over the field coils, armature surface, and commutator, then through the armature, to be expelled at the pinion-end. In a multiple-ventilated motor (fig. 41), the intake is at the commutator-end and the stream of air divides, passing in multiple over the field coils and through the armature. In a mixed or series-multiple ventilated motor there are intakes at both ends of the motor, some of the air passing over the field coils and armature face and the whole passing through the armature. In each case the air is expelled through openings in the pinion-end frame head. When such motors were first introduced fears were entertained that they would deteriorate rapidly through the admission of water and other foreign matter with the air. The fears, however, have proved groundless, matter entering the motor being ejected with the air. The self-ventilated type is preferable to that having an external blower where it is capable of dissipating the heat; and fortunately, with continuous current motors, there has never been found need to use the external blower in motor coach operation, where its use is particularly undesirable.

Dynamical Characteristics.—The dynamical capacity of a railway motor is best represented by its characteristic speed, tractive effort, and efficiency curves. These for typical modern motors are shown in figs. 44 and 45. The speed is that of the train, and the tractive-effort that at the rims of the driving wheels. The efficiency and speed curves are drawn to correspond with a motor temperature of 75° C., this being approximately the average working temperature of the motors. The characteristic curves are made up for definite voltage, gear reduction, and size of wheel. If they are required for other gear reduction or size of wheel the correct curves are easily deduced, inasmuch as the train speed varies directly as the diameter of the driving wheel and inversely as the ratio of gear reduction; whilst the tractive effort varies directly as the ratio of gear reduction and inversely as the diameter of the wheel, the

efficiency remaining sensibly unchanged with change of gears. The correction for change in voltage is but little more complicated; for the tractive effort at any current is practically independent of the voltage, whilst the speed varies as the counter-electromotive force. The resistance of the motor winding being known, the drop of voltage in it at any current is immediately deduced; and that in the brush contact may be obtained from appropriate special tests, but for most

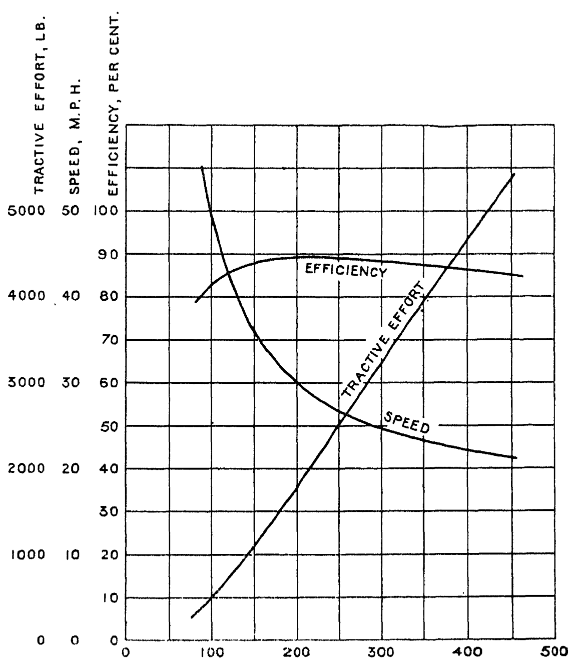


FIG. 44.—Characteristic Curves of G.E. 212 Railway Motor. 63/20 gear, 40 in. wheels, 600 volts.

purposes may be assumed independent of the current at about 2 volts. The difference between the motor terminal voltage and the total resistance drop gives the counter-electromotive force. Thus the motor of fig. 44 has a winding resistance of approximately ·095 ohms; and at 200 amps. and 600 volts the train speed is seen to be 30·0 m.p.h. Accordingly at the same current but at 300 volts the train speed will be diminished in the ratio of 600 − 19 − 2 to 300 − 19 − 2 or of 579 to 279, its value accordingly being 14·5 m.p.h.

WEAR OF DRIVING WHEELS.—The effect of the wearing of the driving wheels is a matter of some interest. Fig. 46 shows train speed and tractive effort curves for motors having the characteristics of fig. 44, but fitted with wheels of $41\frac{1}{2}$ and $38\frac{1}{2}$ inches diameter respectively; if now both sizes of driving wheels coexist in the same train it will be seen that the motor on the larger wheels continually takes greater current than the other. Thus at 30 m.p.h. the motor with $41\frac{1}{2}$-inch driving wheels takes 214 amps and that with $38\frac{1}{2}$-inch wheels 190 amps, the corresponding tractive efforts being 1,910 and 1,720 lbs. respectively. With equal numbers of the two sizes of

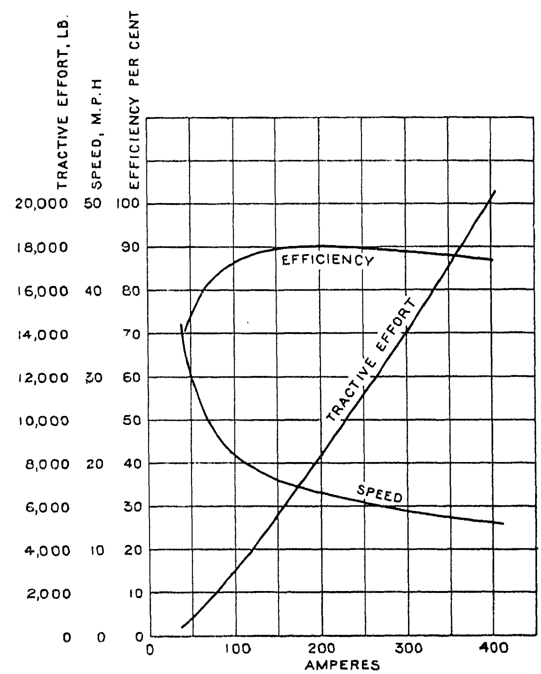


FIG. 45.—Characteristic Curves of G.E. 253 Locomotive Motor. 82/18 gear, 52 in. wheels, 1,500 volts.

driving wheel on the train, the mean tractive effort per motor is 1,815 lbs. at this speed. The motors on the larger driving wheels naturally run somewhat hotter than those on the smaller, as they do more than their share of the work, being most unfavourably circumstanced in this respect when one truck has new wheels and all other driving trucks in the train

worn wheels. This point should be kept in mind when the normal heating is high. The effect of running driving wheels of different sizes in a train is thus quite appreciable, although rarely objectionable in ordinary operation.

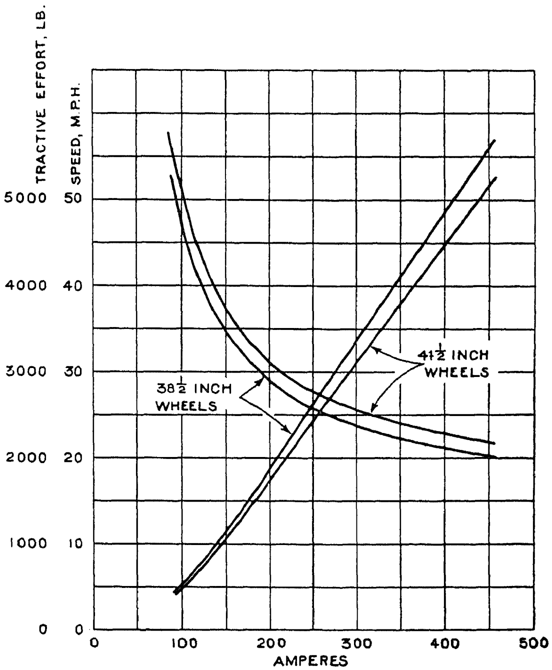


FIG. 46.—Effect on Characteristics of Wear of Wheels.

UNSUITABILITY OF SHUNT MOTOR.—The conclusions arrived at in the last paragraph would have been quite different if a fixed field motor, such as a shunt motor, were under consideration. The speed curve of a shunt motor at constant voltage is given in fig. 47 with $41\frac{1}{2}$-inch and $38\frac{1}{2}$-inch wheels respectively, and it will be seen that if the two sizes of driving wheels coexisted in a train running at 30 m.p.h., the motor on the $41\frac{1}{2}$-inch wheels would take about six times the current of the other; whilst at speeds in excess of $30\frac{1}{2}$ m.p.h. the motor on the smaller wheels would actually act as a brake on the train. This is one of the reasons that render such motors inapplicable to the needs of railway work. Another reason is that any sudden fluctuation in voltage cannot as suddenly vary the

exciting field of the motor. This results in a violent change in armature current, which is likely to slip the wheels or cause the motor to flash over at the commutator before the field and train speed can adjust themselves to the changed voltage. In the series motor the change in armature current cannot take place without a corresponding change in the field strength, and this circumstance limits the fluctuations to a moderate amount. The effect of sudden change of voltage on a series motor can readily be seen by computing the speed curves for the two limiting voltages. These will resemble the speed curves of fig. 46, a single tractive effort curve being, however, common to the two. Taking for example the upper of the two tractive effort curves in this figure as representative of the motor, and taking

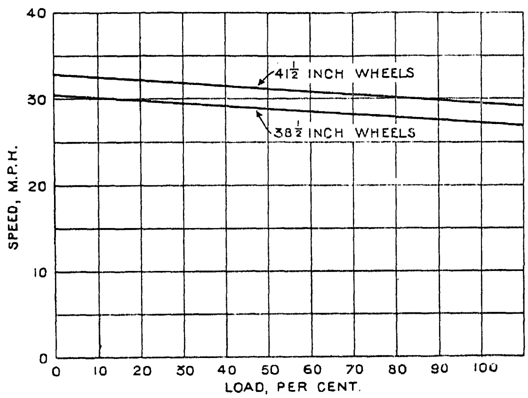


FIG. 47.—Speed Curves of Shunt Motor.

the two speed curves as representing these before and after a rise in voltage, it will be seen that if the train is running at 30 m.p.h. there is a rise in current per motor from 190 amperes to 214 amperes, the tractive effort per motor rising at the same time from 1,720 lbs. to 2,080 lbs. Considerable and rapid variation in voltage is unavoidable in some parts of a railway system, and in fact the power supply to the train may at times be entirely interrupted and restored after the lapse of a short period. Shunt motors could not be operated successfully under such conditions.

Determination of Dynamical Characteristics.—The dynamical characteristics of a motor even when derived

entirely from test are, in certain respects, put on a conventional basis for the sake of definiteness in the results. The speed of the motor at any voltage and current is readily observed; but it varies a little with temperature and should be taken when the motor is warm and then, if necessary, corrected to correspond with a definite temperature, which is usually taken as 75° C. The tractive effort is deduced from the efficiency, but considerable care is required to obtain a thoroughly representative efficiency curve. The most obvious method of obtaining efficiency is to measure the input at the terminals electrically and the output at the axle mechanically, the latter by the means of a Prony brake. This however is a difficult test to make on a motor of large power; and, unless great care and skill are available and the mean of many observations taken, the results are likely to be disappointing. The value of the result, moreover, does not warrant so difficult a method; for the gear losses are affected by the state of the gears, whether new, worn smooth, or badly worn, whether binding anywhere or running with sufficient clearance, whether well lubricated or otherwise. Inasmuch as these conditions have no reference to the motor itself, although it is practically necessary to include gears in the presentation of its characteristics, it is not expedient to adopt a difficult method of test where a simpler one can be found to serve the purpose, even if this depends on principles less easily justified. The usual method of determining efficiency is by a form of Hopkinson test. For this purpose a testing stand is used comprising an axle running in three bearings and fitted with equal gears on either side of the centre bearing; the motors, two in number, ride on this axle exactly as they would on car axles, their noses being supported by suitable brackets bolted to the base. Views of the arrangement are shown in figs. 48 and 49.

METHODS OF TESTING.—In making an efficiency test, the motors are connected as shown diagrammatically in fig. 50; one motor, G, is operated as a generator, being excited in series with the other motor, M, which operates as the motor. The generator armature is connected in series with a load booster B, between the line terminals, the degree of excitation of the booster determining the load. A second booster B_1, which may be designated the line-booster, serves to adjust the voltage between motor terminals to the required value.

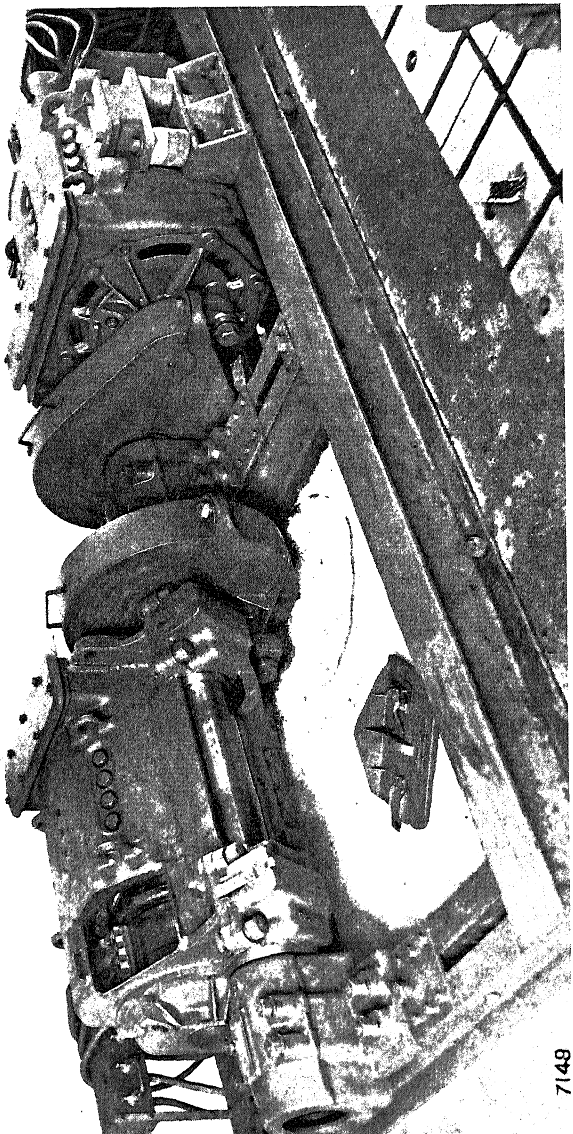


FIG. 48.—Railway Motor Testing Stand.

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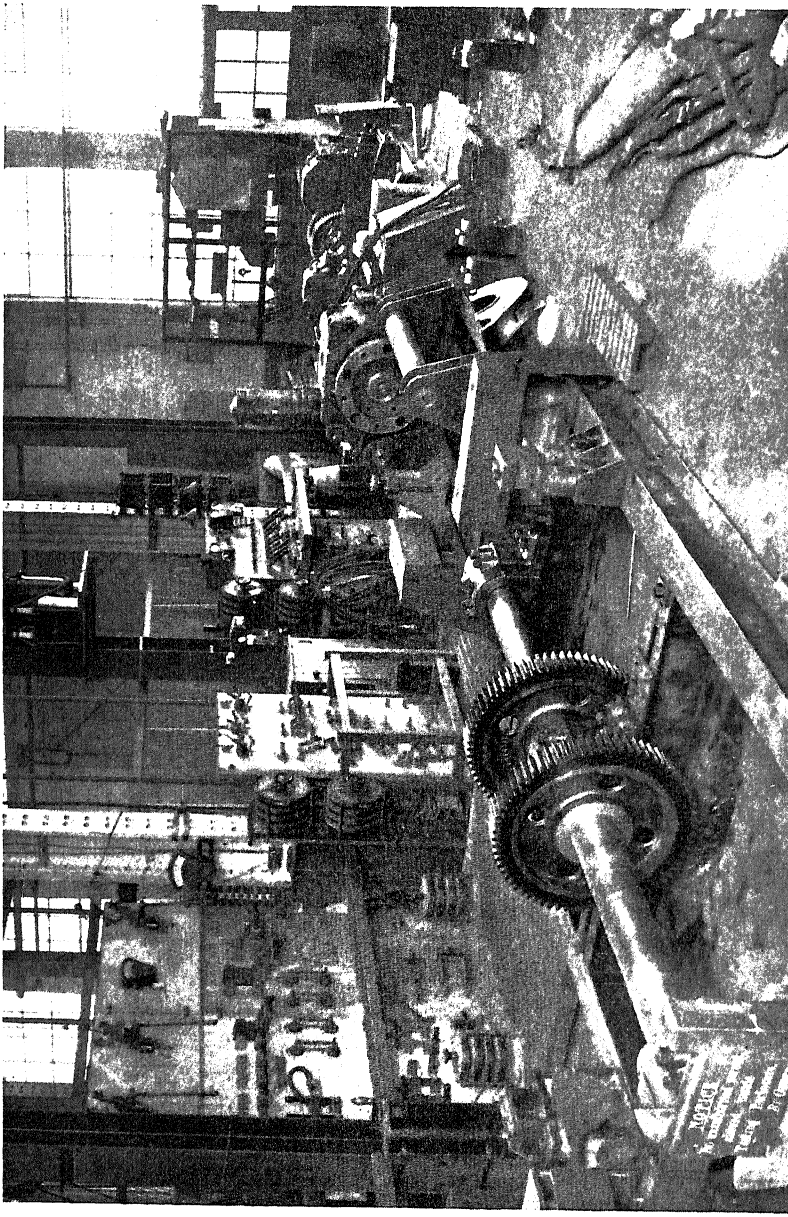


FIG. 49.—Railway Motor Testing Gear.

The motors are first run on load until moderately warm, and are then stopped and the resistance of armature and field windings measured. On re-starting, the motor-voltage is held at its prescribed value, and a series of simultaneous readings are taken of line-voltage V, line or make-up current c, motor current C, and load booster or make-up volts v. The make-up current and voltage are measured by means of low reading instruments and the loss is accordingly determined with greater accuracy than would be possible if the more obvious method of deducing it from readings of input and output were followed, since this involves taking the difference of two large quantities, both of which may be subject to error. On completing the series of readings, the motors are stopped and the resistances of their several windings again measured. A mean between the resistances found before and after the efficiency readings may fairly be assumed for the several resistances during the tests, although, if the series is a long one, intermediate measurements of resistances may be deemed desirable. Such sets of readings are taken for each motor and for each direction of rotation. Since the main circuits include not only motor and generator windings, but also certain cables, switch contacts, and switch-board connections, it is necessary, if accuracy is desired, to determine the resistance of these. This is readily accomplished by inserting short-circuiting jumpers in place of the motor windings and using the load booster to supply current for taking the voltage drop between the appropriate points in the motor and generator circuits.

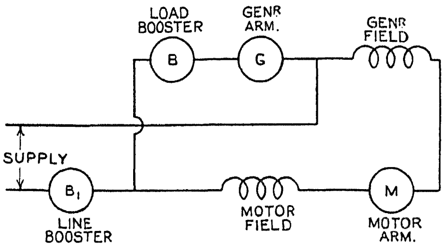


FIG. 50.—Simplified Diagram of Connections for Testing Railway Motors.

Appropriate switches are provided to facilitate the necessary connections, a diagram of suitable switchboard wiring being shown in fig. 51.

THE CALCULATIONS.—In terms of the observed quantities the input to the motor circuit is VC, and the output of the generator circuit is $(V-v)(C-c)$, so that the total loss of energy in both machines, including their connections, is :—

$$W = VC - (V - v)(C - c) = Vc + vC - vc \quad . \quad (1)$$

From the resistance measurements the corresponding losses are immediately deduced; and the brush contact resistance loss can be computed with sufficient accuracy by allowing 2 volts total brush drop in each motor, independently of load, since any error that may be introduced by this assumption is carried to the residual losses and corrects itself, except for a negligibly small quantity. If now the resistance losses for both motor and generator circuits be subtracted from the total, there remain the core losses, the load losses (due to distortion of field), the brush friction losses, the losses in armature bearings, axle bearings and stand bearings, the gear losses, and the wind-

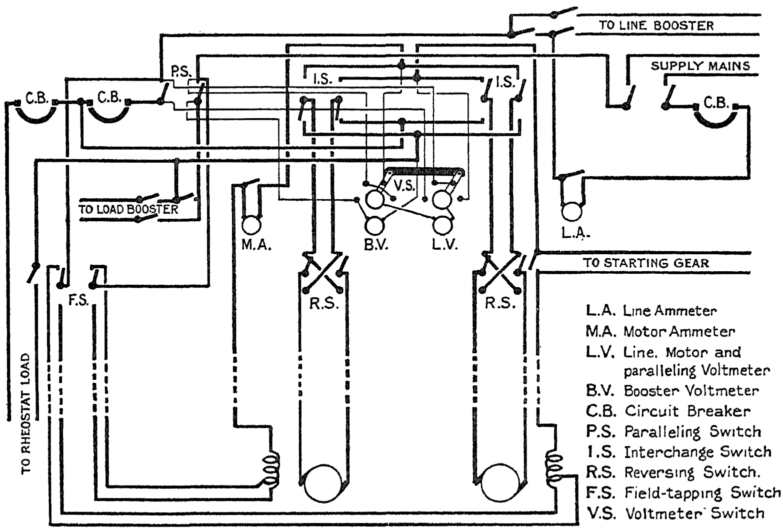


FIG. 51.—Wiring of Testing Stand.

age and vibration losses. These residual losses are divided by two, and the half charged to the motor. The stand bearing losses are not properly chargeable to the motor, but as they are insignificant and it is practically impossible to separate them they are included as motor losses. The efficiency of the motor is then deduced from the several losses, the resistance losses being taken to correspond with the standard copper temperature of 75° C.

Schedule of Calculation.—A suitable schedule for the calculation is given in table 6, and in the example chosen the

calculations are made for a motor having two field strengths. The resulting efficiency represents an average between the two machines tested together. If it were desired to distinguish between them, the difference between their resistances would have to be taken into account; and it could only be assumed that the difference between the residual losses is in accordance with the no-load core loss tests. Usually the motor speed is

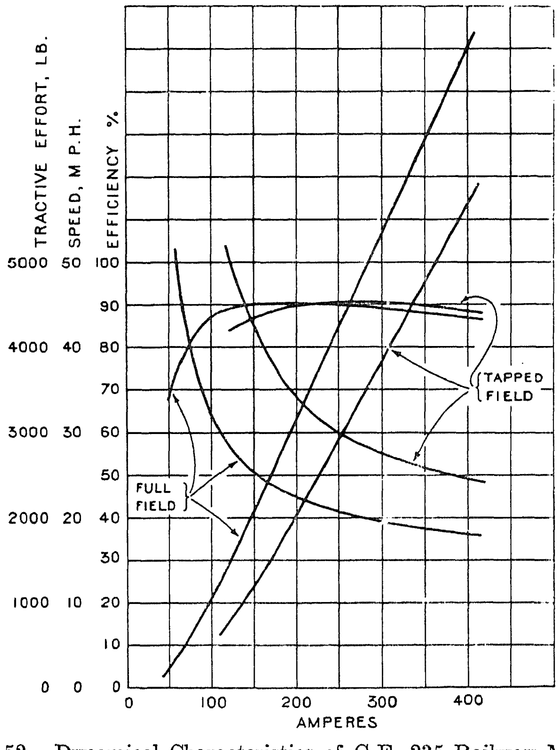


FIG. 52.—Dynamical Characteristics of G.E. 235 Railway Motor. 70/22 gear, 42 in. wheels, 775 volts.

observed at the same time as the efficiency readings are taken, although it has no bearing on them, and may be taken independently if so desired. From the motor speed, the gear reduction and size of wheels, the train speed is immediately deduced, and from the train speed and efficiency the tractive effort at the wheels is found for any input. Fig. 52 gives speed, tractive effort and efficiency curves as deduced from many sets of readings such as recorded in table 6.

TABLE 6

Efficiency and Loss Calculation

Type, G.E. 235. Voltage, 775. Gear, 70/22. Wheels, 42 inch. Rotation, C.
Motor No. . Arm. No. . Genr. No. . Arm. No. .

	Motor Circuit.		Genr. Circuit.	Motor at 75° C.	
	Tapped Field.	Full Field.		Tapped Field.	Full Field.
Resistances :					
Exciting field .	·0309	·0606	—	0·317	·0634
Commutating field . .	·0296	·0296	·0268	·0274	·0274
Armature .	·0506	·0506	·0492	·0501	·0501
Generator field	·0305	·0612	—	—	—
Board . .	·0058	·0058	·0021	—	—
Total . .	·1474	·2078	·0781	·1092	·1409

	Tapped Field.				Full Field.			
Motor current C	300	250	200	150	300	200	125	75
Difference volts v	53	34	23	18	93	61	34	22
Line current c	39	37·5	35·5	35	28	23	21	25
Line volts V	790	785	780	778	795	790	785	778
Vc	30,800	29,420	27,680	27,210	22,250	18,180	16,490	19,450
vC	15,900	8,500	4,600	2,700	27,900	12,200	4,250	1,650
$-vc$	−2,060	−1,270	−820	−630	−2,600	−1,400	−710	−550
Losses . . .	44,640	36,650	31,460	29,280	47,550	28,980	20,030	20,550
C^2R Motor circuit	13,270	9,210	5,900	3,320	18,700	8,310	3,250	1,170
C^2R Generator circuit . .	5,320	3,530	2,110	1,030	5,780	2,450	850	200
Brush C^2R (2 volts drop) .	1,120	920	730	530	1,140	750	460	250
Core loss., fric., etc.	24,930	22,990	22,720	24,400	21,930	17,470	15,470	18,930
Core loss, etc., one motor .	12,465	11,495	11,360	12,200	10,965	8,735	7,735	9,465
C^2R motor windings . . .	9,830	6,815	4,370	2,460	12,680	5,635	2,200	790
Brush C^2R (2 volts drop) .	600	500	400	300	600	400	250	150
Total loss . .	22,895	18,810	16,130	14,960	24,245	14,770	10,185	10,405
Efficiency % .	90·15	90·3	89·6	87·15	89·55	90·45	89·5	82·05
Speed r.p.m. .	690	770	883	1,090	500	572	708	1,050
Light running loss . . .	11,910	12,910	13,560	14,180	11,510	10,750	11,370	14,990
Gear and load loss . . .	13,020	10,080	9,160	10,220	10,420	6,720	4,100	3,940
Do. per motor	6,510	5,040	4,580	5,110	5,210	3,360	2,050	1,970
Do. % of input .	2·8	2·6	2·95	4·4	2·25	2·2	2·1	3·4

Where only one motor is available from which to deduce the characteristic curves, or where a suitable testing stand is lacking, the speed curve should first be obtained by loading the motor on any machine capable of taking the load. Next the light running input should be observed at various excitations and the corresponding speeds, the resistance losses should be computed from the observed resistances, and after allowing for brush contact drop, the remaining losses—often classed as gear losses—should be estimated as a percentage from the results of other motor tests at corresponding loads. Thus an approximate efficiency curve is obtained from which with the speed curve, the tractive effort can be deduced.

Losses.—Reverting to table 6, the fourth line from the foot of the table gives the no-load core loss in combination with the brush friction, windage, and armature bearing friction losses for the two motors. This is obtained when the motors have been removed from the testing stand by observing the power input to the armature of each motor, with the excitation of the efficiency test and at the corresponding speed, the motor meanwhile running light without the gears. If this quantity is also subtracted from the losses, the remainder comprises the load losses, the axle bearing and stand bearing losses and the gear losses. These have hitherto generally been reckoned as gear losses, although at the light load end of the curve the axle bearing losses are considerable; and at the heavy load end the load losses. That the gear losses themselves are not nearly as large as they are thus accounted is shown by the fact that the gears do not usually become very hot even after considerable running at heavy loads.

Load and Gear Losses.—In order further to separate the losses, input-output tests may be made in the manner described above, but with the two armature shafts coupled together directly at their pinion ends by means of a flexible coupling. Proceeding with the calculation exactly as in table 6, subtracting from the total losses the resistance losses and light running losses, there remains the load loss. The difference between this and the gear and load loss together given at the foot of table 6, is the gear loss, including therein the axle bearing and stand bearing losses. The load loss is considerable at heavy loads, particularly with the weakened field. The

direct-coupled stand may be employed for all load tests in which the gears are not important, and is particularly useful for the continuous heat runs from which the service capacity of a motor is estimated. A view of such a stand, having one motor and half-coupling in place, is to be seen in the middle distance in fig. 49.

No-load Core Loss, and Friction.—The no-load core loss is best determined by driving the motor at constant speed with its armature circuit open and with various exciting currents, a small auxiliary motor, having constant excitation, being used for the purpose of driving. The input to the small motor when there is no excitation of the main motor is subtracted from the input with excitation; and the result, with a small correction for difference of resistance loss in the auxiliary motor armature, gives the no-load core loss at the particular excitation and speed. If readings are taken for a number of speeds and results for each speed plotted against excitation, the core loss for any voltage and current can be deduced by cross plotting from the speed curve. The core loss test is a difficult one to make; and the results are often disappointing. As the interest is generally in a type rather than in an individual motor the mean of several series of tests should be used. If on a particular motor the core loss is subtracted from the light-running input the remainder is the friction loss in the motor. This will depend on the condition of the brushes, the brush pressure, the direction of rotation, and the lubrication; the mean of the results from several motors should accordingly be taken and plotted against speed. The friction loss is, however, preferably obtained in a similar manner to the core loss, by subtracting the light-running input to the auxiliary motor alone from its input when driving the unexcited railway motor. It is necessary to separate the friction and core losses, for one of the chief purposes to which the results are applied is that of estimating the heating of the motors in service, and the two forms of loss accompany one another only during the time that power is applied, the friction loss existing alone whilst the train is coasting. Fortunately the friction loss varies approximately as the speed, within the limits of use; being in the case of the motor of fig. 52 about $3{\cdot}2 \times$ r.p.m. watts. The mean friction loss in service is accordingly that at the mean speed.

The total motor loss can therefore be readily divided into the following elements :—(1) Copper loss, (2) Brush-contact resistance loss, (3) Core loss, (4) Load loss, (5) Brush friction loss, (6) Armature bearing friction and windage loss, (7) Gear and axle bearing loss. In obtaining representative values for

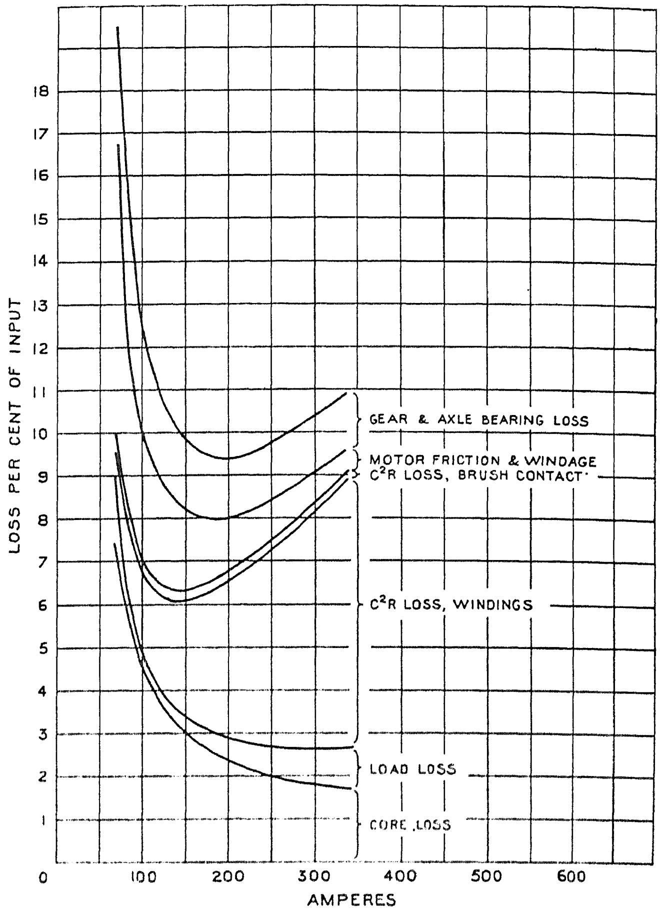


FIG. 53.—G.E. 235 Railway Motor, Losses, Full Field, 775 Volts.

these losses it is not advisable to rely on a single series of tests on a particular pair of motors, for small errors in the readings or changes in the circumstances may lead to very wrong estimates of individual losses. Many tests are required to yield characteristic values, and if practicable these should be

made on a number of different motors. Figs. 53 and 54 give segregated losses for the motor whose characteristic curves are given in fig. 52. The results are deduced from tests on a large number of motors and may be taken as representative of the type. The large load-loss when the motor is operating with weakened field is worthy of note, particularly as this is one of the losses tending to heat the motor, and accordingly one which affects its service capacity.

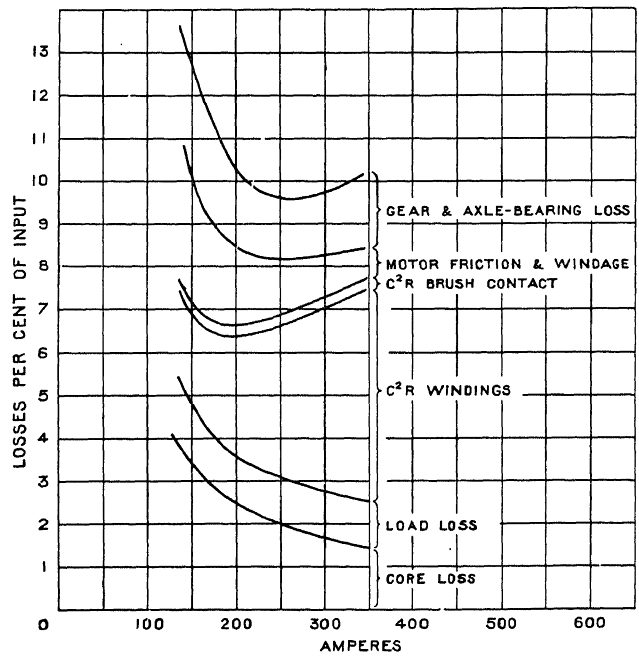


FIG. 54.—G.E. 235 Railway Motor, Losses, Tapped Field, 775 Volts.

Rating of Motors.—For most classes of electrical machinery it is practicable to devise simple shop tests which shall enable an adequate judgment to be made of the performance of the apparatus in service, and this not only as regards speed and dynamical characteristics, but also as regards heating and capacity for dissipating internal losses, features on which successful operation for protracted periods so largely depends. In the case of such machinery, the power rating, as deduced from the shop tests on a recognized basis of temperature rise,

is closely related to the service capacity, and when this rating is known a fair estimate can be made of the duty of which the machine is capable. The rating is therefore an important characteristic of such a machine, conveying, in concise form, much valuable information to those able to appreciate its significance. In the case of the motors employed for propelling railway trains and tramcars, however, the rating is of small value, having little connection with the service duty; for the conditions of service are very different from anything that can be realized in practicable shop tests, and it is difficult —one might say impossible—to correlate the operations which constitute service, with a definite power load to which the appellation "rating" can appropriately be applied.

The duty of a traction motor in service consists of a series of quite irregular cycles normally constituted as follows :—

(1) A period of acceleration at heavy current and partial voltage.

(2) A period of running at full voltage, during which the car or train accelerates with decreasing current.

(3) A period of running without power, during which the car or train decreases in speed.

(4) A period of rest.

The several periods vary between more or less wide limits, depending on the distance between stopping places, the time available, the load carried and the gradient profile of the road. The heating of the motor in service depends principally on two factors, viz. :—The average power loss in the motor itself, and the average power dissipated per unit temperature rise; for a given motor the first of these is a function of the current and voltage, and the second usually a function of the train speed and of the armature speed, both having reference to the cycle of operation constituting the actual service.

Nominal Rating.—The power rating of a traction motor, as usually understood, was formulated in the first instance by the American Institute of Electrical Engineers, and has been generally adopted. It has been rendered more precise by successive revisions, and now reads as follows :—"The nominal rating of a railway motor shall be the mechanical output at the car or locomotive axle, measured in kilowatts, which causes a rise of temperature above the surrounding air, by thermometer, not exceeding 90° C. at the commutator, and 75° C. at any other

normally accessible part after one hour's continuous run at its rated voltage (and frequency in the case of an alternating current motor) on a stand with the motor covers arranged to secure maximum ventilation without external blower. The rise in temperature, as measured by resistance, shall not exceed 100° C. The statement of the nominal rating shall also include the corresponding voltage and armature speed."

It will be seen that, whilst the service capacity, as limited by the heating, depends principally on the motor losses and on the dissipative capabilities of the motor, both assessed under service conditions, the rated capacity depends principally on the motor loss during an artificial run and on the capacity for heat of the motor. In the rating test run, no very great proportion of the heat escapes, most of it being used in heating the mass of the motor; mere increase of mass, though it have no mechanical or electrical value, and no effect on the service capacity, increases the rating. The service capacity, as measured by the dissipation of heat, is increased very largely by allowing a free circulation of air through the motor, from the outside; the current required to give 75° C. rise in a one hour's run however varies but a few amperes whether the motor be entirely closed-in or arranged to permit free circulation of the outside air. It is clear therefore that there is no relation between the nominal rating and the service capacity; the two are functions of different factors, and are not comparable.

In the early days of electric traction, when the nominal rating was first formulated, a traction motor was understood as a totally enclosed continuous current series motor of some 20 to 40 horse-power nominal rating, designed and constructed for propelling a tramcar. With these limitations there was no large margin for variation, and although surprising results in the way of temperature rise in service were occasionally remarked, there was reasonable expectation that motors of equal rating and similar dynamical characteristics would be capable of equal service, and that the weight of car which a motor could propel in a given service would be roughly proportional to its rating. It was soon apparent, however, that rating and service capacity did not go hand in hand. As motors increased in power, the weight increased at a greater rate than the dissipating surface, and thus the service capacity

by comparison fell behind the rating. The commutation limits, moreover, which were of course set with the service, rather than the rating in view, caused a reduction in the accelerating current by comparison with the rated current. Thence arose the practice among manufacturers of assessing the rated power of the larger motors at a figure considerably below the nominal rating, more, that is, than was necessary to allow for differences between motors of a type. Thus the G.E. 69 motor used so extensively on the Underground Electric Railways of London, rates strictly at something over 240 horse-power at full voltage. It was, however, rated commercially at only 200 horse-power, this figure representing approximately its service capacity as compared with other motors of similar type. Similarly the DK–4A motor used on the Liverpool–Southport line is rated at 150 horse-power, although its nominal rating is stated to be about 190 horse-power. Since these motors were produced, however, large changes have taken place in motor design; the interpole has permitted a wide extension of commutation limits, whilst improved methods of ventilation have very much enhanced the capacity of the motor for dissipating heat. As a result of these improvements, recent motors show greatly increased service capacity as compared with rating, and frequently employ an accelerating current much in excess of the rated current. The maximum load permissible for short periods is indeed now determined rather by the area of brush contact than by the heating of the motor. Of recent years also the alternating current railway motor has come into being, and this again having quite different thermal characteristics from the continuous current motor, does not accord with the latter in the relation of rating to service capacity. The comparison of traction motors on the basis of their nominal rating is now therefore, more than ever, misleading, and no significance should be attached to this rating, other than is contained in its definition.

Determination of the Heating of Motors in Service.— The heating of a railway motor in service, or even in closely specified service tests, is not susceptible to very accurate estimation on account of the number and uncertainty of the factors affecting the results; and no single figure is adequate to express the service capacity of the motor. If a standard

schedule could be devised and arranged to be run in a certain manner, the weight of train that a motor would carry continuously through the schedule with a definite temperature rise might be taken as a measure of its capacity. The commercial determination of this capacity would, however, be impracticable; and moreover motors compared on the standard schedule would not necessarily preserve their relative capacities on other schedules. No pretension is made, therefore, to expressing the service capacity of a motor by a single figure; but in practice curves are obtained from which the heating in a specified service can be deduced with a fair degree of accuracy. The tests from which these curves are computed consist of a series of continuous heat runs, that is to say, heat runs continued at uniform input until the temperature is constant. These are best made on a direct-coupled testing stand with the motor arranged, as regards ventilation, as it is to be employed in service. The voltage and current are selected to give a temperature rise of armature as measured by thermometer, of the order of 75° C., and at the same time to give a suitable armature speed. A number of such runs are made, at different armature speeds, corresponding to the whole range of average armature speed, that may occur in service. When the temperature has become steady, readings to determine the losses in the motors when under the load should be taken, just as in the efficiency test. Immediately after the run, both machines are opened and temperatures are taken of field coils, armature surface, commutator, frame, and surrounding air, whilst resistance measurements are made of the several circuits of the motors.

Losses Involved in Thermal Capacity Tests.—Hitherto it has been deemed sufficient in making an estimate of the losses tending to heat the motor, to include only resistance loss, no-load core loss, and friction loss; the latter loss being computed as brush friction alone, since this, the dominant factor, is approximately calculable. The load loss and windage loss are accordingly neglected, and the bearing loss is considered capable of dissipation without heating vital parts of the motor. As long as motors are similar in the matter of the ratio of the losses neglected to those included, and as long as the losses considered in using the data are the same as those considered in obtaining it, this procedure does not lead

to large error. It has, however, been shown that under certain circumstances, the load loss becomes quite appreciable, and since this varies with the excitation, its neglect may give rise to inaccuracy in a result. On grounds of correct scientific method, moreover, it is advisable to assemble and use data on a true, rather than on a fictitious basis, for only thus can comparable results be anticipated. The losses which result in the heating of a motor can very readily be measured, during the heat run; whilst the corresponding losses pertaining to the conditions of actual service are, as explained above, determinable with equal facility by means of special tests.

Thermal Dissipation Curves.—The results of the set of continuous heat runs are best presented in the form of a thermal dissipation curve, giving the total watts loss per degree rise of temperature plotted against armature speed as abscissa, this being the chief variable on which the motor temperature for a given heat dissipation depends. Points on the curve may be deduced from the readings both for the motor and the generator; and, in the author's experience, these are generally found in close agreement. The temperature used in deducing the thermal dissipation curve is the highest recorded temperature as measured by thermometer, and this is generally an armature temperature. A separate curve may, if desired, be obtained corresponding to temperature by resistance, but usually this does not show quite the same consistency as the other, for, the resistances being small, their accurate measurement under ordinary test conditions is somewhat difficult. Fig. 55 gives the thermal dissipation curve for the motor whose dynamical characteristics are given in fig. 52, under the conditions of ventilation that would be used in service. The motor has series fan ventilation; a totally enclosed motor would show much less variation in dissipation with armature speed. In computing the curve of fig. 55, the losses determined by direct test have been employed. The data for obtaining these losses are given in fig. 56, and this, or equivalent data, should be employed in making use of the dissipation curve.

If from such data as are given in fig. 56 the losses corresponding to any definite load be deduced, the temperature resulting from continued operation at that load on the testing stand is given by the dissipation curve of fig. 55. Moreover, since the dis-

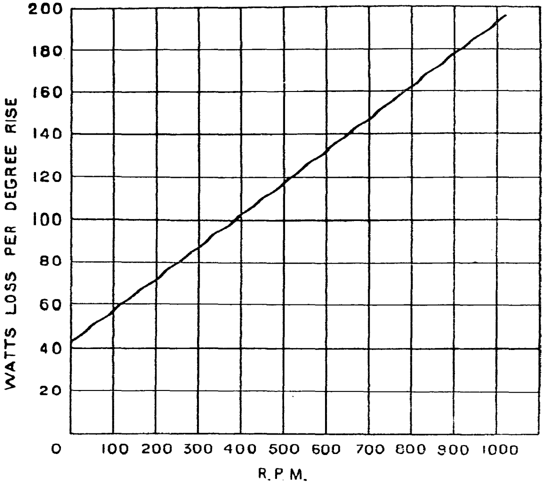


Fig. 55.—G.E. 235 Railway Motor, Heat Dissipation Curve.

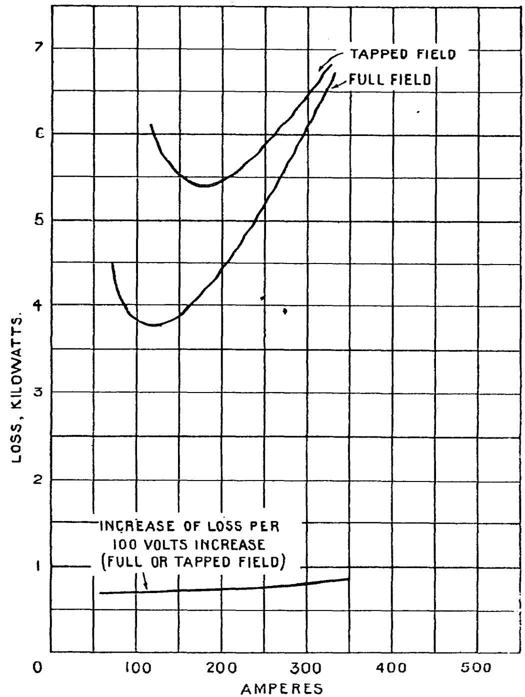


Fig. 56.—G.E. 235 Railway Motor, Core and Load Losses at 775 Volts.

Addenda to Fig. 56.

Resistances (75° C.):	Full Field.	Tapped Field.
Armature	·0508	·0508
Exciting Field	·0636	·0316
Commutating Field	·0279	·0279
Total	·1423	·1103

Brush Drop, 2 volts. Motor friction and windage, $3{\cdot}2 \times$ r.p.m. watts.

sipation curve when plotted against speed is sensibly a straight line, if the load passes through cyclical changes, the mean dissipation is the dissipation at the mean speed, and it is accordingly permissible to employ the same dissipation curve for such cycles of load as occur in service. In order to determine the temperature rise of the motor in a given service, however, it may be necessary to apply a correction factor to allow for the difference between the heating on the testing stand and the heating in service. It is fortunate that this correction can be found once for all from tests on any motor of the general type, without reference to the particular design, for the tests necessary to determine it are difficult and expensive. A length of track is required on which definite schedules can be run on still dry days until motor temperatures become constant. By means of suitable recording instruments the voltage and current at all times can be found; and from these the loss in the motor can be deduced, and the watts dissipated per degree rise of temperature determined. The temperature rise for a given loss may be expected to be lower in service than on the testing stand, on account of the cooling of the frames. The ratio of the temperature rise in service to the corresponding figure obtained from stand tests for the same motor with the same losses, at the same mean armature speed, gives a "schedule factor," which may be applied to any motors of the same general type.

Schedule Factor for Enclosed Motors. The schedule factor is, however, only of importance in the case of totally enclosed motors, for which the correction may be considerable. It usually lies, indeed, between 75 and 85 per cent., the lower figure corresponding to the higher schedule speed. It is, however, not strictly the same at the same speed for all motors of a given type; for the heat dissipated from a totally enclosed motor depends on the general temperature of the frame, whereas the schedule factor correction is based on armature temperatures. Accordingly, a second co-ordinate is required to express the results properly, and this may be taken as the ratio of the frame temperature rise to the armature temperature rise as determined in the stand tests. The ratio depends on the size and design of the motor, and on the armature speed as well as on the general arrangement for circulating the air inside the motor. However this is a refinement the import-

ance of which has disappeared with the practical supersession of the totally enclosed motor.

Schedule Factor for Ventilated Motors.—Ventilated motors, having a through draught of air, although doubtless affected by dissipation from the frame, have most of the heat carried away by the draught, and are moreover subject to other influences, beside which the effect of this dissipation is negligible. It would appear that the wind, combined with the motion of the train, is generally able to interfere with the draught, and so to stall it that some of the motors run somewhat hotter in service than would be expected—others running cooler. Altogether there is less consistency between service temperatures of ventilated motors than in the case of totally enclosed motors; and the maximum temperature found for the hottest motor of a train may be even higher than that found in the stand test for the same losses. It is accordingly not expedient to apply any correction factor to allow for heat dissipated by the frame in the case of such motors; but to take the results of stand tests as applying approximately to service conditions.

Limitations of Theory.—In such a subject as this it should be understood that no workable theory could possibly take account of all the circumstances; and whilst the above may be useful as a guide, in that it takes account of the principal influences, it must not be expected to give other than approximately correct results, even when special circumstances are absent. As a matter of fact special weather conditions, such as wind and rain, generally have the effect of cooling the motors, particularly if these are enclosed, and are of little interest inasmuch as the highest temperature likely to be attained in normal service is the usual objective of investigations in this subject.

The Heating of Motors in Service.—The above methods are capable, if used with judgment, of giving motor temperatures fairly in agreement with the results of carefully executed service tests; but actual service is usually affected by a number of circumstances which it would be impracticable to cover in such tests. It has already been shown that the size of the wheels which a particular motor drives affects the proportion of the load carried by the motor, and thus affects its heating. In addition to this the differences between motors may cause

variations of temperature of a few degrees from the standard. Inefficient driving also has the effect of heating the motors abnormally. Signal-stops and delays of any kind which necessitate the making up of time are fruitful causes of high temperature, and of course excessive weight of train, which the exigencies of the traffic sometimes impose, has a similar effect. The gradients on certain sections of a railway may be generally adverse, and excessive train resistance, whether in the train itself or arising from external causes, is likely on occasion to increase the temperature of the motors. Altogether it is not good practice to choose a motor which in normal operation is estimated to reach a temperature near to the limit which the insulation will stand continuously, and in these latitudes an armature temperature rise of about 65° C. usually strikes a good balance between excessive weight and initial cost of motor, with the concomitant increased energy consumption on the one hand, and excessive maintenance cost on the other. Certainly there are operators who prefer to face frequent re-winding of armatures for the sake of low equipment weight rather than carry greater weight in order that re-winding may not be needed oftener than every eight or ten years, but general good practice favours more moderate temperatures, less on account of expense of maintenance, which may in fact be offset by other saving, than because of the risk of having the motors break down in service, which may result in considerable direct or indirect loss. There is usually less justification for requiring the temperature in service to be abnormally low, for the possible saving in maintenance expense is negligible ; whilst the greater weight to be carried about is a constant source of expense. It is the realization of this latter fact which constitutes the chief claim of the motor having a definite circulation of air from outside. Such a motor for a given service can be designed to be lighter than the totally enclosed motor without sacrifice of qualities which make for low maintenance and operating cost, and the engineer with a full knowledge of the problem would hardly now consider the totally enclosed motor for any service in which the weight of equipment is a matter of moment. In view of the space limitations, and the cost of carrying additional weight, it is considered good practice to operate railway motors at higher temperatures than is advisable in stationary motors. The insulation of these motors is usually

of such nature that the so-called " hot spot " temperature may be assumed as high as 140° C., or even higher, and whilst the armature temperature, as measured by thermometer, may be considerably lower than this, the exigencies of abnormal service do not usually require the normal service rise to be less than 65° C. The above remarks apply particularly to the geared motor; for the gearless bipolar motor is usually allowed to run somewhat hotter in order to permit of reduction in the weight of the armature.

Importance of Thermal Characteristics.—The consideration of the heating characteristics of a motor is quite as essential in the determination of its possibilities for service as that of the dynamical characteristics ; and in fact the heating in service is usually made a limiting feature in the design. It is a matter involving considerable labour to determine, even approximately, the temperature rise of a motor of known thermal characteristics in any given service ; but it is necessary labour and is amply repaid if it leads to a choice of motor which is neither inadequate nor excessive.

Tapped Field Motors.—It is now fairly common practice to make railway motors having their exciting field coils in two or more sections, and to arrange the control so that sometimes part and sometimes the whole of the field is used. The motor is started with full field and when the rheostat has been cut out of circuit the field is tapped, thus giving increased speed to the motor. This is undoubtedly a valuable provision in its place ; but it is not in every case worth the extra complication. The matter is easily misrepresented, particularly as in tapped field operation it is usually sought to work a motor near to the limit of desirable peripheral speed. By making comparison with the same motor used with full field, thus using a smaller ratio of gear reduction in order to give the same maximum train speed, it is possible to show considerable saving in energy. One advantage of the system, indeed, is that it sometimes permits of a better showing being made with a particular motor. If, however, it is adjudged desirable practice to run to a certain armature speed, advantage from running to this limit will accrue whether tapped field operation is adopted or not, and if motors are compared which are designed to run at the same

limiting armature speed for the same limiting train speed, thus having the same ratio of gear reduction, the advantage from tapped field operation is small, as regards saving of energy. Of the two motors, that designed for tapped field operation is usually somewhat the heavier, whilst several extra contactors are required to control the operation. Thus the equipment is a little heavier, more costly, and more complicated; whilst the saving in energy per train mile is small, although quite appreciable. The cases in which tapped field operation can be justified on account of saving in energy are comparatively few, provided motors are designed for the service required, and a claim of unduly low energy consumption for such motors should be subjected to scrutiny in order to determine whether it conceals risks that ought not to be assumed. Although the provision is, in many cases, hardly worth the complication in urban service, there is considerable scope for tapped field operation in high speed service with few stops. Here it is a distinct advantage to be able to vary the power at a given speed in order to meet the requirements of variation in train resistance due to wind, varying weight of train, or other cause. Considerable power is required at high speed, for the power is jointly proportional to the speed and the train resistance. On the other hand, very rapid acceleration is unnecessary, and it is particularly undesirable to continue the acceleration to a high speed on account of the high peak of power taken. A very satisfactory method of control for this class of service is accordingly obtained by accelerating on resistance at full field, then weakening the field until the desired speed is obtained with sufficient power to carry the train. With this method of operation the peaks of power are kept relatively small, a matter of considerable importance in all but heavy suburban services, where the substation load is equalized by the large number of trains running. It may here be advisable to show why the field is weakened by tapping rather than by shunting. This is because with shunted fields any sudden change in voltage, such as occurs, for instance, when the circuit is broken and re-established at a section insulator, causes the variation in current to evade the field and pass through the shunt—a condition very severe on the commutator and liable to cause a flashover—unless the shunt is so highly inductive as to have the same time-constant as the field.

Regeneration.—It is sometimes, particularly on hilly routes, very desirable to be able to use the train motors as generators to feed back energy to the line and at the same time limit the speed of the train on down-grades. Various schemes for regeneration have been proposed and tried; but they have generally employed the motor as a shunt generator, to which there is the same serious objection as has been shown to exist to the use of the shunt motor. The chief requirement in regenerating, as in running, is that the field current shall always vary with the armature current in such manner as to oppose the rise of the latter. A very satisfactory way of effecting this in the case of a locomotive is shown diagrammatically in fig. 57. The field of the motor is kept in series with its armature; but at the same time is separately excited from a small generator specially provided for the purpose, and driven by a motor from the line, or in other suitable manner. This generator is excited by means of a shunt field, whilst the main current is caused to pass through series coils on the field in such direction as to oppose the shunt excitation when regenerating. The effect of this is that the fields of the motors are not rigidly excited but vary in such manner as to oppose variation of armature current. In operation the regenerated current on any braking notch varies until a certain speed is reached, determined by the shunt excitation; and the motors may then be taking power or delivering it according to the needs of the train as affected by the gradient. Regeneration by the separate excitation of the motor fields is not new, but the features which render the above system of commercial value lie in the controlling devices which safeguard the motors so as to secure their satisfactory operation.

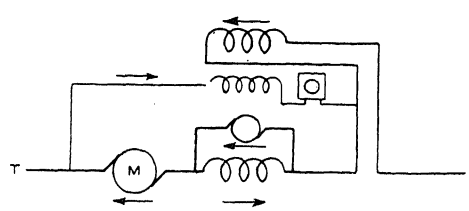


FIG. 57.—Connections for Regeneration.

Flashing at Commutator.—The phenomenon of flashing-over at the commutator as met with in railway motors, arises chiefly from the exacting nature of their conditions of operation. The motors are usually designed with such stability

that it is almost impossible to flash them over on the testing stand. With the motor running at any service speed, indeed, it may be found quite feasible to interrupt the current for two or three seconds and re-establish it suddenly by applying full voltage to the terminals. In fact the flicker taking place at the brushes may under this test be barely perceptible, whilst double voltage can generally be thus thrown off and on without flashing the motor over. The same motor, however, may be found to flash over readily in service, quite apart from any defect such as would cause a large rush of current to the armature only. There are of course defects in a motor, such as high bars or loose commutator, which tend to cause flashing, but it indicates poor workmanship to find these sufficiently serious to give trouble on the testing stand at normal voltage. The cause of flashing may be anything able to raise the brush from the commutator for an instant while it is carrying current, and the chief cause is undoubtedly roughness of the track. It is a matter of common observation that flashing usually takes place at high speed, that is, when an irregularity, such as high rail joint, gives a considerable blow to the wheel. Mr. Priest has recorded * that motors are more likely to flash over when on a frozen road-bed and attributes this to the greater rigidity of the unevenness of the track. It is found moreover that a motor is more likely to flash over in service when running ahead of the axle than when the axle is ahead of the motor—that is, the motor on the second axle of a truck is the more likely to give trouble. There are possibly two reasons for this. In the first place, with the motor on the second axle, the pinion tooth, when driving, is above the gear-tooth; and accordingly an upward blow given to the wheel is transmitted immediately to the armature. With the motor on the first axle, however, the pinion tooth is below the gear-tooth, and a blow on the wheel has to take up the clearance between teeth before it can affect the armature. In the second place, the ordinary motor is constructed so that, when facing the axle, the commutator and pinion ends are respectively on the left hand and right hand sides, so that flashing over is more likely to take place when the motor is rotating in a clockwise direction as viewed from the commutator-end. Many motors are constructed with one brush box at the highest point of the com-

* *General Electric Review,* Nov., 1913, vol. 16, p. 815.

mutator and one at the right hand side. Since, however, the latter can hardly be affected by a blow on the wheel, the flashing is likely to start at the former; and, when the rotation is clockwise, the arc is carried by the shorter path between the brush holders, thus having a better chance of establishing a short circuit. Mr. Priest has made the observation * that if the positive brush is jerked away from the commutator as little as a hundredth part of an inch a flashover is likely to occur; but the negative brush may be jerked off several times as much without greater likelihood of flashing. If, accordingly, the motors are so connected that the top brush is always negative when the rotation is clockwise, the chances of flashover are minimized. A contributory cause tending to make a motor liable to flash over is too low a brush pressure, since this allows the brush to be raised from the commutator too easily. Unsuitable brushes again may cause the trouble, those containing a large proportion of natural graphite, although usually giving a good surface to the commutator, being more likely to lead to flashing than dense brushes composed principally of graphitized gas carbon.

High Voltage Continuous Current Motors.—Continuous current motors for high voltage have not hitherto differed in general design from those for lower voltage. The number of commutator bars is increased so as to make the volts per bar about the same as has been found good practice at lower voltage. The flashing and creepage distances are increased, the commutator cones, with their insulation, being made to project farther from the commutator; whilst the thickness of insulation to ground is of course everywhere increased. High voltage continuous current motors are often wound for a half or even a smaller fraction of the line voltage, and run two or more in series, being of course insulated for the full line voltage. Thus the motors of the C.M. & S.P. locomotive of fig. 197 are run two in series on 3,000 volts, whilst those of the locomotive of fig. 198 are connected at least three in series for the same voltage. The highest voltage for which motors have been wound for commercial operation appears to be 1,750 volts, for use on a 3,500 volt line, but experimental double armature motors have been made to run at 2,500 volts per motor. With

* Ibid.

a distributed compensating winding on the field structure it should be permissible to increase the voltage per commutator bar and thus render higher motor voltages practicable.

Bipolar Gearless Motors. The bipolar gearless type of motor which is used with much success on the New York Central and Hudson River Railroad locomotives, and which has even been proposed for motor car work, merits attention. In this type the armature is built up on the driving axle of the locomotive, and the poles are carried on the locomotive truck. The pole faces are vertical and for the most part flat, sufficient space being left between them to allow the armature to be removed when necessary by lowering it with its wheels and axle. The centre portion of the pole face is recessed a little, giving a short section of uniform air gap. The magnetic circuit is completed through the locomotive side frames and other steel work, reinforced by a suitable yoke. The brushes are on the horizontal diameter of the commutator where the vertical motion of the wheels is unable to affect them injuriously. The motor is of course without commutating poles, but having of necessity a large interpolar space the difficulties of commutation are not great as the armature field is weak in the commutating zone. The motor is closed in below by means of a suitable plate, and may be ventilated in much the same manner as a geared motor. The excitation required being large, the electrical efficiency is somewhat lower than in the geared type, but the absence of gears makes the overall efficiency high, whilst the simplicity of construction especially commends it to operators wherever the service is of a nature to warrant its use. Figs. 58 and 59 show longitudinal and transverse sections of the bipolar gearless motor; and fig. 60 gives characteristic curves.

Motors for Collective Drive. Motor units of large power, carried in the locomotive cab, have not yet become a practical necessity where continuous current motors are in question, and, as the balance of advantage is considerably in favour of independent motors for the several driving axles, extended consideration of the type is unnecessary. The problem of their design, however, presents fewer difficulties than are involved in the more common types as most of the restrictions of space are absent. In general construction the

motor follows the lines of the stationary motor, but includes those features of rigidity of binding and robustness of structure which have been found necessary from experience with the usual types of railway motor. Such motors are used on the Pennsylvania Railroad locomotive of fig. 204.

The Single-phase Motor.—The single-phase commutator motor, as a later development, naturally contains many of the constructional features suggested by experience with the

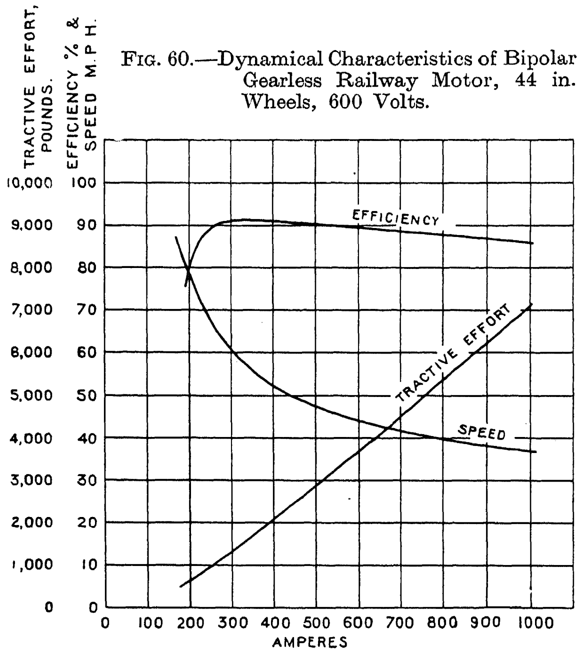


FIG. 60.—Dynamical Characteristics of Bipolar Gearless Railway Motor, 44 in. Wheels, 600 Volts.

continuous current motor. When made for gearing to a driving axle it has a cast steel frame, which, however, now serves no purpose in the electrical design, and is therefore perforated for the sake of lightness. This is frequently split, the alternative being to have a very large frame head aperture at one end for the purpose of allowing the introduction of the field laminations. The field winding is in part at least distributed in slots in the laminations. The armature is generally similar in appearance to that of the continuous current motor, though the commutator is usually larger and the number of segments greater, whilst the slots are often

partially closed. It is moreover usually of the multiple drum type and provided with equalizers. The brushes are much more numerous than in the case of continuous current motors, there being usually a brush holder for each pole, and in some types of motor intermediate holders in addition. The air gap is smaller than in the continuous current motor, being the result of a compromise between the designer, who is hampered by the necessity of keeping down the exciting field turns, and the operator who desires as large a gap as is practicable for mechanical reasons. It is with the object of bringing the equivalent gap as nearly as possible to the dimensions of the mechanical gap that the slots in stator and rotor are often partially closed.

Single-phase commutator motors may be divided into two general classes, viz. series motors and repulsion motors. The series motor is an adaptation of the continuous current series motor to the requirements of alternating current operation. It is in the main of one type, with a number of minor modifications, principally in the means adopted to secure satisfactory commutation. The repulsion type includes the repulsion motor proper with the Latour and Winter-Eichberg motors, the Deri motor, and other modifications involving the same principle. The main distinction between the two types, from the operating view-point, is that whereas the repulsion type works satisfactorily from a certain comparatively low speed to about synchronism, above which speed the difficulties of commutation increase, the series type works best from about synchronism upwards. All single-phase commutator motors are compensated in the sense that both field and armature carry distributed windings such that the fluxes due to the currents in them neutralize one another in so far as they are co-axial. Every such motor has, or may be considered to have, three windings, viz. an exciting winding, an inducing winding, and a compensating winding.

The compensated series motor is represented diagrammatically in figs. 61, 62 and 63, in which the inducing winding I, is on the armature, whilst the coaxial compensating winding C, having the same magnetomotive force as the armature winding, together with the quartered exciting winding E, are on the stator. In fig. 61 the compensating winding is connected directly in series; in fig. 62 it is short-circuited on itself,

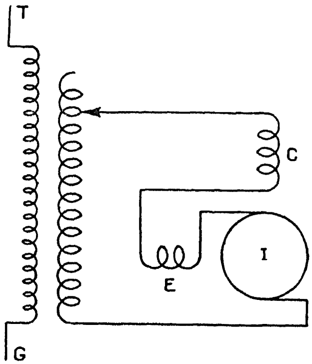


FIG. 61.—Compensated Series Motor, Series Compensation.

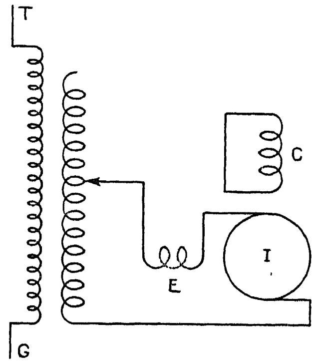


FIG. 62.—Compensated Series Motor, Induced Compensation.

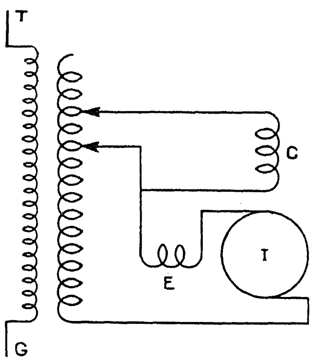


FIG. 63.—Compensated Series Motor, Induced and Shunt Compensation.

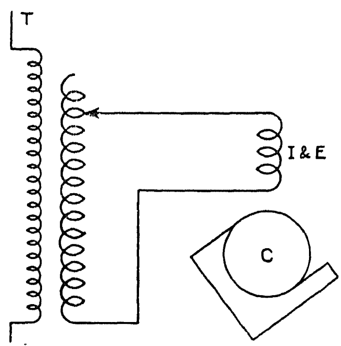


FIG. 64.—Repulsion Motor.

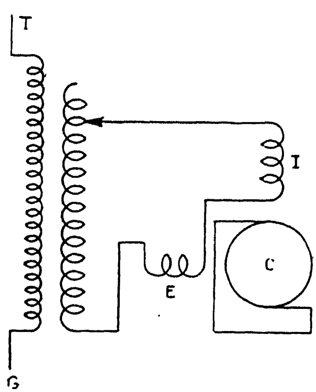


FIG. 65.—Repulsion Motor.

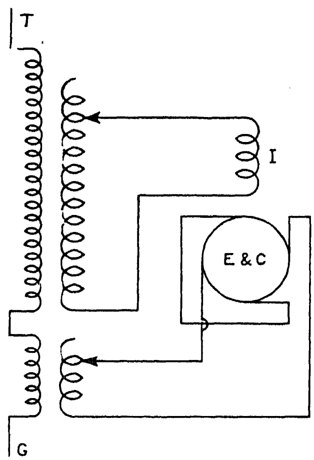


FIG. 66.—Winter-Eichberg Motor

and excited by induction from the armature; in fig. 63 the compensating winding is given an additional excitation in shunt with the motor in order to assist commutation, as will be explained later. The simple repulsion motor is represented diagrammatically in fig. 64 or in the equivalent form of fig. 65. Here the armature brushes are short-circuited, the armature current being induced by the field I. The torque per ampere can be varied by shifting the brushes, that is by varying the ratio between exciting and inducing field-turns. In the Latour-Winter-Eichberg motor (fig. 66) the main armature brushes are also short-circuited and the coaxial inducing winding is as in the simple repulsion motor, but the cross exciting field is produced by a series current fed to the armature through quartered auxiliary brushes. The Alexanderson type has the repulsion connection (fig. 65) for starting; but is a series motor of type fig. 63, when the period of initial acceleration has been passed.

Commutation in Continuous Current Motors.—The chief difficulties of design of the single-phase commutator motor centre about the problem of commutation. In the continuous current motor the field due to the armature current is practically a stationary one, for as the armature in revolving carries its field round, the commutation of the current keeps restoring it, and the combined effect is that of a stationary field. Without interpoles therefore the e.m.f. in the short-circuited coil is the same as if the armature conductors were cutting the armature field. If therefore interpoles are provided which furnish a line of force through the short-circuited coil to replace each one taken out as the coil cuts the armature field, the reversal takes place non-inductively and without e.m.f. in the coil. The commutating field has accordingly to neutralize the armature field at the level of the armature conductors (including slot field as part of the armature field). The commutating field winding is required to provide a little greater magnetomotive force than the armature, in order that it may neutralize this at the level of the armature conductors. In railway motors the excess is usually between 15 per cent. and 25 per cent., but if the motor is satisfactory for its purpose a very exact adjustment is immaterial, and no perceptible change in the commutation can be detected with

quite a considerable change in commutating field, since the brushes are able to take care of the commutation as long as certain limits of slot reactance or of e.m.f. in the short-circuited coil are not reached. If then an approximately correct interpole winding is supplied the machine will commutate satisfactorily at all speeds and loads, since the commutating field is always proportional to the armature field.

Commutation in the Single-phase Motor. In the single-phase motor, on the other hand, the exciting field flux, which threads the short-circuited coil, is in a continual state of variation, resulting in an alternating electromotive force in this coil which varies as the frequency and as the total field flux. As this e.m.f. is independent of the armature speed, it cannot at all times be counteracted by the movement of the short-circuited coil in any imposed field, for such field would have little effect when the motor is moving slowly. This e.m.f. is able to set up a very large current in the short-circuited coil. For instance, if the exciting field had fifteen series turns, a low resistance turn embracing the whole flux of this field would carry a current opposing almost as great a magnetomotive force, and therefore of value approaching fifteen times the field current. The effect of this current is to neutralize part of the field, the turn acting like the short-circuited secondary of a current transformer. If now resistance be inserted in the turn one effect is to put up the primary or field volts, so that the current changes more slowly than in the inverse ratio of the resistance. Actually, the magnetic leakage between field and short-circuited coil reduces the induced current in the latter below the figure given above, but nevertheless a large current of this nature has to be dealt with by the brushes while the load current is being commutated; and though it diminishes as the speed rises it is always there and unavoidable. The commutation of the load current presents no undue difficulty, but it is this inseparable and considerable local current which, although sometimes giving but small evidence of its presence in visible sparking, is nevertheless very destructive to commutator and brushes. It is this also that requires the single-phase motor to be a compromise at almost all points, making it a very high development of the designer's art.

Shunt Commutating Coils.—The e.m.f. in the short-circuited coil is in quadrature with the exciting field; and a commutating field to be effective in neutralizing the induced e.m.f. by the motion of the conductors must accordingly be in quadrature with the main field. This is obtained in the series motor by exciting the commutating field in shunt with the line as shown in fig. 63, in which the shunt excitation is impressed as part of the compensating winding. A similar expedient is adopted in the Alexanderson motor.

Influence of Commutating Difficulties on Design.—In the single-phase motor, then, it is necessary at times, and particularly at low speed, to put up with worse commutation than would be tolerated in the continuous current motor; but there is a limit to this, and it is accordingly necessary also to work with lower flux per pole than would otherwise be desirable, in order to reduce the e.m.f. in the short-circuited coil. The product of the total flux and the ampere-conductors on the armature is, however, a measure of the torque, which must be considered to be prescribed. Hence the tendency is towards a large number of poles, and a large number of armature conductors, whilst it is expedient to employ lower starting torque than might otherwise be considered desirable. With large ampere-conductors, however, either a large diameter of armature must be permitted or a greater density of conductors than the best practice would approve.

The greatest permissible e.m.f. in the short-circuited coil is, according to Mr. Lamme, 6 to 8 volts if resistance leads are used between armature coil and commutator, and about a half of this if these are dispensed with. But this e.m.f. approximates to, and is in fact usually somewhat greater than, the volts per commutator bar at full voltage, although it is less than a half the limiting figure which experience has evolved for continuous current motors. Hence the tendency is towards large commutators, with a large number of bars and low armature voltage. These motors are usually designed for an armature voltage of from 200 to 300 volts, and the limit is imposed by considerations of practicability rather than of desirability.

The reactive voltage of the motor is practically the exciting field voltage; and accordingly this voltage must be kept low in order to reduce the wattless component of the current.

The field voltage varies as the number of poles, the turns per pole, the flux per pole and the frequency. With the product of the number of poles and flux per pole determined by the torque, and with the frequency prescribed, the reactive voltage can only be kept low by keeping the exciting turns low for the flux required, that is by employing a small air gap and working at low saturation. Compensated series motors usually have some 20 per cent. to 25 per cent. of the number of exciting turns that would be used on continuous current motors of like capacity and voltage. The reactive voltage also clearly limits the possible capacity of the motor, particularly when the power voltage is limited by considerations of size of armature. It should be noted that both the e.m.f. in the short-circuited armature coil and the reactive component of the voltage vary as the frequency; and accordingly a reduction in frequency eases the difficulties of satisfactory design throughout.

Limitations of design may be inoperative over certain ranges of speed or capacity and be felt keenly over other ranges. The requirements of railway work may indeed make the limitations of design of the single-phase commutator motor so oppressive at ordinary frequencies, that sacrifice of valuable features is inevitable, and reduction of frequency to 15 or 16 cycles is practically imposed if the motor is to be considered seriously for the work. When however to the natural limitations of design, are added restrictions, requiring the motor to work in approximately the same space as the continuous current motor of equal capacity, the difficulties of the designer are further increased and the wonder is perhaps less that the motor has been found wanting in features generally considered indispensable, than that so successful a compromise has been devised to meet such exacting conditions.

In starting the single-phase commutator motor it is necessary either to use a very weak field, with consequent low torque per ampere, or to employ a stronger field and resistance leads between commutator and armature coil. Resistance leads are generally designed to reduce the induced current in the short-circuited coil to approximately the same value as the load current, a condition which results in minimum commutation losses. The resistance required in the leads is usually four or five times the resistance of the coil and brush contact.

Resistance leads are most necessary in motors which may be called upon to start heavy trains, in which accordingly the low speed period may be unduly prolonged, and the starting torque required, high. Where however conditions are less exacting and the period of acceleration is short, an inferior initial commutation may be tolerated. When resistance leads are not employed the initial acceleration is usually effected by using a field sufficiently weak to avoid excessive e.m.f. in the short-circuited coil; and at the same time employing a very heavy armature current to obtain as high a torque as practicable in the weak field. This of course causes larger loss in the motor than when resistance leads and more normal currents are used. In the Latour-Winter-Eichberg system, the motor is started with comparatively high voltage on the stator and low voltage on the auxiliary brushes, which in this system correspond with the exciting field terminals. As the motor gains speed the auxiliary voltage is raised, the accelerating current being at the same time kept from falling unduly by increase of stator voltage. The Alexanderson motor possesses several features of interest. The armature is wound with a short pitch winding, commutation taking place in the fringe of the exciting field; and this requires the compensating winding to extend only over the pole face. The compensating winding has double the number of turns of the armature, so that when, in starting, it is acting as the inducing winding it causes an armature current to flow of value double that taken from the line; thus furnishing the required large starting torque with comparatively weak field. Of course any other ratio than two between compensating turns and armature turns may be employed if desired. When the motor has been started as a repulsion motor, the series connections are made and the distributed stator winding is then used as a short-circuited compensating winding; but since, as in the ordinary series motor, this winding is able to act as a commutating winding also, it is given an auxiliary excitation in shunt with the motor, thereby effecting its purpose in the manner explained above. Experience indicates that the repulsion type of motor is inferior to the series type for general railway work, though its commutation is somewhat superior at low speed; and Alexanderson's modification in combining desirable features from both, those of the repulsion motor for starting

and those of the series motor for running, would appear to have advantages over the ordinary compensated series type.

Dynamical Characteristics of Single-phase Railway Motor.—Dynamical characteristic curves of a typical single-phase motor are shown in fig. 67. The changes produced in these characteristics by variation in gear or size of wheel present no difficulty, but these due to variation in voltage

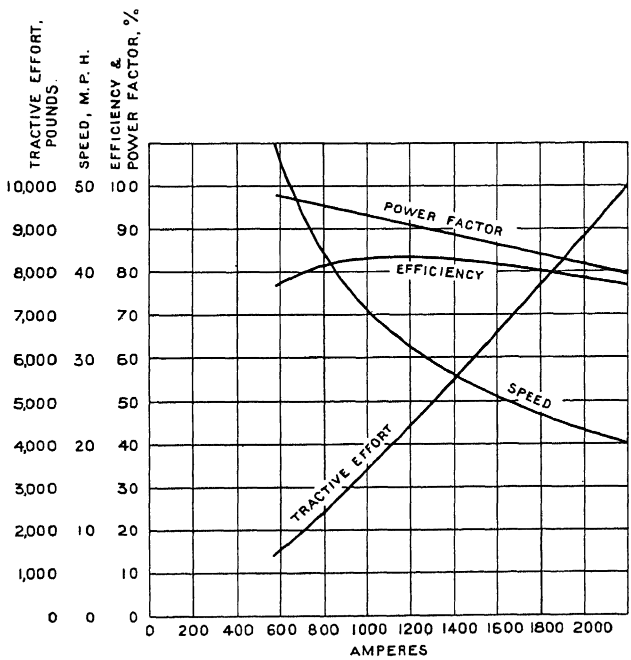


FIG. 67.—Dynamical Characteristics of Single-Phase Railway Motor. 79/34 gear, 63 in. wheels, 300 volts, 25 cycles.

are a little more involved than in the case of the continuous current motor. In the compensated series motor, if curves for voltage v are given, and if for current c the power factor is k, then kv is the power component and $v\sqrt{(1-k^2)}$ the reactive component of the voltage. The latter quantity is independent of the applied voltage, so that if the speed is required for voltage v' and current c, k' being the corresponding power factor :—

$$v'\sqrt{(1-k'^2)} = v\sqrt{(1-k^2)}$$

or

$$k' = \frac{\sqrt{v'^2 - v^2 + k^2v^2}}{v'}$$

If the resistance of the motor is r:

$$\frac{\text{Speed at voltage } v' \text{ and current } c}{\text{Speed at voltage } v \text{ and current } c} = \frac{k'v' - cr}{kv - cr}$$

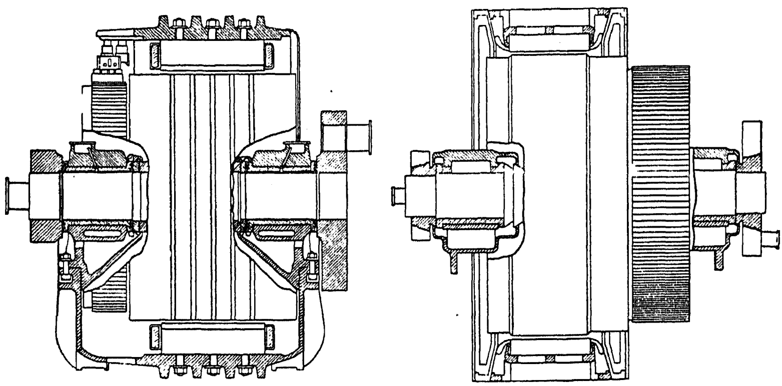
These equations determine the power factor and speed; whilst the tractive effort is sensibly independent of the voltage. The effect of change of frequency on speed can be determined in a similar manner, for the reactive component of the voltage is proportional to the frequency.

Variation of Power with Speed.—The power of the single-phase commutator motor varies much less with speed than that of the continuous current motor, a feature which makes the former type less suited for service in which a high rate of acceleration is desirable. Its dynamical characteristics in fact approximate to those of the steam locomotive, being well suited for long distance work in fairly level country, but showing to less advantage where gradients are severe or stoppages frequent. A consequence of the comparative uniformity of the power input of the single-phase motor—a feature readily noticeable in service records—is that the effect of gradients on speed is more marked than in the case of the continuous current motor. For example, comparing the motors of figs. 44 and 67 and supposing them hauling such trains as would make the maximum speed reached on level track, say, 45 m.p.h. in each case: if each train strikes such a gradient as requires the tractive effort to be three times as great as that at free running speed on the level, the continuous current train slows down to 29 m.p.h., or by 35·5 per cent., whilst the single-phase train slows down to 25·5 m.p.h., or by 43·5 per cent.

Heating.—The heating of single-phase locomotive motors presents no peculiar features other than that from the large amount of heat to be carried away it is generally impracticable to dispense with forced draught. When the motors are carried on the axles, however, the heat is necessarily so localized on account of undue density of armature conductors that if

employed in heavy service large maintenance charges must usually be faced.

The single-phase commutator motor requires the following conditions to render it suitable for railway work. In the first place, the frequency should be low: the accepted compromise in this respect is about 15 cycles; for below this the transformers become unduly heavy and the separate pulses of power are felt unpleasantly, whilst above it the difficulties of design of the motor become onerous. In the second place, the motor should not be unduly crowded. It is naturally very much larger than the continuous current motor of equal capacity, and if it is designed to occupy approximately equal space it can only be at great sacrifice. Fig. 68 * is interesting as showing



A B

FIG. 68.—Comparative sizes of 2,000 H.P. continuous current motor of Pennsylvania Railway (A) and 800 H.P. single-phase motor Lötschberg Railway Locomotive (B).

the relative sizes that continuous current and single-phase motors assume when restrictions are not imposed on them. The service capacities of these two motors may, it is true, not be in the ratio of their ratings, but comparison on the basis of service capacity would probably be still more in favour of the continuous current motor. Locomotives with motors carried above the underframe, and driving groups of axles, are accordingly natural to the single-phase system, in which individual driving imposes undesirable restrictions on the design of the motors. In the third place, the service should be worked with

* See *Journal Inst. E.E.*, vol. 52, p. 386.

comparatively low rate of acceleration, for it is expensive in cost of equipment, in power and in maintenance to emulate the rate of acceleration practicable with the continuous current motor, a conclusion which may be expressed by the dictum that the economical rate of acceleration is lower with the single-phase commutator motor than with the continuous current motor.

Polyphase Motor.—The polyphase motor has the great advantage of dispensing with the commutator with its attendant brush gear ; and these are always a source of weakness in a motor which is not under continual observation. The merits of the induction motor with regard to its ability to work for long periods with little attention are well known, it being in this respect the first among motors. It can be designed without difficulty or sacrifice for any frequency, and any voltage that may be considered desirable. Thus it is not necessary for the motor to deal with large currents, and the frequency may be chosen to suit the convenience of generating plant. Its torque is uniform, like that of the continuous current motor. Its speed varies between narrow limits, and is kept within bounds on down-gradients by the machine acting as a generator and delivering power to the line. Its wound rotor, provided with the usual slip rings by means of which resistance can be introduced in the armature circuit, enables a high torque to be maintained until full speed is reached. Its chief constructional defect, for the present purpose, is that a small air gap is a necessary feature of successful design, the more so the higher the frequency ; this is very undesirable in a railway motor.

Dynamical Characteristics of Polyphase Railway Motor.—The dynamical characteristics of a three-phase railway motor are given in fig. 69, the curves being those of the motors employed for the Cascade Tunnel electrification. The modifications required to adapt the curves for other gear reduction or size of wheel offer no difficulty ; and operation at other voltage is of no particular interest, since voltage control of speed is impossible with this type of motor.

The constant speed characteristic of the polyphase motor is an objection to its use in railway service, for reasons that have been sufficiently explained already. If two of the Cascade Tunnel locomotives were coupled together for hauling a train,

one having 60 inch wheels and the other 58 inch, that on the larger wheels would take about 1,000 h p greater load than the other. With a 4 inch difference in wheel diameter between the locomotives, that on the smaller wheels would practically never take load, and indeed would often generate and thus act as a brake on the train, driven by the other locomotive. By

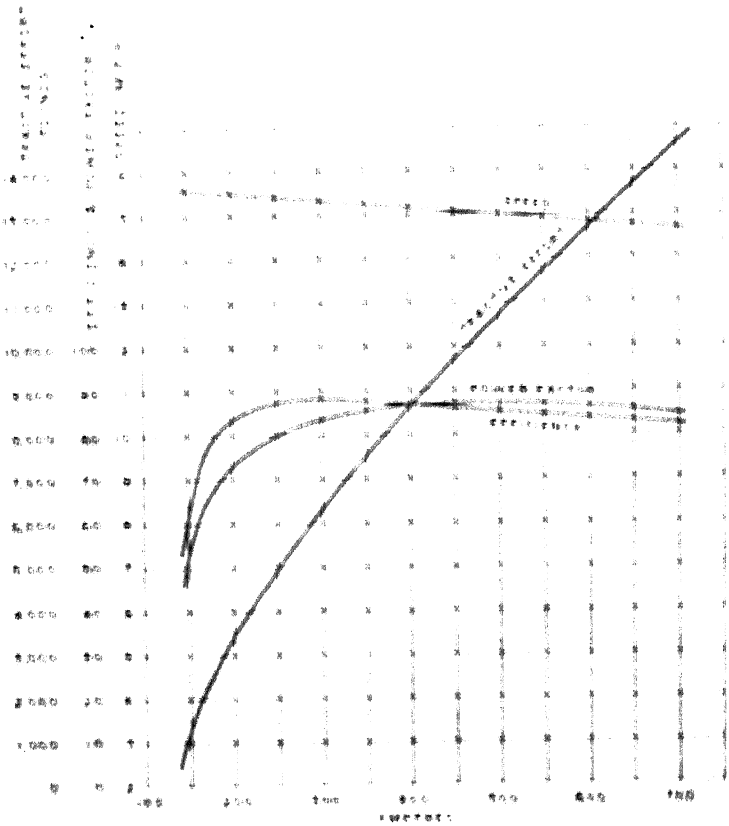


FIG. 69 Dynamical Characteristics of Three Phase Railway Motor (G N R Cascade Tunnel) 91 19 gear, 60 in. wheels, 500 volts, 25 cycles

putting resistance in the rotor circuit of the motors which drive the larger wheels the loads could be equalized for a particular weight of train, and a particular gradient; but it is impracticable to keep them so equalized under the varying conditions of service. Thus the ideal of using the whole adhesion of all driving wheels to provide tractive effort cannot

practically be realized when the wheels are unequal. These considerations effectively limit the use of the polyphase system to circumscribed areas and to railways in which it is convenient always to work trains by a single locomotive, for it would be impracticable to allow a minor consideration, like the wear of driving wheels, to interfere with the working of the traffic. For a similar reason, the use of multiple unit trains is impracticable in this system.

Split-phase System.—An objectionable feature of the polyphase system is that it requires two overhead lines for power supply, and these are difficult to instal suitably, particularly at special work. A modification, which avoids this feature, is the single-phase-polyphase or "split-phase" system, in which the supply is single-phase and the driving motors polyphase—a suitable phase converter being carried on the locomotive for supplying current to the motors in the correct phase relation. The split-phase system was due to Mr. E. F. Alexanderson, and in the form proposed by him two-phase driving motors were employed, power being supplied to one phase directly, and to the other from the windings of the phase converter. The phase converter is practically a light running two-phase induction motor, having one phase connected with the supply, and the other with the corresponding phase of the driving motor. If, however, the windings of the phase converter and motor were simply in parallel, the current taken by the motor would so displace the second phase as to render the arrangement ineffective for its purpose. Mr. Alexanderson accordingly connects the corresponding second phases of motor and converter in series with one another, and, in order to preserve the correct phase relation in the motor, introduces into the circuit a component voltage in the phase of the supply. The arrangement will be understood from the diagram (fig. 70), in which however a three-phase motor is used in much the same manner as described. This diagram shows the system as used on the locomotives of the Norfolk and Western Railway. The phase converter makes better use of material than the motor, since it can be designed for limiting peripheral speed, and moreover usually feeds several motors. Its weight is therefore much less than the aggregate weight of the motors.

STARTING AND HEATING OF POLYPHASE MOTORS.—In the polyphase system the train is started by inserting resistance in the secondary circuits of the motors and reducing it as the speed rises. Sometimes, particularly when stops are infrequent, and only one running speed is desired, as in the Great Northern (Cascade Tunnel) locomotives, this is the whole process of

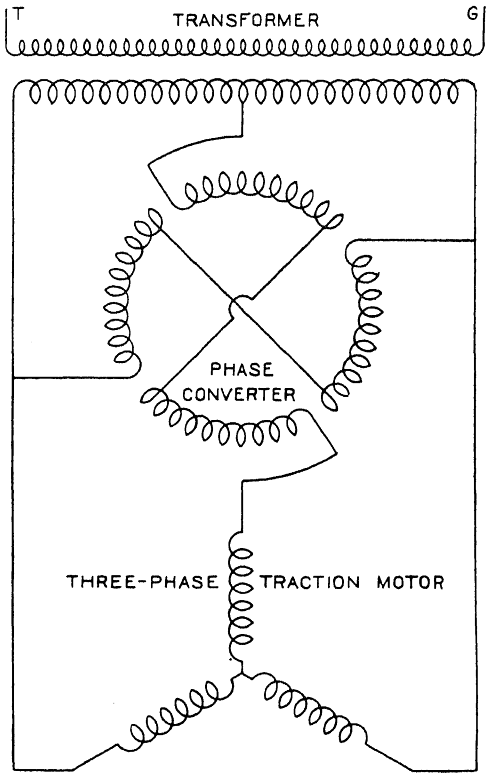


FIG. 70.—Connections of Split-Phase System.

starting; but more frequently cascade or concatenated control is employed in some form or other. Thus if all motors are similar they may be grouped in pairs so that with one stator winding connected to the line the corresponding rotor winding is connected with that winding of the second motor whose stator winding is closed on itself through a resistance which is cut out as the motors gain in speed. The limiting speed with similar motors concatenated is approximately a half that

with the motors in parallel on the line. The polyphase motor is fairly efficient when running near to synchronous speed, and where stops are infrequent the service capacity is comparable with that of the continuous current motor. The heating is well distributed, and, as the system is essentially one for locomotive operation, there is usually no difficulty in getting rid of the heat developed.

CHAPTER IV

MOTOR CONTROL

The apparatus now used for the control of train-driving motors, whilst not differing in principle from that employed for similar purposes in other departments of the electrical engineering industry, has become highly specialized and of considerable complexity. Among the causes that have contributed to this result, doubtless the chief is the need for reliability under the exacting conditions of a service in which shock and vibration are normal features, and supervision of the apparatus is, at best, occasional. With frequent starting the need for saving energy arose and led to the development of series-parallel control, now almost universally used with continuous current motors. The necessity of dealing with larger and larger currents as the art progressed taxed the resources of the control; and the desirability, first appreciated by Mr. Sprague, of operating trains of motor and trailer coaches in suburban service from a single driver's cab led to the invention of the multiple unit system of control, now in general use even for locomotives. The drum type of controller, such as is used on tramcars, was used on many of the early locomotives (e.g. the Central London Railway locomotives, the Quai D'Orsay terminal locomotives, the Buffalo and Lockport locomotives, etc.). Such controllers, however, soon tended to become unwieldy in size and unduly heavy to work; and the necessity for locating them where adequate space was available and applying power to operate them, was early realized. Mr. Sprague saw the possibility and advantage of controlling this power from a central point whilst employing it to operate controllers simultaneously on the several motor coaches of the train, in whatsoever combination these might be assembled. The system so developed is the basis of all modern locomotive

control systems, which consist essentially of two main elements: one, a motor controller for making the various power connections required, and the other a master controller, which is the instrument by means of which the driver controls the working of the motor controller.

Motor Controller.—In its first stages the motor controller was of the drum type, the shaft of the drum being turned by a motor. With increase in current, separate switches or contactors were devised to deal with it, these being actuated each by a separate engine, either electro-magnetic or pneumatic. The tendency now appears towards a combination of the two; the contactors are retained, some of them being actuated by separate engines, and others grouped and actuated mechanically by means of a cam shaft driven by a suitable engine, either electro-magnetic or pneumatic. The cam shaft carries also a controller drum by means of which the connections for the currents which control its motion are changed; for it may be said that in all cases the ultimate control is electric.

The motor controller may be taken to comprise all controlling apparatus which deals with main-circuit-currents. It consists of an assemblage of contactors, for making the successive connections required to bring the train to speed, a reversing switch for each motor, with such rheostats and other auxiliary apparatus as the system of operation requires. The cut-out switches, by means of which defective motors are rendered inoperative, are, in multiple unit trains, frequently inserted between master controller and motor controller; and really effect their purpose by rendering the motor controller inoperative. In locomotives, however, the cut-out switches are now often located in the motor circuit, consisting in fact, in the larger locomotives, of a suitable assemblage of contactors. In the continuous current system of operation, the motors are almost invariably grouped in pairs, which by suitable operation of the motor controller can be connected in series or in parallel, one or more such pairs being associated with each motor controller. The means adopted for applying force to close the contactors and bring about other mechanical movements in the control system, whether electromagnetic or pneumatic, are not of primary importance from the point of view of motor control, for either system can be made to work satisfactorily

ther can be applied to furnish a given combination and ce of connections. It is easier to obtain a large force atically than electromagnetically, and contactor finger res, reversing forces, etc., are therefore usually made hat greater in the former system : against this advantage e set some risk of freezing of valves in cold weather, due er having been carried over with the air. Sometimes ontactors involved in a particular operation (e.g. in g from series to parallel grouping of motors) are actuated oup by means of a pneumatic engine and cam shaft, as locomotive control of fig. 86, in which the twelve transfer es numbered 27 to 38 are so operated, and the remainder control is electromagnetic.

TACTORS.—The contactor is a switch of construction riate and adequate for carrying and, on occasion, ng the motor current : it is used in an upright position, hen out of action is kept open by gravity, assisted in ases by a spring : it is closed by forces governed directly irectly by the master controller. The contactor is ed to close with a wiping motion of the tips, in order od contact between them may be secured. The movable inged to its operating lever and thereby rendered capable ependent motion against a spring, which also serves to ate the opening of the circuit when power is cut off from erating mechanism. The pressure between contacts is made of the order of 15 to 20 lbs. per inch width. the contactor is intended to open under power, the arc ured in an incombustible chute, being generally blown ds by a strong magnetic field occasioned by a special ut coil connected in series with the main contacts, sometimes deflected to a copper disc and then ruptured action of its own field : in some recent types the arc is eflected to a pair of diverging horns, along which it , thus avoiding the burning of the main tips.

RLOCKS.—Many contactors carry auxiliary switches as interlocks, whose function is to render the making ain connections contingent on the making or breaking r connections. Thus the main circuit contactors carry nterlocks as require the main circuit to be open if the r is to be actuated, and possibly in other contingencies : ntactor connecting motors in series is interlocked with

that connecting them in parallel in such manner that neither can be closed unless the other is open; some of the contactors involved in the transition between series and parallel may also carry interlocks to ensure that the various operations take place in the order required. The cut-out switches, too, are frequently interlocked with some of the contactors, so as to make appropriate changes in the method of control when motors are rendered inoperative. In automatic methods of control, interlocks are freely used to secure the required sequence of the operations. In cam-operated systems the interlocking is for the most part mechanical, the correct sequence and timing of the action being determined by the rotation of the cams: this constitutes one of the chief advantages of such systems.

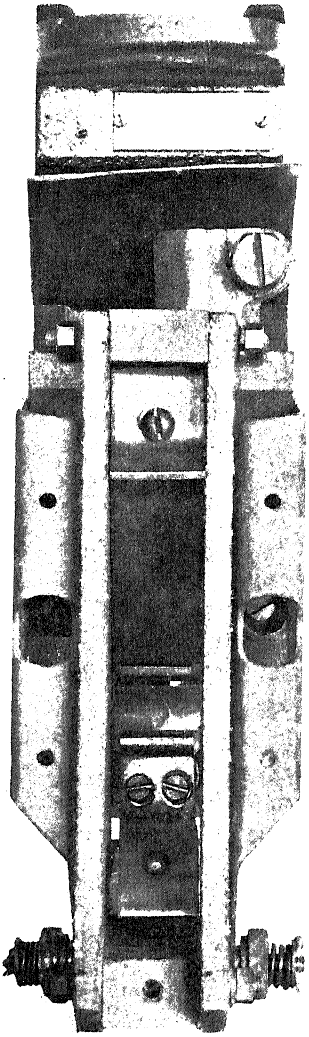


FIG. 71.—Electromagnetically operated Contactor. Front View.

Figs. 71 and 72 give views of low-voltage electromagnetically operated contactors. These contactors are wound with enamelled wire, which yields a high space factor, besides standing heat and vibration. Fig. 73 shows a line breaker unit designed for electropneumatic operation. Fig. 74 shows a cam-operated contactor with arc-chute removed.

The alternating current contactor is constructed on the same general principles as that for continuous current; but when electromagnetically operated from the alternating supply, naturally has a laminated magnetic circuit. In order to prevent

chattering, due to the pulsating force on the plunger, part of the pole is surrounded by a short-circuited conductor, called a "shading coil," in which a current is set up by induction. The effect of this is to throw the magnetic field in the shaded portion of the pole out of phase with the remainder, and thus force is kept on the plunger continuously as long as the current is on.

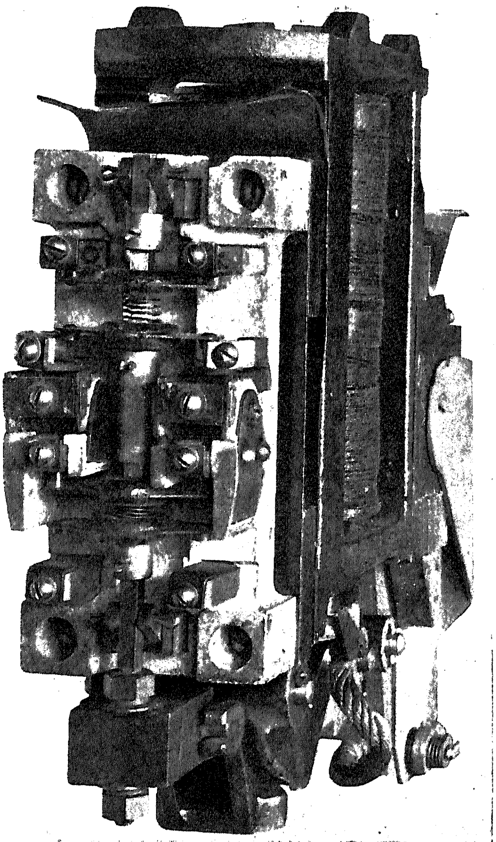


FIG. 72.—Electromagnetically operated Contactor. Back View.

Alternating current contactors have usually to deal with large currents on account of the low voltage of the motors.

REVERSERS.—The reverser in multiple unit train operation usually takes the form of a rocker-arm or reversing cylinder

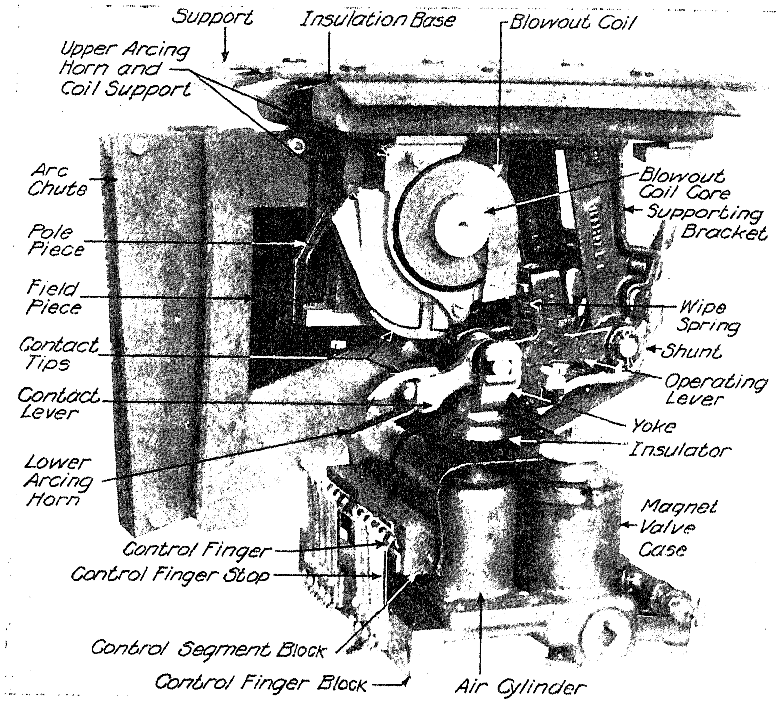


Fig. 73.—Line Circuit-Breaker, pneumatically operated.

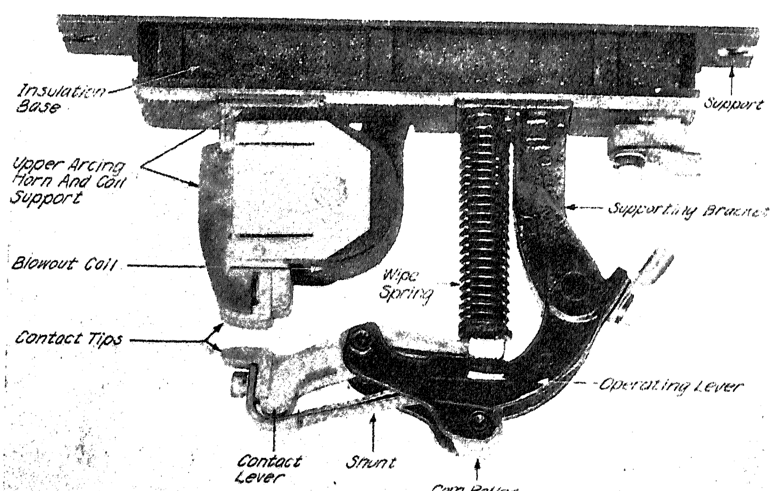


Fig. 74.—Cam-operated Contactor.

carrying the necessary contact segments and engaging with suitable fingers for reversing the group of motors comprising a control unit. This is thrown by a suitable electromagnetic or pneumatic engine operated by means of the master controller. Where large motors are used, however, and particularly in locomotive work, it is now common practice to employ an assemblage of contactors for reversing purposes. The reverser, in whatever form, is always interlocked with the line contactors in such manner that it cannot be thrown if the

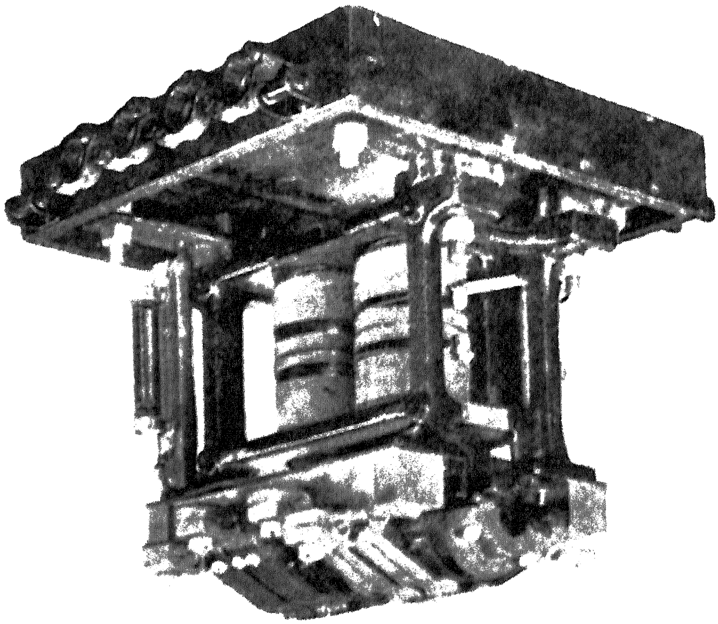


FIG. 75. Electromagnetically operated Reverser.

main circuit is closed, and must be thrown before the motor controller can be operated. Reversal may be carried out either on armatures or fields, the modern tendency being towards reversal of fields. Fig. 75 shows an electromagnetically operated reverser as used in multiple unit train control.

AUTOMATIC CIRCUIT BREAKERS.—Automatic circuit breakers, opening on overload, are included in the motor circuits. In some systems these are special switches, held in by mechanical locks, which are released on overload by the action of a series

tripping coil. The circuit breakers are interlocked with the control system, but are otherwise independent of it, special two-way switches in the driver's cabins being used for setting and tripping them. They can be tripped by the driver whatever the position of the master controller; but reset only when the handle is in the "off" position and then only when the main circuit is open. In other systems one of the line contactors is used as overload circuit breaker, in conjunction with a separate relay (see fig. 76), which, on tripping, opens the energizing circuits of the contactors. As in the former

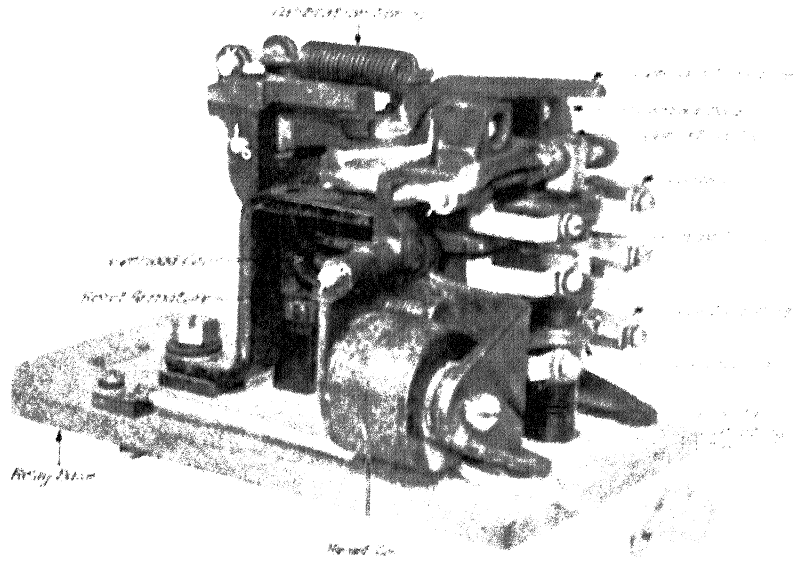


Fig. 76.—Overload Relay.

system, the breakers are interlocked with the master controller, and the relay is set by means of a separate switch. The driver's trip is, however, no longer necessary since he can open the breaker without tripping the relay. Like the former also this system is particularly applicable to multiple unit working, the relays or breakers being set throughout the train by the single action of the driver, and the breakers opened simultaneously at will. In locomotives the remote control features are sometimes dispensed with, the relays or circuit breakers being set by hand.

STARTING RHEOSTATS.—The rheostats used for starting the motors in the continuous current system are composed of cast iron grids of special composition, calculated to give uniformity, flexibility, and comparatively small temperature co-efficient of resistance. They are constructed, assembled, and installed so as to allow as good a natural circulation of air as practicable, being often carried on the underframe of the motor coach or locomotive. The grids are made so that any one of them can be removed and replaced in case of breakage, without dis-assembling the whole rheostat. They are, of course, insulated from the frame, and the whole rheostat is mounted on insulators suited to the supply voltage. In multiple unit trains in which the rheostat is in use only for short periods, the grids are run at much higher current density than it would be practicable to employ continuously, particularly on the earlier points of the control. In locomotives intended for general service, and particularly in goods locomotives, since the period of accelera-tion may be an extended one, the rheostats are best designed to carry the accelerating current almost continuously where practicable. Continuous current locomotives frequently have to carry ballast to give sufficient adhesive weight, so that, provided space can be found, there is usually no objection to a large number of rheostats on the score of weight, and little on the score of expense.

OTHER MOTOR-CONTROL GEAR.—Of other apparatus in the main circuit little need be said. Suitable instruments are provided for the driver's guidance. Isolating switches, of the knife-blade type, are included in circuit with each control group of motors. In multiple unit trains it is not uncommon, but by no means universal practice, to run a bus-line through the train so that in passing gaps in the conductor rails power for the motors can be picked up by any shoes that happen to make contact. Fuses are provided in the several main circuits, viz. shoe circuits, bus-line circuits and motor circuits, intended to isolate the parts in case of short circuit.

Hand and Automatic Control.—There are two general types of multiple unit control, viz. hand control and automatic control. In the former the master controller segments complete the circuits which cause the motor controller to operate, subject only to safeguarding interlocks. Each

arrangement of the motor control switches corresponds with a definite position of the master controller handle, and successive arrangements can be brought on quickly or slowly at the will of the driver. In automatic control, on the other hand, a few final arrangements only are governed by the position of the controller handle, whilst intermediate arrangements are effected by the motion of the motor controller itself, governed, however, by the operation of a current-limit-relay, which makes the final connection required to bring about a succeeding arrangement only when the current has fallen to the value for which the relay has been set. The action of the relay is, indeed, generally delayed still further by means of a time element, the practical effect of which is to make the mean accelerating current dependent on the rate of acceleration, increasing it, for instance, in starting with heavy load, or against gradient.

Automatic control was developed to meet the requirements of multiple unit trains in heavy urban service, for which it is particularly suitable. It ensures uniformity in the accelerating current and thereby favours low energy consumption and small heating of motors; it requires less expertness on the part of the driver, leaving him little opportunity for abusing the motors, and by relieving him of responsibility in this direction makes better use of his faculties for securing rapid transit. This form of control is, however, not suited to conditions where the limits of adhesion may frequently be exceeded, for if the wheels are once skidded the current immediately drops, and succeeding points are taken as rapidly as the time elements in the current relays will permit, thus tending to enhance the skidding. It is therefore particularly applicable where the power is distributed over many axles, and less so when it is concentrated on few, since in the latter case the working is usually much nearer to the limit of adhesion. In locomotive control, in which it is desirable to adjust the accelerating current to suit the needs of load and gradient, and often to keep the tractive effort near to the limits of adhesion, hand operation is the preferred practice, although calling for greater skill and experience on the part of the driver.

Master Controller—HAND CONTROL.—The master controller used for hand-controlled multiple unit trains re-

sembles a small tramcar controller in appearance, construction and operation. Its main cylinder carries segments to correspond with the several control steps or notches and is biased towards the notches by means of a star wheel and pawl. Its reversing cylinder, though interlocked mechanically with the main cylinder, so that it can only be moved when the latter is in its "off" position, is distinct therefrom and is worked by a separate and removable handle. Fig. 77 shows a typical master controller of this kind, whilst fig. 78 is a wiring diagram for a motor coach driven by two motors; the system having an earthed return, and the coaches being provided with a bus-line.

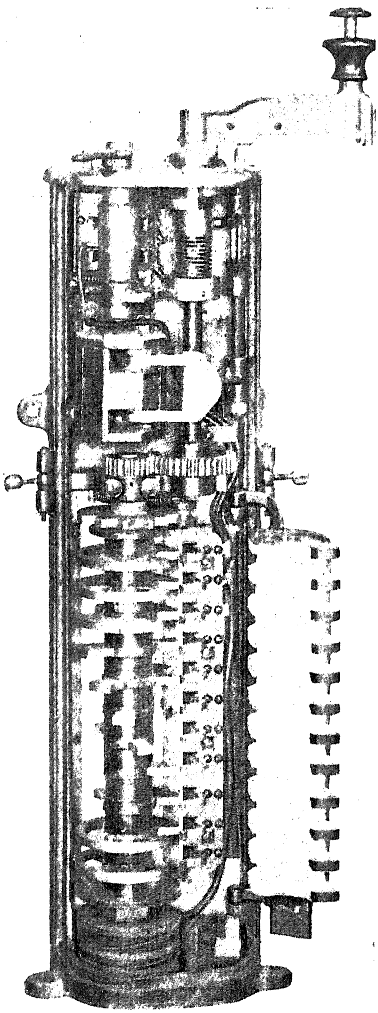


FIG. 77.—Master Controller, Hand Operated Multiple Unit system.

AUTOMATIC CONTROL.—The controller used for automatic relay control of multiple unit trains is smaller than the hand controller. It may be provided with separate reversing handle and otherwise be outwardly similar to the hand controller (see fig. 79); or it may be provided with a single cylinder for both forward and reverse motions, this being spring-biased towards the off position. The notches on the automatic controller dial plate are usually seven in number, viz.: (1) off, (2) switching or first series, (3) series, (4) first parallel, (5) parallel, (6) reverse switching, (7) reverse series. With

such a combination the first series notch causes the line contactors to close, connecting the motors, in series with all the starting resistance, between the lines. The second series notch causes the resistance contactors to be closed in correct sequence, under the control of the current-limit relay, until the full series position is reached; if, however, after automatic notching has commenced, the handle be returned to the first series notch, the automatic action ceases but the contactors do not reopen, retaining the existing condition until the handle has been brought to the "off" position. The first parallel notch permits the motor controller to pass through the transition from series to parallel connections, and also acts in arresting the progress of the motor controller in the same manner towards the later parallel combinations as does the first series notch towards the series combinations. The second parallel notch allows the motor controller to cut out resistance until the motors are connected in parallel between the lines. The controller handle may be, and usually is, brought directly to the last notch, and the acceleration of the motors takes place in due course without effort on the part of the driver. If the motors are intended to work with tapped fields, additional notches may be introduced in the controller, and this is the usual practice with hand-control. In automatic control, however, it is more usual to make the necessary connections automatically, consequent on the operation of a current relay, which causes the closing of the appropriate circuits when the current in the full parallel connection falls to a certain predetermined

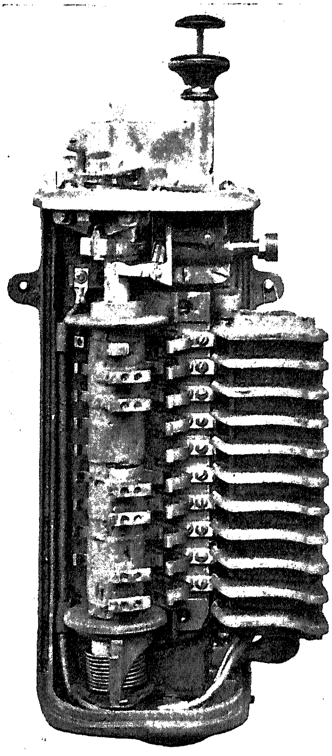


FIG. 79.—Master Controller Relay-automatic system.

value. Fig. 80 is the wiring diagram of a motor coach fitted with relay automatic control.

NOTCHING RELAY.—The current limiting relay (usually called the "notching relay"), used in relay-automatic control systems, deserves mention. As shown in fig. 81, it is fitted with two shunt coils and a series coil, the latter carrying the current of one of the motors. The contactor coils are actuated in series with one or other of the two shunt coils in such manner

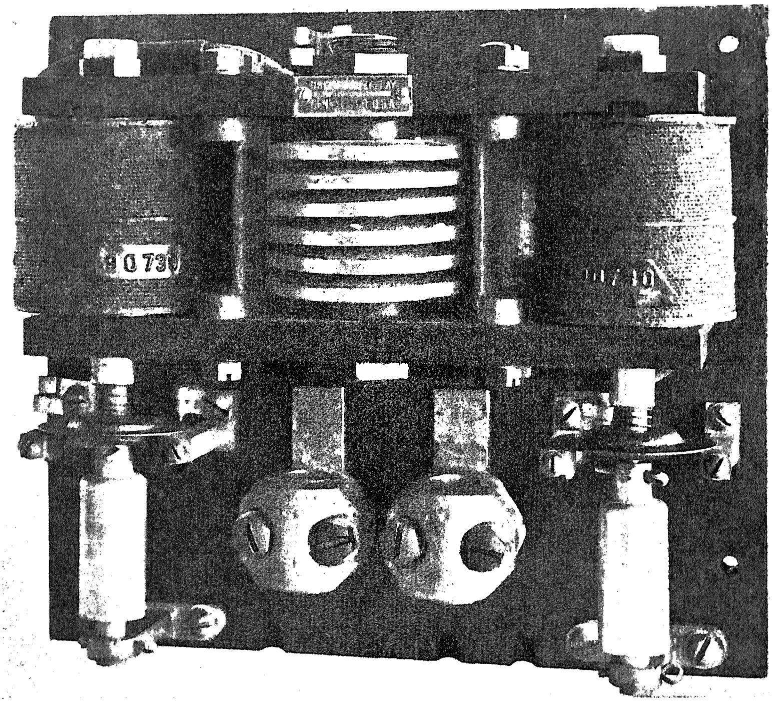


FIG. 81.—Notching Relay for Automatic Control.

that, if one is energized in series with one shunt coil, the next one to close will be energized in series with the other. Each plunger in its lowest position closes a switch in the circuit of the coil of the other plunger, so that both cannot be energized at the same time. Either plunger when raised is acted on by the main current coil, which retains it in its raised position (its shunt coil circuit having been opened by the closing of the contactor), until the current has dropped to a predetermined

value. The general method of operation is as follows: As each contactor closes, it transfers its operating coil from a lifting to a holding circuit, opening also the connection with the first relay shunt coil. The succeeding contactor is now ready to be energized in series with one of its own interlocks and the second relay shunt coil, closing when the main current has diminished sufficiently to allow the first plunger to complete the circuit. The various operations can be traced in fig. 82, and the large number of interlocks needed will be noticed in the control diagram, fig. 80.

Cam-operated Systems.—In cam-operated systems, the successive control circuits are energized from a small auxiliary

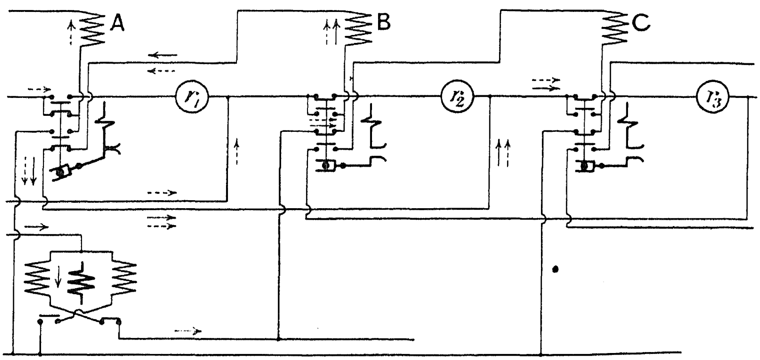
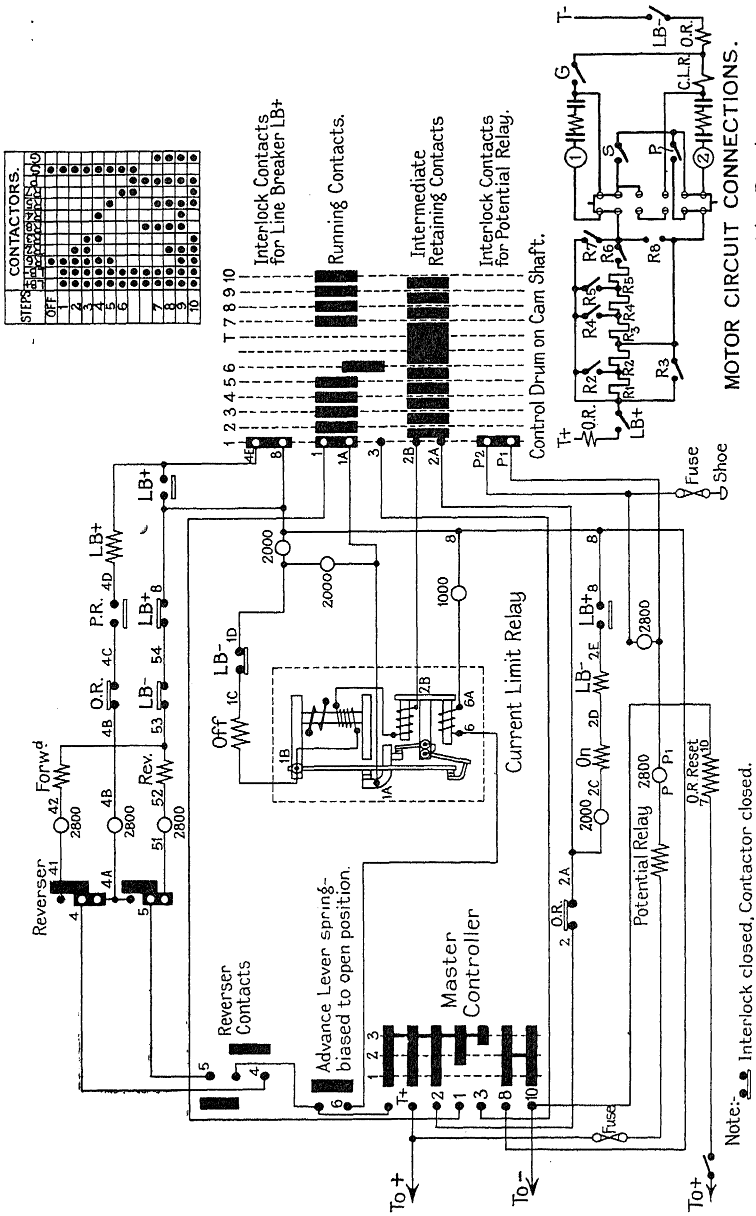


FIG. 82.—Diagram of succession in Automatic Relay Control. (Full line arrows show closing circuit for Contactor B, Contactor A being closed. Broken arrows show holding circuit for Contactor B, after closing.)

drum controller, carried on the cam shaft, thus obviating the need of interlocks on the succession contactors. The working of the cam shaft is governed by a notching relay, which permits a succeeding step when the current has fallen to a predetermined value. The arrangements can be traced in the wiring diagrams, figs. 83 and 84, which refer to a pneumatically operated cam-control system. In this the notching relay has two magnetic circuits and three shunt coils in addition to the main series coil. The series coil is the current-limiting device, and a lifting coil on the same core ensures action. The holding and by-pass coils are used in connection with an "advance lever," and by means of this device the control can be advanced step by step without reference to the magnitude of the main current,



O.R.=Overload Relay. P.R.=Potential Relay. LB=Line Breaker.

Note:- Interlock closed, Contactor closed.
Interlock closed, Contactor open.

FIG. 84.—Simplified Connections for Pneumatic Cam Control.

so long as it does not trip the overload relay. The master controller has three steps or notches : on the first the reversers are thrown, the line breakers closed and the motors connected between the lines in series with one another and with the starting rheostats : on the second the control is advanced step by step to the full series position : on the third it is further advanced until full parallel is reached. The master controller is provided with three handles, a reverse handle, an operating handle and the advance lever mentioned above. Certain interlocks, generally of the finger and segment type, are provided on the line-breakers, to prevent the throwing of the reverser when these are closed or the forward movement of the cam shaft when they are open. A fuller description of the system shown in figs. 83 and 84 is given later. Generally, the chief advantage of a cam operated system lies in the simplicity of the wiring scheme, which enables a defect to be traced with little difficulty,—a very valuable feature.

POTENTIAL RELAY.—With automatic control, when no main bus line runs through the train, it is necessary to arrange that the motor controller becomes inoperative on any coach not at the time taking energy, since otherwise the notching relay, limited by no main current, would act as quickly as the time elements would permit on the dead coach, resulting in an excessive current when voltage was restored. The desired result is effected by means of a potential relay which opens the control circuits and causes the motor controller to return to the "off" position if voltage is lost on any coach. The potential relay also minimizes the chance of flashing-over of the motors in crossing a gap in the line conductor at high speed.

Locomotive Controllers.—In locomotive control, as has been stated, it is the preferred practice to operate by hand. The locomotive controller usually accelerates in a large number of steps, by which the tractive effort can be finely graduated ; it also provides as large a number of running points as possible, in order to work efficiently at various speeds. The master controllers are therefore often much larger than those used on multiple unit trains, and differ greatly from them in appearance and construction. Fig. 85 shows the master controller used on the Chicago, Milwaukee and St. Paul locomotives of fig. 197. It has three operating handles, A, being the handle for regulat-

ing tractive effort and speed during motor running, B, the handle for controlling regenerative braking and, C, the reversing

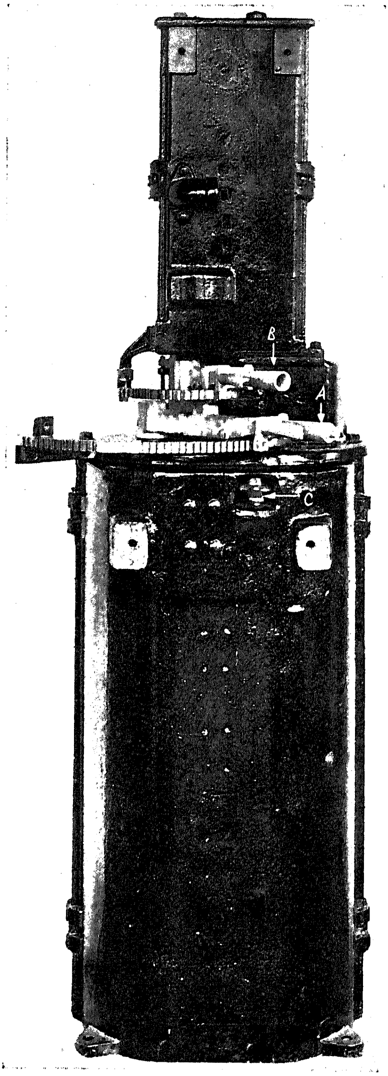


FIG. 85.—Locomotive Controller. Chicago, Milwaukee and St. Paul Railway.

handle. The lower portion of the controller has to do with the power steps, the small inverted controller being used solely

in connection with the regenerative braking features. Fig. 86 gives much simplified connections and sequence of this control in motor operation; whilst fig. 87 gives connections used in regenerative braking. Other locomotive control diagrams are given in figs. 88 and 89.*

Safeguards.—The control usually embodies certain pro-

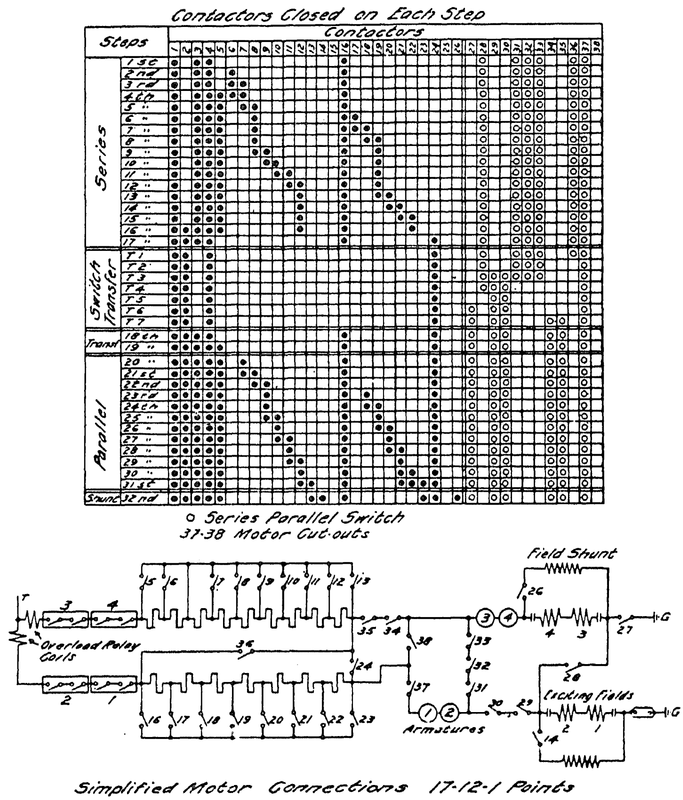


FIG. 86.—Diagram of Motor Operation, Chicago, Milwaukee and St. Paul Locomotives.

visions against accident, particularly in the case of multiple unit trains, in which there is generally only one man in the driver's cab. For instance, the control circuit is opened, and power cut off

* "Control Equipments for Direct-current Locomotives on Interurban Railways," by R. Stearns, *General Electric Review*, Nov., 1913, vol. 16, p. 854.

from the train, if the driver removes his hand from the operating handle of the controller: this is usually effected in hand control by a spring-opened switch, closed by pressure on a button in the grip of the controller handle, and in automatic control by the spring fly-back action of the handle already re-

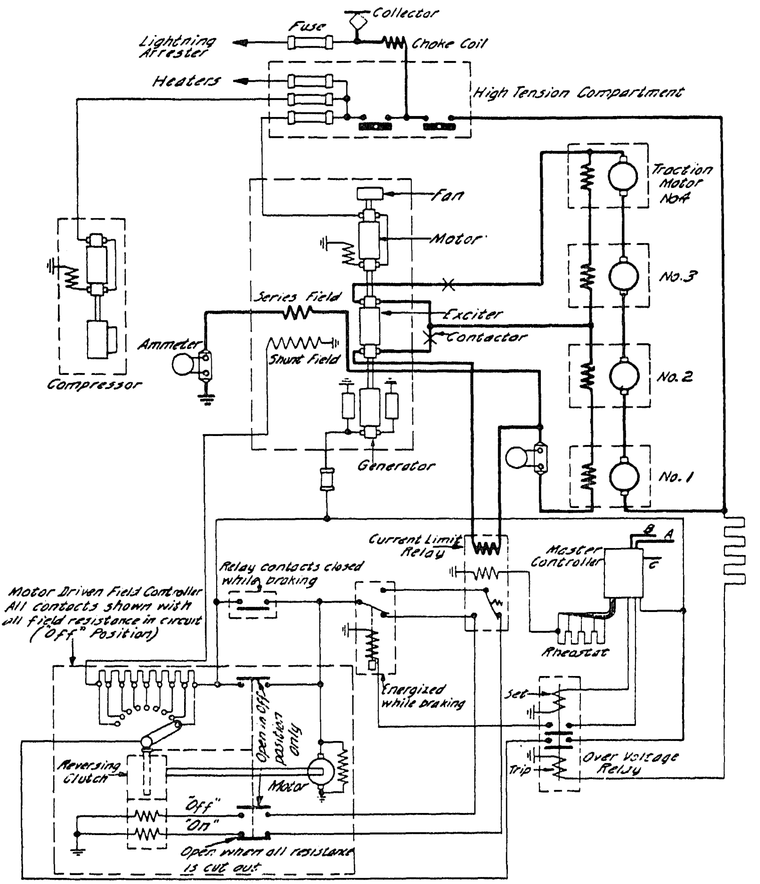


FIG. 87.—Simplified Diagram of Regenerative Electric Braking Connections.

ferred to. It is generally arranged also that brakes are set when power is thus cut off. A pneumatically operated switch, the contacts of which are connected in series with the main control wire, is often provided, being so devised as to open on reduction of pressure in the train pipe. It thus prevents the train being

started until sufficient air pressure to apply the brakes has been attained, and opens the control circuit when the brakes are applied, whether voluntarily, by driver or conductor, or by the action of an automatic trip devised to apply them if the signals are over-run.

POWER FOR WORKING CONTROLLER.—In the electromagnetic system of control, it is usual to operate the motor controller by means of current taken directly from the line in the low voltage continuous current system (800 volts is the highest so employed hitherto); or from a dynamotor or motor generator in high voltage continuous current systems. The dynamotor is a small continuous current converter, consisting of an armature, having two windings and two commutators, rotating in a suitable field. The controller is fed from one armature winding, whilst the two armature windings are usually connected in series between the lines. The action is therefore somewhat similar to that of an auto-transformer. The train lights are often fed from the same machine, and occasionally a blower for cooling the motors is coupled to the dynamotor, which then acts also as a motor to the extent required. In electropneumatic systems a battery is often used for working the magnet-valves. In alternating current systems, the controller operating current is taken from a low voltage tap of the transformer.

Transition.—The transition from series to parallel connection in the continuous current system presents some difficulty, and no method has yet been devised which meets all requirements of practicability and convenience for both multiple unit trains and locomotives. Three general methods have been used to effect the transition: (1) That of opening the main circuit altogether while the rearrangement of connections is being effected, (2) that of short-circuiting and afterwards open-circuiting one only of the motors in transition, (3) the so-called "bridge" method, in which power is applied continuously to both motors and resistance drop is balanced against counter-electromotive-force during transition. The first method, although common in early controls, both for locomotives and multiple unit trains, is now obsolete. The other methods are however employed both for locomotives and multiple unit trains. Fig. 90 shows the sequence of con-

nections during transition where a motor is short-circuited; whilst fig. 91 shows a more modern variant of the same: the object of introducing a resistance in the circuit of the motor short-circuited is to prevent the generation of considerable current in the circuit, due to the residual field and resulting in burning of the series contactor. Fig. 92 shows the sequence

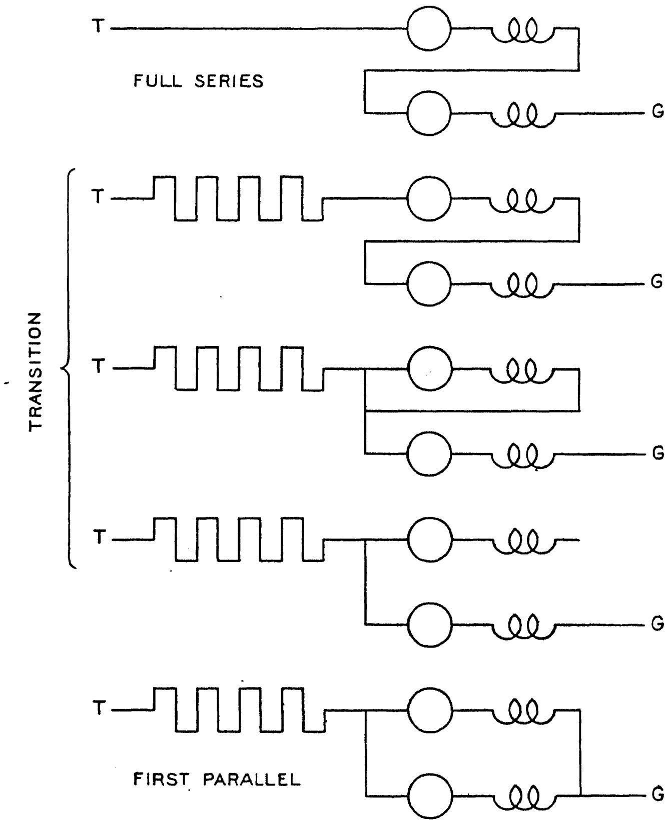


FIG. 90.—Sequence in Single Motor Transition.

of connections in bridge transition. In the method of figs. 90 and 91, if the rheostat resistance during transition is adjusted to the value appropriate to the first parallel notch, an excessive current flows through the active motor while the other is cut out, and if the resistance is increased in order to reduce this excess, the first parallel notch is taken at too

low a current. In the first case the tractive effort of one motor becomes excessive, tending to cause slipping of wheels, and in the second case the tractive effort on the train falls off considerably during transition. In multiple unit trains, if there is a good margin of adhesion, transition may be made with somewhat low resistance in the rheostats, but in locomotive

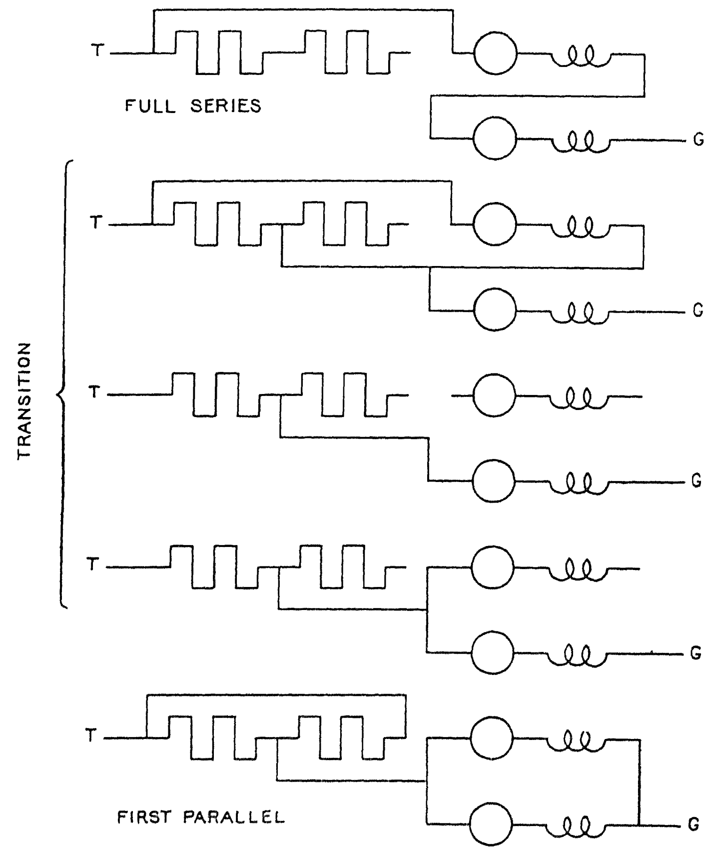


FIG. 91.—Sequence in Single Motor Transition.

work this cannot be allowed, and the halving of the available tractive effort must be tolerated,—an extra parallel notch, with the appropriate transition resistance, being inserted to minimize the jerk. In fig. 91 it will be noted the throwing-in of the second motor and reduction of resistance are intended to take place simultaneously. Transition by means of the bridge

connection gets over these troubles, but introduces others, the chief of which is the burning of the bridge-contactor tips, due to transition not taking place at the predetermined current. In multiple unit trains, particularly where automatic control is used, the rheostats can be adjusted to make transition at the appropriate current for acceleration, in which case the bridge contactor is not required to break an appreciable

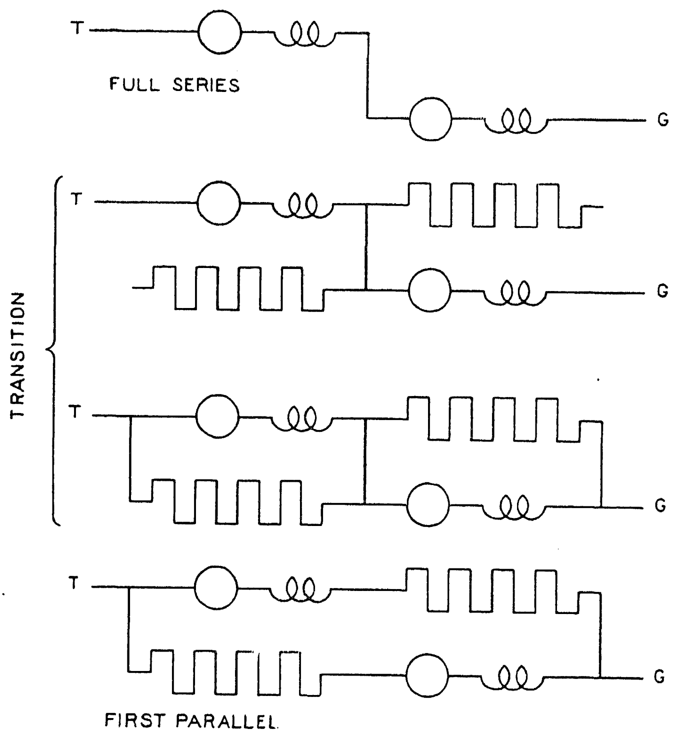


FIG. 92.—Sequence in Bridge Transition.

current, but even then, when a coach passes a gap in the conductor rail, all contactors are dropped and picked up again in proper order, and transition may take place with very little current in the motors, so that the bridge contactor is required to open on a large current. In locomotive work, in which the accelerating current is varied to suit the load, transition seldom takes place at the predetermined value of the current. The transition in which a motor is short-circuited is somewhat the

simpler and allows greater latitude in selecting rheostats, whilst employing their capacity to better advantage than in the case with bridge-transition. Where four motors are used, with the three groupings, series, series-parallel, and parallel, it is advantageous to combine the two methods of transition, using one in passing from series to series-parallel and the other in passing from series-parallel to parallel, as in fig. 88.

Study of a Typical Control.—The subject of train control is an extensive one to which much attention has been given, for not only must operation be reliable, but the train must be safeguarded in any event that can be foreseen. Much could therefore be written in explanation of the methods and devices used, but it will suffice here to take a typical control for study and endeavour succinctly to show how it acts and why it is

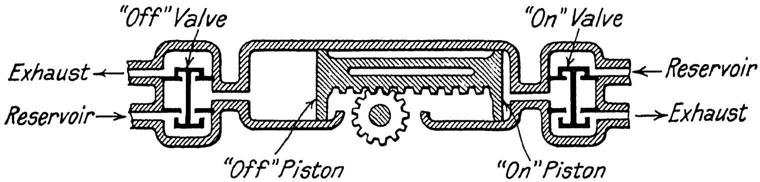


FIG. 93.—Diagram of Operating Cylinder, pneumatic cam control.

arranged as it is. For this purpose the multiple unit train control of fig. 83 will be considered. This is a modern pneumatic-cam-control for a system with insulated return, as used by the London Electric Railways: a simplified diagram of connections is given in fig. 84.

The air for working the controller is taken from the main reservoir through at least 15 feet of cooling pipe, which drains towards this reservoir, and through a hair strainer, to an auxiliary control-reservoir, thence through an insulating joint to the several control cylinders. The pipes, which are galvanized, are pounded and blown out before connection to remove all detachable matter. The main pneumatic engine consists of a cylinder containing a double-ended balanced piston which rotates the cam-shaft by means of a rack and pinion. Its motion is controlled by means of two valves, known respectively as the " on " valve and the " off " valve. It is shown diagrammatically in fig. 93. In the " off " position of the controller handle, when the valves are not actuated, the

" on " valve connects its side of the piston to exhaust, whilst the " off " valve connects its side to the control reservoir. The reverser and line breakers are worked by separate engines, controlled by valves similar to the " on " valve. In normal operation the " on " valve is actuated as soon as the controller handle is moved from its " off " position and remains lifted until this position is regained; the " off " valve on the other hand is actuated intermittently, a notch being taken each time it is lifted, until the full parallel position is reached. The cam shaft is biased towards the notches by means of a star-wheel and pawl.

The power supply for working the control is taken from the live contacts of the main switches, through a double pole control switch and a pair of fuses; thence the positive line passes through one way of a two-way switch to the controller, whilst the negative line passes directly to the controller. The other way of the two-way switch is for the purpose of energizing a control wire, 7, which runs through the train for setting the overload relays: thus in the act of setting these relays power is cut off from the controller circuits, opening the line breakers, releasing the valves of the main engine, and dropping the potential relay, which does not pick up again, even if power be re-applied to the control, until the cam shaft has returned to its " off " position. The interlock contacts P_1 P_2, short-circuited in the " off " position of the control drum, are bridged also by a resistance of 2,800 ohms, which prevents the relay picking up, but suffices to hold it up when once closed.

Function of Several Control Wires.—There are nine wires in the control bundle. No. 1 actuates the " off " valves on notches, through the contacts of the current limit relays. No. 2 actuates the " off " valves in transition between notches, through the lifting coils of the current limit relays; it also sets the negative line breakers and actuates the " on " valves. No. 3 controls the transition between series and parallel, Nos. 4 and 5 actuate the reversers and set the positive line breakers. No. 6 actuates the current limit relays through the medium of the advance lever. No. 7 is for setting the overload relays. No. 8 is a common return wire. No. 10 is the connection to the negative shoe, and this wire is characteristic of the system with insulated return.

Succession to Switching Point.—In the " off " position

of the cam shaft, the contractors S and R6 are closed. In fig. 84 the reverser is shown thrown for forward motion: if it were desired to reverse, the necessary circuit would be made on the first control point from T through 5, 51, 52, 53, 54 and 8, provided that both line breakers are open. The coils marked "Forw[d]." and "Rev." represent the actuating coils of the reversing magnet valves. As shown for forward motion, however, a circuit is made on the first control point through 4, 4A, 4B, 4C, 4D, 4E and 8, which picks up the positive line breaker, provided that the overload and potential relays are both in, and that the control drum is in its "off" position. The positive line breaker in closing makes another connection from 4E to 8, which maintains the circuit when the control drum has been rotated; it also completes a circuit from wire 2, through 2A, 2C, 2D, 2E and 8, which picks up the negative line breaker and actuates the "on" valve. The main circuit is now complete, with motors and grid rheostats in series, and this is all that happens on the first point.

Succession to Running Points.—In passing to the second point the wire 1 is energized, actuating the "off" valve through 1A, 1B, 1C, 1D and 8, and causing the cam shaft to rotate. With the rotation of the control drum the circuit through 1 and 1A is opened, but the "off" valve is still actuated through the circuit 2, 2A, 2B, 1B, 1C, 1D and 8. This circuit, passing through the lifting coil of the current limit relay, breaks the contact between 1A and 1B which is held open, after the circuit through the lifting coil has been opened by the rotation of the drum, as long as the current in the series coil exceeds the value for which the relay has been set. Thus, a portion of the rheostat having been cut out by the closing of contactor R_2, the rotation pauses on the second notch until the motor current has decreased to its predetermined minimum, when rotation starts again and the action is repeated until the sixth notch is reached, at which stage rotation ceases. In passing to the third point of the controller the wire 3 is energized and takes the place of 1 in actuating the "off" valve, restarting the rotation of the cam shaft, and, having passed through a transition period during which the connections shown in fig. 91 are made successively, taking the parallel notches in the same manner as the series notches were taken. The sequence in which contactors are closed or opened by the rotation of the

cams is shown in the diagram at the top of fig. 84, the location of the contactors being shown in the motor circuit connections at the foot of this figure.

ACCELERATION.—In case it is desired to accelerate the train notch by notch, at a rate faster or slower than the automatic devices are set for, this can be done by means of the advance lever, by which the by-pass coil is energized through the wire 6. This actuates the "off" valve by completing the appropriate circuit through 1A, and the armatures of the hold and by-pass coils, to 1B: in transition from notch to notch this circuit is broken by the attraction of the holding coil armature, and the magnet is devised so that the by-pass coil retains this armature once it has been attracted by the holding coil. Thus the action pauses on the next notch until the circuit through 6 is opened by the release of the advance lever, upon which it can be re-applied immediately. The advance lever is used also in testing the control apparatus.

Single-phase Motor Control.—The single-phase commutator motor is accelerated by increase in terminal voltage, either gradual, by means of an induction regulator, or stepped, by contactors connecting to successive transformer taps. In the former method, the voltage regulator, if it is to provide for the whole variation in voltage, must be capable of carrying the accelerating current, and of raising and lowering the voltage by a half of the range of variation: the apparatus needed for effecting this is large and expensive. Lydall * has devised a combination of contactor and induction regulator control, in which a regulator of comparatively small size, whose range is that of the voltage between successive taps, is used, but which nevertheless allows acceleration at uniform tractive effort. The arrangement is used on the Prussian State Railways: however, the advantages of acceleration at uniform tractive effort are not sufficient to warrant great expense and complication in obtaining it, and most single-phase locomotives employ a contactor system only for voltage variation.

In accelerating continuous current or polyphase motors, sections of rheostat are short-circuited successively in transition between notches, and no untoward difficulty arises from the practice. In the acceleration of single-phase commutator

* See *Journal of Institution of Electrical Engineers*, vol. 52, p. 390.

motors, however, there is a live electromotive force between successive taps of the transformer, and in transition from point to point it is necessary to ensure both that the circuit be not opened and that the section of transformer winding be not short-circuited. This is effected by means of a suitable reactance, sometimes called a "preventive coil," which is connected between the taps. In some cases this remains between the taps in use and the motor connection is taken from its centre, as in fig. 95, in which Dr 1 represents the preventive coil: in some cases, in fact, as in the Kiruna-Riksgränsen goods locomotives, three coils are used, connected to successive taps at one end and to a common point, with the motor connection, at the other. In other cases the preventive coil is only used during transition, the motor connection being ultimately made directly to the transformer taps. Fig. 94 shows the principle of the latter arrangement: the motor is connected to the tap (*d*) of the transformer through the contactors D and F; in order to pass to the tap (*c*), contactor C is closed, D is opened, E is closed and F opened, the operations being performed successively in the order named; in order to pass to the tap (*b*), contactor B is closed, C is opened, F is closed and E opened, and so on.

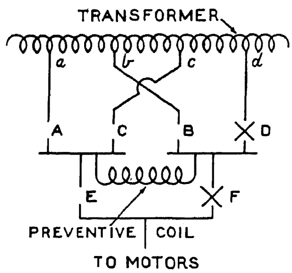


FIG. 94.—Diagram, Preventive Coil.

TYPICAL CONTROL DIAGRAM OF SINGLE-PHASE LOCOMOTIVE.—Fig. 95 shows simplified control and power wiring of a typical single-phase locomotive, being that of the A.E.G. express locomotive, built for the Dessau-Bitterfeld line of the Prussian State Railways.* This locomotive is driven by a single motor rated at 1,000 h.p., approximately. The current is taken from the line at 10,000 volts pressure, through a choking coil and through an oil switch, provided with auxiliary contacts in series with a resistance in order to reduce the shock on switching-in, thence through the primary of the main transformer and that of a current transformer to earth. The secondary of the main transformer is provided with eight taps whereby seven different voltages can be impressed on the motor terminals. There are twelve contactors in all, of which four

* See Appendix, p. 406.

are connected two and two in parallel to the motor leads, introducing the preventive coil between taps, and the other eight to the transformer taps. An auxiliary winding on the main transformer provides power for working the control. An auto-transformer is connected between the motor terminals whereby appropriate shunt voltages depending on the speed are applied to the compensating winding and armature for the purpose of improving commutation; the motor is in fact started as a repulsion motor with armature short-circuited. The master controller has two drums which can be revolved independently of one another: the upper drum regulates tractive effort and speed and the lower one adjusts the commutating field. The driver operates the control drums by means of two hand-wheels, placed one above the other on a pedestal fitting, St, one of these being located at each end of the cabin: the upper hand wheel turns the upper drum, and the lower, which is marked with speed, turns the lower drum, being adjusted by the driver in accordance with the reading of a speed indicator before his eyes. The main switch is provided with an overload tripping device, but in case it is desired to cut off power suddenly, a tripping circuit is provided from the control supply, completed by a press-button, Du, or by the application of the brakes, through a special switch, AS_1.

CONTROL CALCULATIONS

Number of Controller Points.—The number of the controller points used in accelerating the train is governed by radically different considerations in the cases of multiple unit train and of locomotive operation. In the former, the tractive effort rarely reaches the limit of adhesion and is, even at minimum accelerating current, much larger than is required to overcome the resistances to motion. The number of points is accordingly governed by simplicity in the control system, having due regard for the comfort of passengers, and by consideration of damage to the motors by the maximum current. In the latter case, however, the load hauled may on certain grades be near the limit of the capacity of the locomotive, and as the tractive effort must be greater than the resistance to motion and cannot be greater than the limit of adhesion, it is desirable, in order to get full duty from the locomotive, to make it possible to use a high minimum tractive effort as compared

with the maximum, and to see that the peaks of current are uniform. Accordingly, it is usual to have many more resistance points in the locomotive controller than in that of the multiple unit train. Thus the Butte, Anaconda and Pacific locomotives have 17 points, excluding transition points, 10 series and 7 parallel (see fig. 89). The Detroit River Tunnel locomotive has 24 points, 9 series, 8 series-parallel, and 7 parallel (see fig. 88). The Chicago, Milwaukee and St. Paul locomotives of fig. 198 have 31 rheostat and 8 operating points, there being 9 rheostatic and 2 operating points with all motors in series, 6 rheostatic and 2 operating points with 6 motors in series and 2 in parallel, 8 rheostatic and 2 operating points with 4 motors in series and 3 in parallel, and 8 rheostatic and 2 operating points with 3 motors in series and 4 in parallel. A common number of points for a multiple unit train is 9—5 series and 4 parallel. The first point series is always made to give a low peak, in order to diminish the shock of starting, and to provide a low acceleration for coupling up, or, in the case of a locomotive, for running without train. Indeed, in the case of road-locomotives, it is usual to provide several low peaks to lead up to normal acceleration. After transition from series to parallel also, it may be desirable to provide a low peak, for reasons already indicated. These special points, though they count as controller points, must be considered separately and need not be introduced in the discussion of normal peaks.

Mathematical Discussion of Rheostat Resistance.—In a continuous current equipment, let :—

- V be the line voltage per motor in multiple grouping.
- c a coefficient such that cV is the voltage per motor in any grouping.
- v the voltage for which the speed curve is drawn, being usually V in the present calculations.
- A the motor current.
- s the speed as given by the speed curve.
- m the motor resistance.
- r the resistance of motor plus external resistance chargeable to it and included in the section to which the voltage cV is applied.

The actual speed of the motor is $(cV - rA)\ s/(v - mA)$; if now r is suddenly changed to r' by the action of the controller,

resulting in A and s being changed to A′ and s' since the speed cannot be changed instantaneously:

$$\frac{cV - r'A'}{v - mA'}s' = \frac{cV - rA}{v - mA}s$$

or:

$$r\frac{A}{V} = r'\frac{A}{V} \cdot \frac{A's'}{As} \cdot \frac{v - mA}{v - mA'} + c\left[1 - \frac{s'}{s}\frac{v - mA}{v - mA'}\right] \qquad (1)$$

In order that a calculation may have a wider application than to a particular problem, it is desirable to reduce the several variables and coefficients to ratios between quantities of the same dimensions, that is, to numbers. Hence, write:

$$\left.\begin{array}{lll} r\frac{A}{V} = R, & r'\frac{A}{V} = R', & m\frac{A}{V} = M \\ \frac{A's'}{As}\,\frac{v - mA}{v - mA'} = h, & 1 - \frac{s'}{s}\,\frac{v - mA}{v - mA'} = k(h - 1) & \end{array}\right\} \qquad (2)$$

and equation 1 becomes:

$$R + ck = h(R' + ck) \quad . \quad . \quad . \quad (3)$$

It should be noted that equations 1 and 3 assume no change in grouping between the successive controller points considered; if such change takes place, and c' is the changed value of c, the transition equation is:

$$R + c'k = h(R' + c'k) + c - c' \quad . \quad . \quad (4)$$

Assuming that all normal peaks range between maximum current A′ and minimum current A, if $R_1\ R_2\ R_3 \ldots R_n$ are the successive values of R in any grouping:

$$\left.\begin{array}{l} R_1 + ck = h(R_2 + ck) \\ R_2 + ck = h(R_3 + ck) \\ \quad . \quad . \quad . \quad . \\ R_{n-1} + ck = h(R_n + ck) \end{array}\right\} \quad . \quad . \quad . \quad (5)$$

Multiplying the second of the above equations by h, the third by h^2, the fourth by h^3, and so on, and adding, there results:

$$R_1 + ck = (R_n + ck)h^{n-1} \quad . \quad . \quad . \quad (6)$$

The resistances on the first and last points of any grouping may be considered known, so that equation 6 determines the number of normal points if the limits of current are specified, or the upper limit of current if the number of points and lower limit are specified. Equations 5 give the successive resist-

ances. The case of the shunt motor is included by making $k = 0$.

It has already been indicated that in locomotives, it is usually desirable to arrange the control so that the machine works between given limits of tractive effort, the maximum being determined by the adhesion and the minimum by the requirements of grade and load. Thus, — R_1, R_n, h, and k are known, and equation 6 gives:

$$n = 1 + \frac{\log \dfrac{R_1 + ck}{R_n + ck}}{\log h} \qquad . \quad . \quad . \qquad (7)$$

In the case of series-parallel control, with series grouping:

$$c = 0{\cdot}5, \qquad R_1 = 0{\cdot}5\,\frac{A}{A'}, \qquad R_n = M \qquad (8)$$

These equations, together with equations 2 and 7, determine the number of normal series points. With parallel grouping, the resistance in the first point is obtained from the transition equation 4 by putting:

$$R' = R_1, \qquad R = M, \qquad c' = 1, \qquad c = 0{\cdot}5$$

giving:

$$M + k = (R_1 + k)h - 0{\cdot}5 \qquad . \quad . \qquad (9)$$

whilst in equation 7:

$$c = 1, \qquad R_n = M \qquad . \quad . \qquad (10)$$

Equations 2, 7, 9 and 10 determine the number of normal parallel points.

The case of series, series-parallel and parallel control can be treated in a similar manner; with series grouping:

$$c = 0{\cdot}25, \qquad R_1 = 0{\cdot}25\,\frac{A}{A'}, \qquad R_n = M \qquad (11)$$

Equations 2, 7 and 11 determine the number of normal series points. With series-parallel grouping, the resistance on the first point is given by the transition equation 4, by putting:

$$R' = R_1, \qquad R = M, \qquad c' = 0{\cdot}5, \qquad c = 0{\cdot}25$$

leading to:

$$M + 0{\cdot}5k = (R_1 + 0{\cdot}5k)h - 0{\cdot}25 \qquad . \quad . \qquad (12)$$

whilst in equation 7:

$$c = 0{\cdot}5, \qquad R_n = M \qquad . \quad . \qquad (13)$$

Equations 2, 7, 12 and 13 determine the number of normal series-parallel points. With full parallel grouping, the number of normal points is given by equations 2, 7, 9 and 10, as in series-parallel control.

Inasmuch as the number of controller points in any grouping is an integer, whilst the numbers determined above are generally fractional, it may be necessary to vary the prescribed limits of motor current to a suitable extent to give equal peaks, and the limits will usually be slightly different with different grouping of motors. The necessity for providing switching and transition points, however, gives a flexibility which is equivalent to permitting a fractional number of normal points and frequently relieves the computer of the necessity of further adjustment. There is, in fact, rarely any need for such adjustment in the case of the series points of locomotive control, as two or three initial points intervene before normal points are reached; but the parallel points will sometimes require adjustment to give even peaks. When the number of normal points has been settled, the successive resistances may be determined from equations 5 and 6, either by suitable adjustment of the limits of current, or by the method outlined below. It should be noted that the resistances finally determined in this manner include the motor resistance, and are associated with a single motor.

Automatic Acceleration.—In automatic acceleration, succeeding points are taken when the current drops to a certain minimum. This minimum therefore remains uniform throughout the acceleration; although the height of the peaks may vary with the grouping of the motors. In hand acceleration, governed by the observation of an ammeter, although uniformity either of maxima or minima is hardly to be expected, the tendency is to maintain a uniform average current in each of the several motor groupings. Of the two assumptions, accordingly, viz. that of equal minimum current in the various motor groupings and that of equal mean current, it would seem the former is better suited to the case of automatic control and the latter to that of hand control. Actually, it is convenient to make the calculation as if the minimum current were specified, and to plot the results in terms of the average accelerating current, as this is usually prescribed and the automatic relays set accordingly. Maximum and minimum

limits of current will however be given on the curves in order that full particulars may be available.

Calculation of Universal Curves.—The problem of computing the various rheostat resistances when the number of normal points in each grouping and the minimum current are given, will now be discussed; this is really the problem of solving the set of $n - 1$ equations, 5, when the minimum current A is given. Write the maximum current $A' = A(1+x)$ and write $s' = s(1 - \alpha x)$ so that α is the proportional slope of the speed-current curve, viz.:

$$\alpha = -\frac{A}{s}\frac{ds}{dA} \quad . \quad . \quad . \quad (14)$$

and usually lies between 0·3 and 0·4 with modern motors at rated voltage. Writing:

$$\beta = \frac{mA}{v - mA} = \frac{M}{1 - M} \quad . \quad . \quad . \quad (15)$$

equations 2 give:

$$h = \frac{(1 + x)(1 - \alpha x)}{1 - \beta x}, \quad k = \frac{\alpha - \beta}{1 - \alpha + \beta - \alpha x} \quad . \quad (16)$$

Since R_1 and R_n are known in terms of M for any grouping, equations 6 and 16 determine x, and equations 5 the resistances on successive points. These equations are too complicated to be solved for x directly, and recourse has to be had to indirect methods for obtaining its value.

For series-parallel control and series grouping:

$$c = \tfrac{1}{2}, \qquad R_1 = \frac{1}{2(1 + x)}, \qquad R_n = M \qquad (17)$$

thus equation 6 becomes:

$$\tfrac{1}{2}\left(\frac{1}{1 + x} + k\right) = (M + \tfrac{1}{2}k)\, h^{n-1} \quad . \quad (18)$$

and this equation determines x in this case.

For series-parallel control and parallel grouping:

$$c = 1, \qquad R_1 = \frac{M + \frac{1}{2} - k(h - 1)}{h}, \qquad R_n = M \quad (19)$$

thus equation 6 becomes:

$$(\mathrm{M} + k)(h^n - 1) - \tfrac{1}{2} = 0 \qquad . \qquad . \quad (20)$$

and this equation determines x in this case.

For series, series-parallel and parallel control, and series grouping:

$$c = \tfrac{1}{4}, \qquad \mathrm{R}_1 = \frac{1}{4(1+x)}, \qquad \mathrm{R}_n = \mathrm{M}$$

thus equation 6 becomes:

$$\tfrac{1}{4}\left(\frac{1}{1+x} + k\right) = (\mathrm{M} + \tfrac{1}{4}\,k)\ h^{n-1} \qquad . \qquad . \quad (21)$$

and this equation determines x in this case.

For the same control with series-parallel grouping:

$$c = \tfrac{1}{2}, \qquad \mathrm{R}_1 = \frac{\mathrm{M} + \frac{1}{4} - \frac{1}{2}k(h-1)}{h}, \quad \mathrm{R}_n = \mathrm{M}$$

thus equation 6 becomes:

$$(2\mathrm{M} + k)(h^n - 1) - \tfrac{1}{2} = 0 \qquad . \qquad . \quad (22)$$

and this equation determines x in this case. For the same control with full parallel grouping, the equations are the same as in the case of series-parallel control, and x is determined by equations 16 and 20. It will be noted that equations 21 and 22 are the same as equations 18 and 20, but with 2M written for M; thus the same curves are applicable if sufficiently extended.

Equations 18 to 22 enable the rheostats to be computed, and the results presented in a form applicable to any control having the number of normal points for which the computation is made. Examples of such results are given in figs. 96 to 100, and in these curves the percentage drop in rheostats or in motor is that at the mean accelerating current, as this is the quantity for which it is most appropriate to arrange the rheostat. The variation of the current from the mean is given on each diagram, so that the maximum and minimum value of the accelerating current may be deduced. The curves are plotted for a proportional slope in the speed curve of $a = 0{\cdot}35$, but the variation in the curves with change in this slope is small, being well within the limits of differences between rheostats or possibilities of adjustment. It may be mentioned,

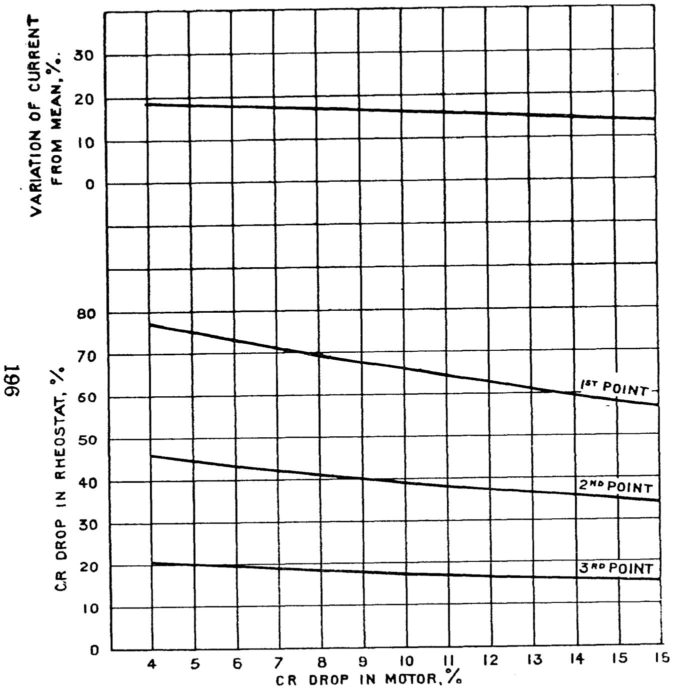


FIG. 96.—Rheostat Calculation. CR-drop at mean accelerating current. Four-point, series.

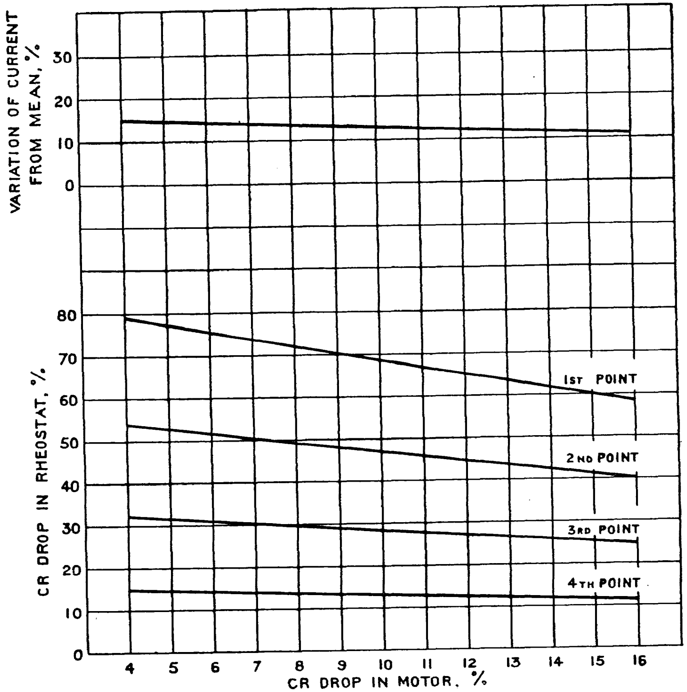


FIG. 97.—Rheostat Calculation. CR-drop at mean accelerating current. Five-point, series.

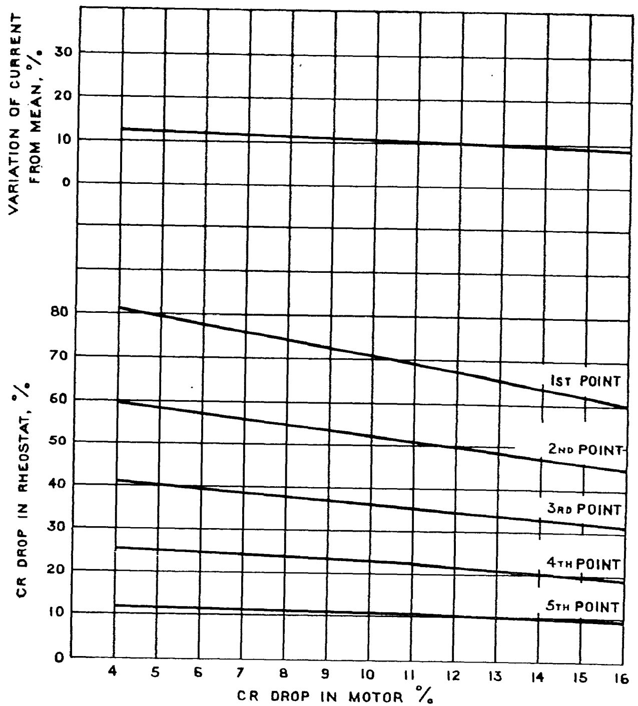


FIG. 98.—Rheostat Calculation. CR-drop at mean accelerating current. Six-point, series.

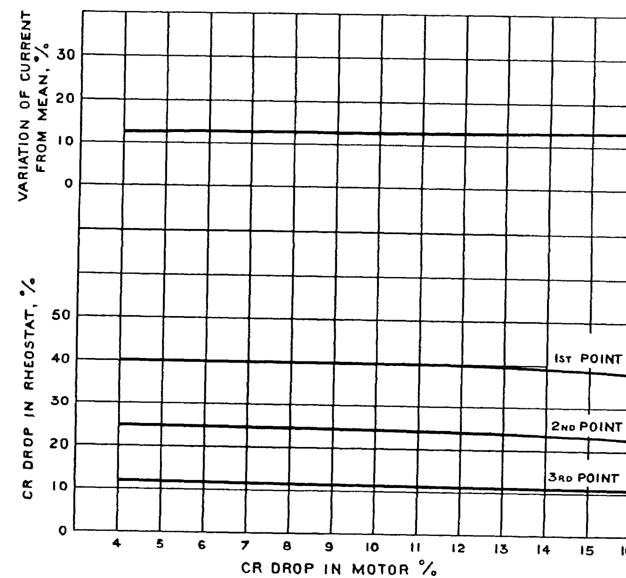


FIG. 99.—Rheostat Calculation. CR-drop at mean accelerating current. Four-point, parallel.

however, that greater slope requires slightly greater drop in rheostats, the excess being greatest for the second point in series, which may vary by 1 per cent. from the value given in the curve. It should be noted that with the same number of normal points in series as in parallel—a desirable arrangement with bridge transition, from the point of view of simplicity in control and fewness of contactors—the range of variation of the accelerating current per motor is considerably

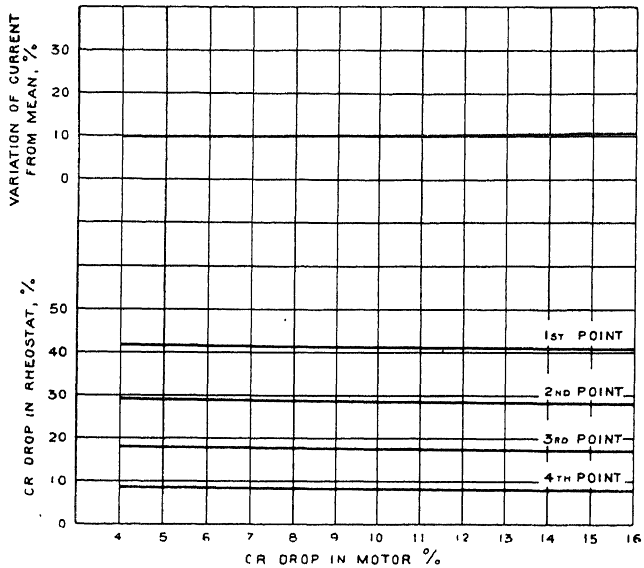


FIG. 100.—Rheostat Calculation. CR-drop at mean accelerating current. Five-point, parallel.

greater in series than in parallel; thus—if succeeding points are taken automatically when the current has dropped to a certain minimum, the mean accelerating current in series will be greater than in parallel. Moreover, a control with uniform range of current throughout requires more normal series points than parallel points, a condition satisfied by the locomotive controls described above.

EXAMPLE OF ACCELERATION CURVE, WITH RHEOSTATIC LOSSES.—Fig. 101 shows an acceleration curve computed from figs. 96 and 99 for four normal points and one switching point in series, and four points in parallel. This figure also shows the loss of power in the rheostats on each step of the controller.

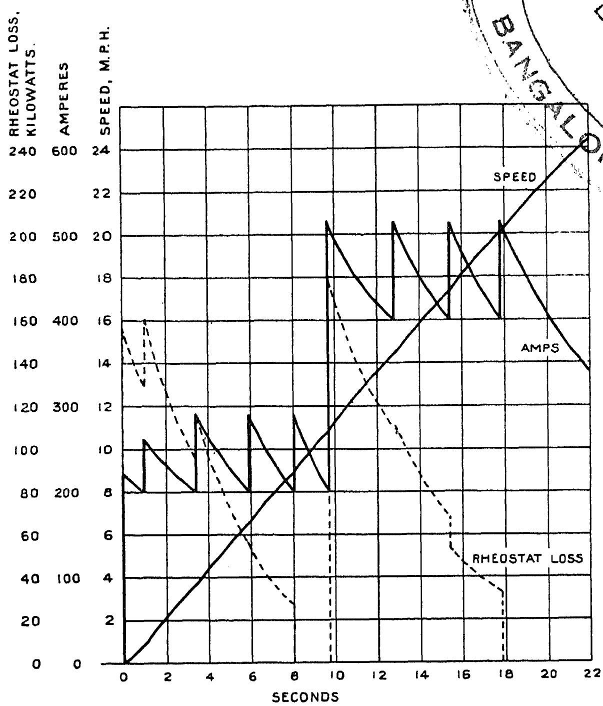


FIG. 101.—Acceleration Curve of Continuous Current Motors.

Fig. 102 shows the energy loss in rheostats with series-parallel control expressed as a fraction of the total input. This curve

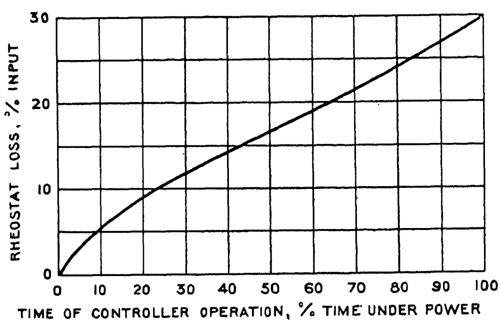


FIG. 102.—Rheostat Loss with Continuous Current Motors.

varies but slightly with variation in the equipment, rate of acceleration or outside conditions, and may be taken as

approximately correct for any case in which series-parallel control is used.

Selection of Rheostats.—The selection of suitable rheostat boxes and grids to give the resistances required on the several control points and at the same time possess thermal capacity and heat dissipating qualities suited to the work in hand, must usually be left to the judgment of the manufacturer, and general guidance only need be given here. In locomotives, and particularly in goods locomotives, the rheostats may be in circuit for considerable periods of time and the final temperature corresponding to continuous operation becomes of importance. In multiple unit trains, and generally in passenger work, on the other hand, where the rheostats are only used intermittently and for short periods, the rate of rise of temperature is the more important feature in governing selection. If A is the cross section of the material of a grid, p the perimeter of the section, and δ^2 the mean square current density, the ultimate temperature rise in continuous operation may be taken as proportional to $A\delta^2/p$. The temperature rise T, at any time t, may be written :

$$T = k\frac{A}{p}\delta^2\left(1 - e^{-k'\frac{p}{A}t}\right) + T_0 e^{-k'\frac{p}{A}t} \quad . \quad (23)$$

where k and k' are constants. With A/p in inches, δ in amperes per square inch and T in Centigrade degrees, $k = 10^{-2}$ and $k' = 10^{-5}$ approximately, the grids being of cast iron.* The limiting temperature for such rheostats in service may be taken as from 200° C. to 300° C. For locomotive work the cross section of the grids may with advantage be made oblong, or several small grids may be used in parallel instead of a larger one : for multiple unit train work, on the other hand, better use is made of the space if the section of the grid is made approximately square.

Voltage Steps in Accelerating Single-phase Motor.—The determination of appropriate voltage steps for accelerating a single-phase motor, so as to keep within a certain range of

* k varies as the specific resistance of the material, which has been taken at 10^5 c.g.s. units. The constants also vary with the ventilation, and are given for natural circulation of air, and with the iron cross section about one-third of the whole.

tractive effort, can be deduced from a system of equations obtained in a similar manner to the corresponding continuous current equations. Thus, in a compensated series motor, if v is the terminal voltage for which a speed curve is given, m the motor resistance, A and A′ the minimum and maximum limits of current, k and k' the corresponding power factors, s and s' the corresponding speeds, and if further V_1 V_2 . . . . are successive terminal voltages in the control k_1 k_2 k_3 . . . . the power factors at these voltages and at current A, and k_1' k_2' . . . . the power factor at current A′, the speed at voltage V_1 and current A is $(k_1 V_1 - mA)s/(kv - mA)$, and so with other voltages and currents. As there is no instantaneous change in speed:

$$\frac{k_1V_1 - mA}{kv - mA}s = \frac{k_1'V_2 - mA'}{k'v - mA'}s' \quad . \quad . \quad (24)$$

As the reactance voltage at any current is for the motor practically independent of the terminal voltage:

$$(1 - k_1{}^2)V_1{}^2 = (1 - k_2{}^2)V_2{}^2 = \; . \; . \; . \; . \; = (1 - k^2)v^2$$
$$(1 - k_1'^2)V_1{}^2 = (1 - k_2'^2)V_2{}^2 = \; . \; . \; . \; . \; = (1 - k'^2)v^2 \quad (25)$$

Thus commencing with full voltage V_n, which may be equal to v, the speed-curve voltage, the tap voltages are determined successively by the equations:

$$\frac{[V^2_{n-1} - (1 - k^2)v^2]^{\frac{1}{2}} - mA}{kv - mA}s = \frac{[V^2_n - (1 - k'^2)v^2]^{\frac{1}{2}} - mA'}{k'v - mA'}s'$$

$$\frac{[V^2_{n-2} - (1 - k^2)v^2]^{\frac{1}{2}} - mA}{kv - mA}s = \frac{[V^2_{n-1} - (1 - k'^2)v^2]^{\frac{1}{2}} - mA'}{k'v - mA'}s' \quad (26)$$

These equations will not usually lead to a normal point with motor at rest, or to $V_1 = [(1 - k'^2)v^2 + m^2A'^2]^{\frac{1}{2}}$, but this is of no consequence, for the starting is a special problem in any case, being often effected with reduced field, or with repulsion motor connections for reasons given in Chapter III.

The greater slope of the speed-torque curves of the single-phase motor as compared with that of the continuous current motor, which makes the variation in speed greater for a given variation in torque, has the general effect of requiring fewer control points with single-phase than with continuous current

motors. On the other hand, since the torque of the single-phase motor pulsates, it is not practicable to work it through as wide a range of average torque as the continuous current motor, unless there is an elastic link in the transmission to the driving wheels. Thus the difference in the number of control points is generally less than would be determined from consideration of the average torque only.

CHAPTER V

DISTRIBUTION SYSTEM

The distribution system of an electric railway is defined by the Standards Committee of the American Institute of Electrical Engineers as "that portion of the conductor system of an electric railway which carries current of the kind and voltage received by the cars or locomotives"; and, although it may be considered more logical to include in so general a term all gear between power-generating and power-using installations, it is in the restricted sense that the expression is used in the present chapter. The distribution system consists of two main elements: viz., the outward and return conductors paralleling the track, and a number of auxiliary elements, e.g. the feeders, the booster installation and the sectionizing switches. The forms taken by the several elements vary greatly, depending principally on the system of operation, on the voltage, and on the limitations imposed by existing structures.

The Live Conductor.—A track conductor which is not grounded should satisfy the general conditions of being, with its support and appurtenances, well clear of the rolling structure gauge at every place, in all sorts of weather; and of being not only clear of all permanent structures itself but so placed that the collecting gear clears such structures everywhere, in all permissible conditions of repair of the parts and all states of oscillation to which the gear may be liable with the running of the train. These conditions are often very onerous when the electrification of an existing steam railway is in question, and sometimes render a somewhat objectionable form of construction necessary, whilst almost invariably adding greatly to the expense.

The Return Conductor.—For return conductor it is usual to make use of the track rails, suitably bonded together for the purpose. These provide a simple means of taking the current from the moving train and make use of the conductivity of the track, thus furnishing a return conductor at comparatively small expense. Sometimes the track rails are reinforced by means of an uninsulated conductor rail, bonded to the track at intervals, as in the Lancashire and Yorkshire Railway electrification. Most of the London underground railways have an insulated return rail, and this arrangement (although lacking the simplicity and convenience of return by track rail) has the merit of avoiding electrolytic troubles, and interference with grounded communication circuits.

THE TRACK RETURN

Track rails have some ten or twelve times the resistance of copper of equal cross section. They are bonded together by means of copper bonds to the extent of 50 to 100 per cent. of the equivalent copper cross section, according to the density of the traffic. The bonds, usually in duplicate at each joint, are preferably put under the fishplates. They are made as short as is consistent with flexibility and are generally some 12 inches between terminals. The greatest practicable measure of flexibility is needed in track bonds in order that they may withstand the vibration of service and the movement of the rails due to changes of temperature: a ribbon type bond, formed of many strip conductors in multiple, edge-on to the rail and kinked to give longitudinal flexibility, is found to give best service. The ends of the conductors forming the bond are welded into suitable copper terminals which, in the process of installing the bond, are expanded mechanically into freshly reamered and cleaned holes in the web of the rails. The track rails thus bonded are connected together at intervals of about 100 yards by means of cross bonds, in order that effective use may be made of the whole available conductivity in reducing the voltage drop. The cross bonds may be of somewhat smaller section than the main track bonds, as their primary function is that of equalization. It is common practice to bond round special work rather than through it, in some cases using special long bonds and in others making use of the continuous portions of the track itself: the cross

bonds required for this purpose being in the main circuit, must be proportioned accordingly. Cross bonds are usually solid, having their terminals formed from the conductor itself. Fig. 103 shows generally the special track bonding at a simple crossover.

With continuous currents, the whole cross section of the rail is useful conductor, and if w is the weight of the rail in lbs. per yard, the resistance of each rail, including bonds, is usually between $4{\cdot}5/w$ and $5{\cdot}5/w$ ohms per mile. With alternating currents, the outer skin of the rail only is effective conductor, and the resistance is accordingly very much greater. Its value in ohms per mile is approximately $1{\cdot}6 \times \sqrt{\text{frequency}}/$ (perimeter of rail in inches), this being the value of the average power loss in the rail divided by the mean square of the current. It is not in this case practicable to obtain the resistance by a measurement of voltage drop inasmuch as the alternating flux due to the rail-current affects the voltmeter lead and makes the voltage reading depend greatly on the position of this lead: accordingly a record of voltage drop conveys little information unless the position of the pilot wire with respect to the rails is defined.

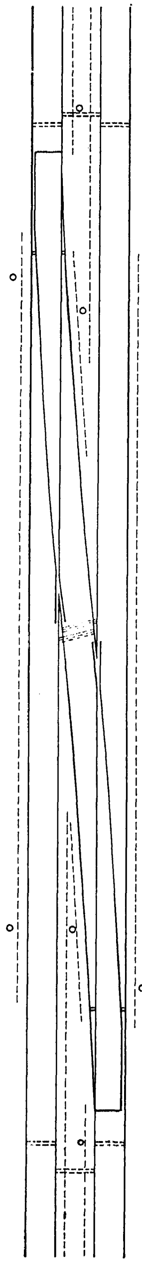


FIG. 103.—Bonding at Simple Crossover.

Leakage e.m.f. and Electrolysis.—The electromotive force tending to produce leakage from the rails to the neighbouring ground is, in the continuous current system, the resistance drop in the rails. In the alternating current system there is an additional electromotive force due to the changing flux which tends to drive the current from the rails, and moreover induces parasitic currents in neighbouring circuits. The leakage currents enter and leave the grounded conductors; but the parasitic currents may complete their circuits within the conductors themselves. The earth currents are usually

very much larger in the alternating current than in the continuous current system, and it is fortunate that an alternating current appears generally to produce less electrolytic destruction than a continuous current of equal r.m.s. value. There is, however, some difference of opinion as to the electrolytic damage caused by the alternating current, some contending that it is about a half that of the continuous current, and others that it is much smaller than this. It probably depends more on the nature and dampness of the surrounding soil than is the case with continuous currents, for the corrosive damage which occurs during the half cycle that the current is leaving the conductor for the ground may, if diffusion is unable to take place, be partly repaired in the succeeding half cycle. If this view is correct, it would seem to follow that the lower the periodicity the greater the corrosion. However, it is probable that the impression that alternating currents are incapable of doing appreciable damage by electrolysis, is mainly due to the fact that they have not been used to any extent under tramway conditions, in towns, coupled with the fact that their use is comparatively modern in this connection, whilst evidence of corrosion accumulates slowly.

Railway conditions, as distinguished from Tramway conditions, are not in general conducive to extensive electrolytic damage in neighbouring gas mains, water mains, cable sheaths, and other grounded conductors, for in the first place the track rails are not very effectively grounded, being carried on wooden sleepers laid on a ballast of broken stones, and in the second, any conductors liable to damage are generally at some distance from the rails, and do not usually follow the railway or join regions between which considerable difference of potential exists: for whatever the differences of potential in the rails, the differences in the earth at such locations as mains usually occupy are small, so that small equalizing currents which moreover generally escape to ground from an extended surface of conductor and so distribute the corrosion, are able to deal with them. The circumstances are therefore, in general, much less likely to lead to appreciable electrolytic damage than those which exist on tramways using the public streets where water and gas mains and the like may run for a considerable distance in close proximity to the imbedded rails. Restrictions imposed on the voltage drop in the rails in this case would

therefore be unreasonable if applied to railways. The Board of Trade, which limits the permissible voltage drop in tramway rails in this country to a maximum of 7 volts, appears wisely to treat railways in accordance with local circumstances, and does not in practice set a definite limit to this drop; to be applied under all conditions; for although the electrolytic damage attributable to the railways is usually negligible, troublesome local problems may arise, to be dealt with according to the local circumstances.

Electrolytic Corrosion.—Electrolytic corrosion of pipes and other earthed structures occurs only where the current leaves the structures, and is accordingly to be feared about the negative portions of mains and the like which happen to join a positive to a negative region. Where the positive region is strictly local, as where a water main crosses the railway, the buried structure may with advantage be insulated locally with a good coating of bitumen or the like; but this is a dangerous expedient for use in a negative region, for any defect in the coating localizes the corrosion and does more harm than good. Sometimes it may be advantageous to destroy the continuity of connection of a main, by means of insulated joints; but this expedient should be used with caution, as its effect is to localize the drop in potential at the joints and possibly to cause corrosion on the positive side thereof. The parts subject to corrosion may, if sufficiently local, be bonded to a suitable renewable shield, designed to take the corrosion which would otherwise fall upon the structure. In some cases the structure may be bonded directly to the negative busbar at the substation; but this expedient, whilst suitable for the protection of structures of local extent, should be scrutinized carefully to see whether it involves connection to mains which may carry a negative potential to a distance and cause the corrosion of other mains approaching them from positive regions. The bonding of structures to the negative busbar is also likely to increase the leakage from the rails.

The aforementioned methods of protecting buried structures against electrolytic corrosion are however of local application and special use; but the restriction of the voltage drop in the grounded conductors is still a necessary safeguard, although the value of the permissible drop should depend upon local

conditions. A railway running in proximity to tidal water, for instance, is liable to return much of its current by way of the earth even with a small drop in the rails, and should accordingly be subject to special attention in this regard, whilst the layout of gas and water mains in urban districts should always be scrutinized carefully in relation to the railway system. A primary requirement in the use of a track return is that the bonding should be maintained in good condition, and many instances of excessive earth-currents have been traced to neglect of this precaution.

Layout of Negative Feeders.—The limitation of voltage drop in the track may, however, have the undesired effect of making this the governing feature in locating the substations; a consummation particularly likely to occur with high voltage systems, since the desirable drop increases in proportion with the line voltage. In districts where electrolytic trouble is feared accordingly it is necessary to consider the economic advantages and disadvantages of employing means for taking current from the rails at selected points between substations. If the route has other tracks than those electrified, it may be found desirable to bond these for use as uninsulated negative feeders, particularly in the neighbourhood of the substations; but it is not usually economical to parallel the track for any great distance with cables. The feeder from the negative busbar is, in railway work, necessarily a heavy conductor, and is generally connected to the track as closely as practicable to the substation unless this proves an undesirable location for a negative connection: in the alternative, the feeder or feeders may be carried, insulated, to a distance, and connected to the track either at a single point or at a number of points situated on as many branches. By these means some control may be exercised over the voltage drop at critical points; but where a drastic reduction in the drop is required, negative boosters, taking current from the rails at some distance from the substations, become necessary.

TRACK BOOSTERS

Boosters Excited by Positive Feeder.—The continuous current negative booster, as used in tramway work, is a generator, excited by current which feeds the boosted section,

and driven by a suitable motor, which is usually worked by power from the main supply. The armature of the generator is connected between the negative busbar and the booster-line,—an insulated conductor connected to the track rails at the point where the current is to be abstracted. The booster is located in the substation. The generator voltage should be equal to the resistance drop in the booster-line, and should accordingly be proportional to the current. The booster should therefore be designed to run at low saturation in order that its characteristic curve may approximate to a straight line through the origin; a certain amount of adjustment is, however, generally provided by means of diverters of the field current. A typical diagram of connections for a negative booster, as used in tramway work, is shown in fig. 104. The method of feeding the line

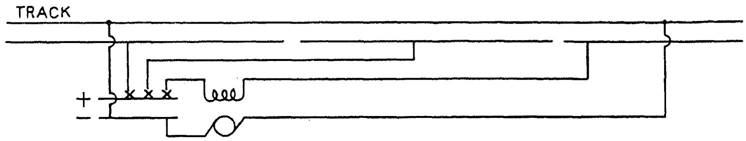


FIG. 104.—Connections of Negative Booster for Tramways.

in sections is however not in accordance with usual railway practice, for which it would prove somewhat expensive in copper, and the more usual methods of feeding are not well suited for the excitation of the booster fields. Usually therefore a compromise has to be made, a section near the substation being fed independently, and a positive feeder being taken, through the booster field, to a more distant point of the conductor rail, as in fig. 105. If the substation is near a railway

FIG. 105.—Connections of Negative Booster for Railways.

station and the track is double, the conductor of the track by which trains come in from the direction of the booster-connection (i.e. the conductor which generally carries but little current for supplying trains near the substation), may be fed through the booster field, and thus act as a feeder as far as the nearest sectionizing switch, as in fig. 106. In this way the strong

excitation of the booster by trains near the substation, causing the track at the booster-line-connection to become strongly negative, is avoided. Boosters of this kind have not been greatly used in railway work, for hitherto little sacrifice has

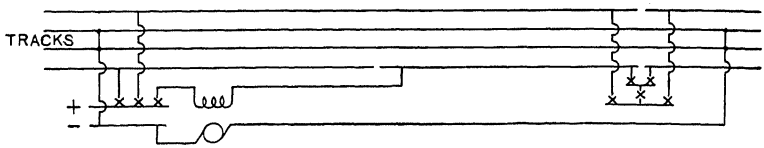


FIG. 106.—Connections of Negative Booster for Railways.

been made in locating substations so closely as to keep the voltage drop in the track to a reasonable amount. The tendency is, however, towards increased voltages; and, as the voltage is raised, a given drop becomes a smaller proportion of the line voltage, so that the substation system is more and more constrained by a limitation of voltage drop in the track rails.

Dynamotor Type.—Another method particularly applicable to high potential continuous current systems is, however, available for reducing the drop in the track rails.* In this, an insulated conductor is provided, connected at intervals to the track rails, and having between adjacent connections a source of e.m.f. just sufficient to overcome the voltage drop in the insulated line. The object is attained by means of a continuous current transformer or dynamotor, having ratio unity if all the return current is to be carried by the insulated line, and having one armature winding in circuit with the positive line conductors—a suitable section insulator being provided for its insertion—and the other winding in the return feeder. The armature rotates in a field excited in series with one of its windings. The dynamotor should be run at somewhat low saturation in order that the speed may not fluctuate greatly with the load. The machines may be put anywhere that may be convenient, the sectionizing cabins being generally appropriate locations, and since they are self-starting and self-regulating they may be left in circuit, without danger, with little attention beyond occasional inspection and oiling. The effect of the arrangement, which is shown diagrammatically in fig. 107, is that a current equal to the line current is compelled to return by the insulated negative line, so that the voltage

* See Patent 9856/1914.

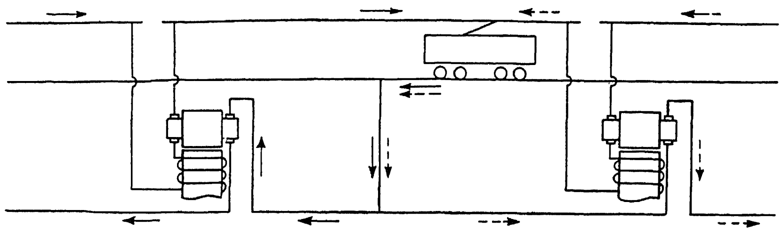


FIG. 107.—Dynamotor Booster for Railways.

drop in the track is merely that between the train and the neighbouring connection with the negative line; the voltage drop in the negative line itself is transferred by the dynamotor to the positive line.

Alternating Current Boosters.—Alternating current systems are able to employ arrangements for taking current from the track rails similar to any of those given for continuous current systems; but that described last, with unsaturated transformers of ratio unity taking the place of dynamotors, is particularly suitable. Fig. 108 shows the

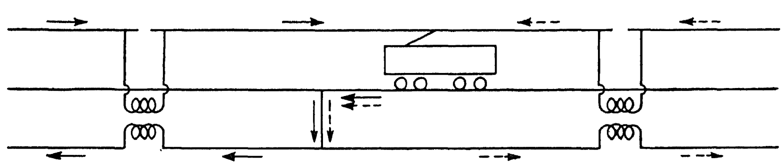


FIG. 108.—Single-Phase Track Booster.

arrangement diagrammatically as applied to the single-phase system; whilst fig. 109 shows its application to the three-phase

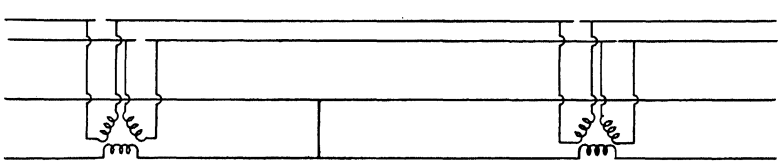


FIG. 109.—Three-Phase Track Booster.

system, and the similarity of these to fig. 107 will be obvious. Another arrangement, in which the insulated line carries the magnetizing current of the transformers whilst the rails themselves carry the load current, is shown in fig. 110. The insulated equalizing line is necessary to success of the latter arrange-

ment, for without it there is nothing to ensure that the drop in the rails will be restored by the series transformers ; and without this restoration the device would be of small value. With dynamotors in place of transformers a similar arrangement could be used in continuous current systems.

The calculation of the effect of a certain booster system on the track drop presents no great difficulty other than that which arises from lack of knowledge of the circumstances, and

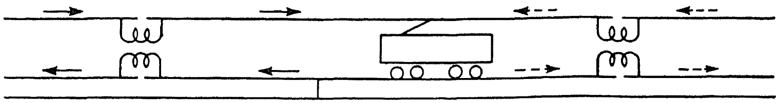


FIG. 110.—Single-Phase Track Booster.

particularly as to what constitutes a typical bad condition of service. The estimation of this condition depends entirely on the circumstances ; but usually one or more trains can be considered as taking their maximum accelerating current, and all other trains on the section as taking their average current. Although this loading may not give the worst condition arising in service, it is of that order and will serve as a basis in comparing booster arrangements.

CONDUCTOR RAILS

In the low voltage continuous current system, power is generally supplied to the trains from an insulated steel conductor rail, of composition specially adapted to give high conductivity. With this object it is made comparatively low in carbon and manganese, a normal composition for the purpose being approximately as follows :—

TABLE 7

Carbon	not exceeding	·05 per cent.
Manganese	,, ,,	·25 ,, ,,
Phosphorus	,, ,,	·05 ,, ,,
Sulphur	,, ,,	·05 ,, ,,
Silicon		trace only

The resistance of such a rail, bonded and installed complete, is usually from seven to eight times that of copper of equal cross section. The figures of an actual case may, however, prove of interest ; the conductor rail used by the London Underground Electric Railways was found to have the following composition :—

TABLE 8

Carbon	·05 per cent.
Manganese	·19 ,, ,,
Phosphorus	·05 ,, ,,
Sulphur	·06 ,, ,,
Silicon	·03 ,, ,,

The resistance of the rail itself proved about 6·4 times that of copper of equal cross section; that of lengths of bonded rail, without special work, about 7·3 times that of copper, and that of the complete rail, including the jumper cables at special work, about 7·5 times that of copper.

The area of cross section of a conductor rail is determined by

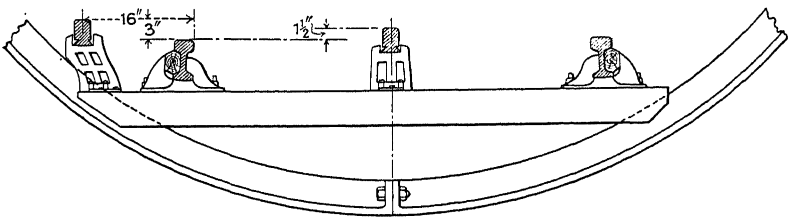


FIG. 111.—Conductor Rails, Tube lines, U.E.R., London.

the conductivity required; but the section may have any shape which provides a contact surface appropriate to the method of collection of current, and is also convenient for installation. An approximately rectangular section is used on most of the London Tube lines (fig. 111): a channel section is

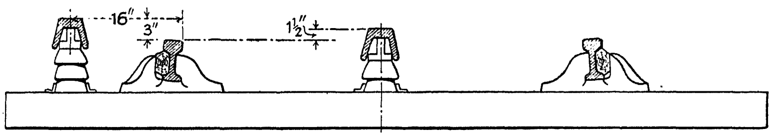


FIG. 112.—Conductor Rails, Hammersmith and City Railway.

used on the Central London Railway and the Hammersmith and City Railway (fig. 112): a specially rolled, unsymmetrical rail (fig. 114) combining the advantages of under-contact and support from below, is employed on the Central Argentine Railway. A specially rolled section is also used for the side contact rail of the Lancashire and Yorkshire Railway (fig. 116). In many cases there is no sufficient advantage in departing from standard rail sections, which are readily procurable;

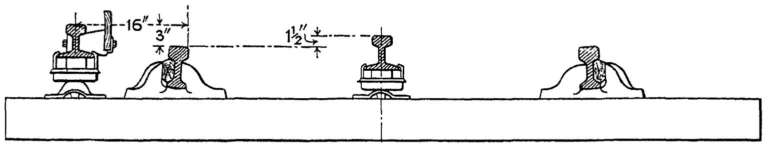


FIG. 113.—Conductor Rails, Metropolitan District Railway.

and the flat-footed or vignoles type of rail is most commonly used where the contact surface is on the top (fig. 113); whilst

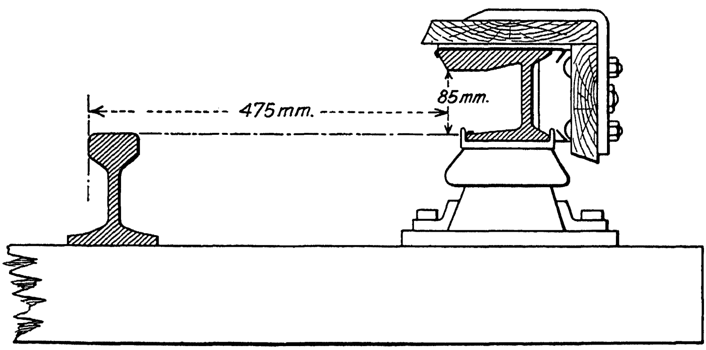


FIG. 114.—Under-running Conductor Rail, Central Argentine Railway.

bull-headed rails are frequently used with under-contact shoes (fig. 115).

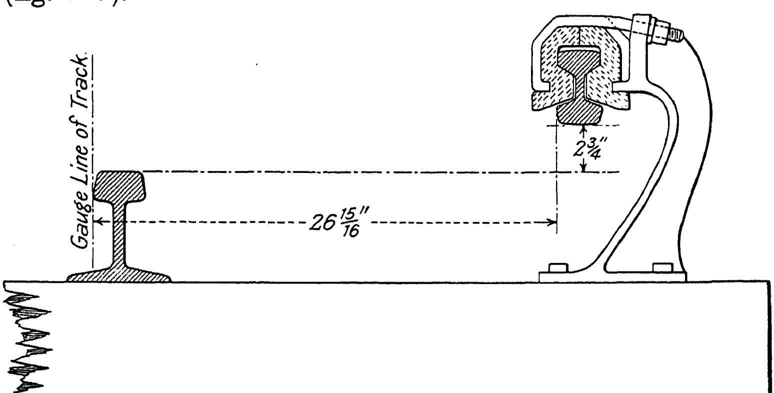


FIG. 115.—Under-running Conductor Rail, New York Central Railway.

The under-contact rail has the advantage of being protected from the weather better than the over-contact type; and particularly of keeping its contact surface clear of ice, which

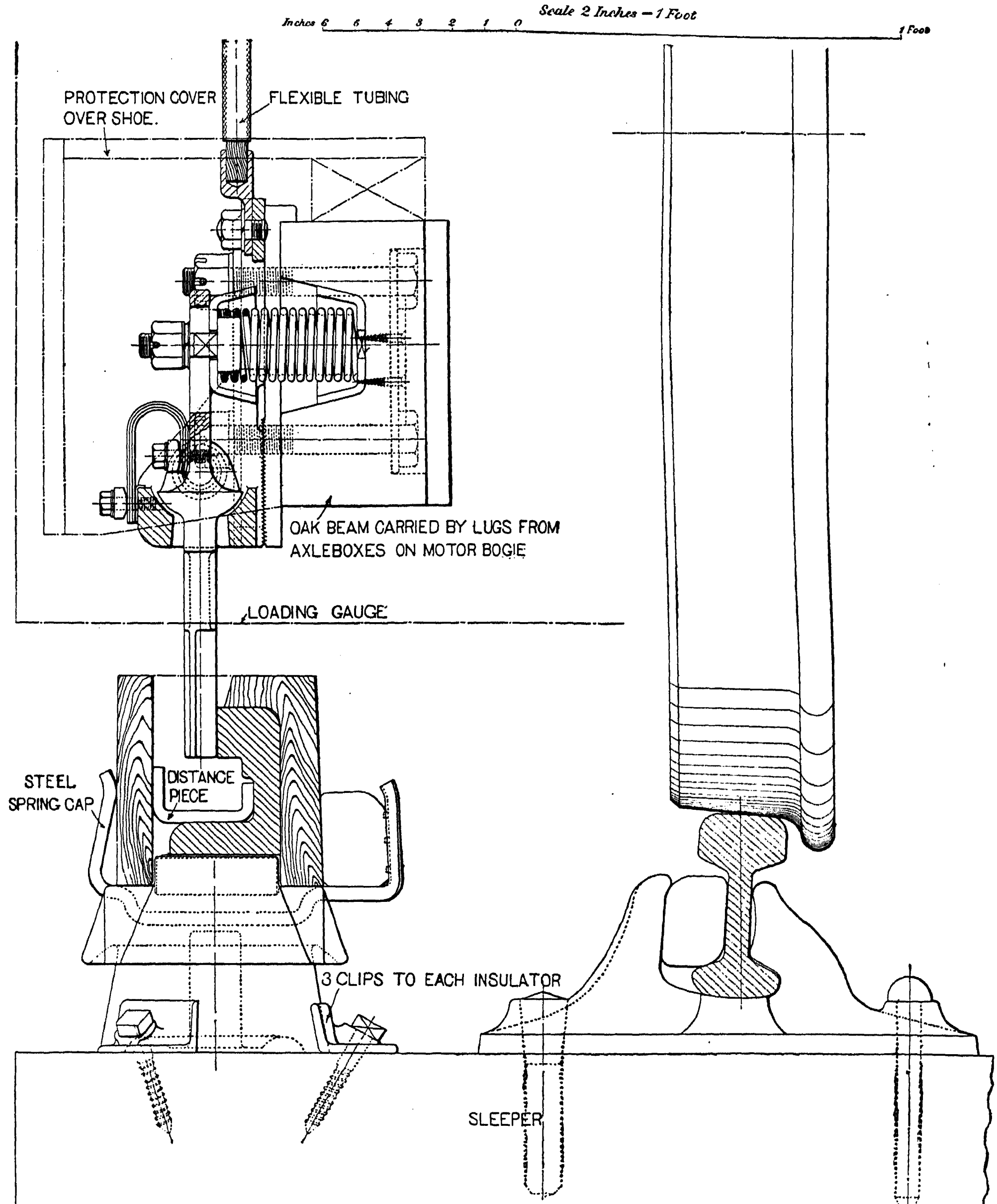


FIG. 116.—Conductor Rails, Lancashire and Yorkshire Railway.

sometimes causes trouble, even to the extent of stopping the traffic, where the contact surface is on the top of the rail. The under-contact rail is also the more easily and completely protected from accidental contact with track tools. The over-contact rail, on the other hand, besides being somewhat the easier to instal, is usually also the more easily designed to suit the available space. On most British railways, the rolling structure gauge at its lower part is only about 6 inches above rail level for a width of 8 feet or more (see fig. 116), and on some, the permanent structure gauge rises 3 inches above rail level. With such restrictions it is practically impossible to design an inverted rail, satisfying the conditions enumerated in the second paragraph of this chapter. A conductor rail, as long as it clears the rolling structure everywhere, and as long as the shoe contact surface clears all permanent structures, may be arranged with local sections of special construction, where the permanent structures would encroach on the rail, or its appurtenances as ordinarily constructed. For these reasons the top contact rail is practicable for British railways, but the under-contact rail generally requires modification in the gauges. Sir J. Aspinall's side-contact rail (fig. 116) appears to combine some of the advantages of both types; although extended experience alone can show whether all trouble from sleet is eliminated by its use.

COLLECTOR SHOES.—Collector-shoes are of two general types, viz., the gravity and the spring type. The former is essentially an over-contact shoe, depending on its weight to keep it in contact with the rail; the latter generally consists of a flap slipper which is pressed to the rail by springs, and can be designed to suit either over-contact, under-contact or side-contact rails. This type allows more complete protection of the third rail; whilst, the moving parts being light, it keeps better contact at high speed than the gravity type. The shoes are usually mounted on oak beams carried on the axle-boxes, these providing the necessary insulation whilst being subject to but small vertical play. Collector shoes are usually designed to have a point of comparative weakness, so that the slipper portion alone may be broken off if any substantial obstruction be met.

Conductor rails are often protected against accidental

contact either throughout their length or at stations and other accessible places only, the degree of protection depending on voltage as well as accessibility. The protective covering is usually of wood, impregnated with creosote, but hard fibre, potware and channel iron have been used to some extent for the purpose. It is designed to conform with the type of rail and collecting gear, but the restricted space available often makes a satisfactory design a matter of considerable difficulty. Where gravity shoes are used, the protection usually consists of boards on one or both sides of the rail, standing a little above the contact surface and supported from the rail itself (see fig. 113). With spring shoes and top contact, the chief protection is supported above the rail, often, where sufficient space is available, by means of brackets mounted on the sleepers. With spring shoes and an under-contact surface the protection is often arranged to cover the top and extend down the sides of the rail, enfolding it closely and being supported by it throughout its length. Other forms of protection are shown in figs. 114 and 116.

The conductor rails are mounted on suitable stoneware insulators carried on the ends of the sleepers, which require usually to be made longer than the standard for the purpose of carrying them. It is necessary to arrange that the rail may have a certain amount of vertical play with respect to the insulator, for otherwise the depression of the sleeper, caused by the passage of a heavy train, puts the insulator in tension and is liable to cause its breakage. In long sections of rail thus loosely held, the alternate expansion and contraction due to temperature changes produces a tendency in the rail to creep bodily in one direction, particularly on gradients, and in the direction of the traffic. It is accordingly necessary to anchor such sections at intervals to large concrete or granite blocks set in the ground, and to allow gaps in the rails between anchorages in order to take up the expansion. The bonding of the conductor rail presents no peculiar features other than arises from the circumstance that, since it is not serving the additional purpose of a running rail, greater latitude is possible in the arrangements, the size of fish-plates and the location of the bolts being adapted to suit the bonding. The bonds used are generally shorter than those required for the running rails. The rails are generally chosen in as great lengths as possible

(about 60 feet) in order to reduce bonding. The intervals in the conductor rails, necessitated by the special work in the tracks, are bridged by means of cables which are brought out to suitable cable terminals and connected to the rails by means of bond terminals. Where such intervals occur it is usual, wherever practicable, to provide a length of rail on the opposite side of the track so that power may be kept on a locomotive continuously as it runs over the route, collector shoes being provided on each side of the locomotive with this in view. Fig. 103 shows the arrangement of conductor rails at a simple cross-over, with the location of feeder terminals. Suitable ramps are provided at the ends of each length of conductor rail, in order that the shoes may be brought to the contact surface without shock. Sometimes, where there is much special track work, as at large terminals, it is not possible to provide contact rails on either side of the track for considerable distances, and a length of overhead rail is used, special collectors being carried on the locomotives to take current from it. The overhead shoes shown in fig. 5 are intended for this purpose.

Sectionizing.—The conductor rail is usually made of sufficient cross sectional area to carry all the necessary current without feeders in parallel; for the cost of additional rail area is generally much smaller than that of separate feeders. The conductor is divided into sections of convenient length, the ends of which are taken to sectionizing switches located in suitable boxes or cabins, where they can be connected together in any desired manner. The principles to be observed in arranging the sectionizing switches are: firstly, that it must be possible to render any section of rail dead as occasion requires; secondly, that it must be possible to render any section alive when any one of the contiguous sections is alive; and thirdly, it must be possible to make certain desirable connections whose nature depends on the circumstances of the case. With regard to the last of these conditions, it is usually considered good practice, and particularly on busy suburban lines, to resort to what is known as single-line feeding, according to which the conductor rails for different tracks are, in normal working, kept entirely separate one from another, for the whole distance between substations. This arrangement, whilst not making the most efficient use of the conductivity of the rails, has the

advantage of localizing a breakdown, a short circuit on one line opening the automatic switches for that line at the substations without interfering with traffic on other lines. Similar results might be attained by having automatic features on some of the sectionizing switches, and this is done where circumstances require it; but as a rule it is preferable to locate all expensive and intricate switchgear in the substations where they can be kept under expert supervision.

The switches ordinarily used for sectionizing are single-pole single-throw knife switches of the requisite current-carrying capacity. Fig. 117 shows diagrammatically a common feeding

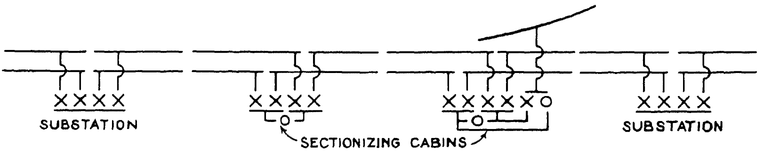


FIG. 117.—Sectionizing of Double-Track Line.

and sectionizing arrangement for a double-track line, the switches marked X being normally closed, and those marked O, open for single-line feeding; on this figure is shown also a spur line with its conductor rail arranged to be fed from any

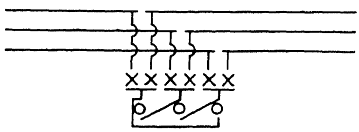


FIG. 118.—Sectionizing of Three-Track Line.

live conductor rail. Fig. 118 shows a usual arrangement of sectionizing switches for a three-track line. The lengths of the several sections will vary greatly among themselves, being principally governed by the length of block sections, for it is expedient to locate the sectionizing switches in or near to signal cabins, where telephonic instruction can be given for operating them, in order that a fault may be located, or a defective section cut out with the least possible delay. The circumstances which require short block sections usually make short conductor rail sections also desirable, and junctions are particularly favourable locations for the sectionizing cabins. Sidings and tracks intended for occasional use generally have

their conductor rails connected to those of main tracks through isolating switches, whether at sectionizing cabins or otherwise.

Limits of Conductor-rail Operation.—The side contact conductor rail used on the Lancashire and Yorkshire Railway is designed for and operates at 1,200 volts, and an inverted or under-contact rail is used by the Central California Traction Company at this voltage. With adequate insulation and protection there appears no reason why a conductor rail should not be used at even higher voltages if economy can be thereby shown. The Michigan Railway indeed put in a conductor-rail installation at 2,400 volts, but it did not prove a success, on account of voltage kicks on change of current, with consequent motor failures, and the pressure has been changed to 1,200 volts. As the voltage increases, the cost of installation necessarily increases also; but for moderate voltages the conductor rail is a much sounder and more lasting mechanical construction than any other form of line conductor of equal cost. The chief objection to it, from the point of view of the electrification of existing steam railways, is that it introduces a new element of danger in the work of an existing permanent way department, and thereby conflicts with the natural conservatism of officials. This objection, however, is likely to appear larger before the electrification has taken place than when use has made the conditions familiar.

OVERHEAD LINE CONDUCTORS

For alternating current and other high voltage systems, it is necessary to carry the line conductors above the loading gauge, suspending them from transverse members of structures erected for the purpose. The line conductors in this form are characterized by great flexibility and the collector gear on the train is given a wide range of action. The location of the line conductors, which in this case are known as trolley wires, depends on considerations to which reference has already been made; but, as long as they clear the rolling structure gauge at all times and places, it is by no means necessary that they shall be everywhere within the permanent structure gauge; it is sufficient that they clear existing structures as met with; since elsewhere the variation in the collector gear permits considerable choice in the position of the contact surface. In this particular, as in its flexibility and overhead location, the

trolley wire is distinguished from the conductor rail, with its rigid contact surface and invariable location with reference to the gauge. Anything in the nature of local rigidity indeed should be avoided, particularly on lines intended for high speeds, for it is at such hard spots that arcs are drawn, which cause rapid destruction of the conductor. Care must therefore be taken that the wire has vertical flexibility at all places, and the method of support in particular must be designed with this necessity in view. For a similar reason, rapid variation in the height of the trolley wire must be avoided where high speed is attained, and the arrangement must be so devised that the inevitable variations due to temperature changes, or those rendered necessary by the presence of permanent overhead structures, take place over a comparatively great distance, the ratio of this distance to the variation in height being made the greater the higher the speed for which the line is to be used.

Catenary Suspension.—The above considerations, together with a desire to reduce the number of main supports, which are apt to obscure the view of signals, have been effective in making the so-called "catenary" type of suspension the universal choice for railway work in the open, in spite of somewhat greater difficulty of installation and adjustment, as compared with the usual tramway forms of construction. In catenary suspension, the trolley wire is hung from one or more messenger cables, usually of steel strand (but sometimes of bronze, as on the Riksgräns line), which follow the line of the tracks and are carried directly on the main supports. The details of the overhead work vary greatly, but two main types may be recognized: viz., the single catenary, which uses one messenger cable, and the double catenary, in which the trolley wire is supported from two cables running side by side. The latter of these constructions, which is used on a few railways of early date, has not been found to possess any advantage to justify its greater cost: it is accordingly no longer used in new installations, and need not be considered further. Single catenary construction takes several forms of which the simplest is chief. Sometimes, as on the New York, Westchester and Boston Railway, a large primary catenary is used, supporting two secondary catenaries, which in their turn support the trolley wire. Sometimes, as on the Shildon-Newport line, the catenary supports an auxiliary straight wire from which the trolley wire is suspended by means of looped hangers; the

object being to secure the greatest possible measure of vertical flexibility throughout the length of the trolley wire. Sometimes, as on the Chicago, Milwaukee and St. Paul Railway, two trolley wires are supported by a single catenary, by means of looped hangers connected alternatively to the two wires. This arrangement was adopted partly on account of the great capacity required and partly to secure flexibility and freedom from arking.

Collector-gear. The design of the overhead work, like that of a conductor rail, depends on the form of collector gear to be used. The trolley wheel and pole, familiar in tramway practice, is used but little in railway work, the three phase Cascade Tunnel locomotives * furnishing an example however. The use of the trolley wheel requires mechanical continuity and uniformity in the trolley wire at all supports, crossings, section insulators and other special work; for defect in this respect may result in the wheel leaving the wire and causing damage to the overhead work. A rolling contact of cylindrical form, capable of reversed motion, is used for current collection on the Butte, Anaconda and Pacific Railway. This, although free from most of the objections to the wheel collector, still restricts the overhead work; its comparative shortness requires the trolley wire to follow the track closely under all conditions of wind and weather, thus limiting the distance between supports; whilst its inertia is greater than is desirable in a single contact collector. Moreover, experience does not indicate any great advantage to set against these disadvantages. Practice is therefore tending toward the use of the sliding collector for all railway work.

The useful length of the sliding collector may be made as great as overhead structures will allow, thus permitting the trolley wire to deviate from its normal position over the track without harm. The contact surface is made renewable, and, in common with all movable parts, is designed to be as light as is consistent with due strength. It is generally carried on a freely jointed structure, built up for the most part of light steel tubes, and called a pantagraph, from its general resemblance to the drawing instrument of that name. For comparatively small currents the contact surface often takes the

* See fig. 194, p. 306.

form of a single bar in the form of a bow, mounted on the pantagraph and erected by means of springs. With this arrangement, small irregularities in the trolley wire are followed by the light bow, the pantagraph responding principally to the larger variations. Sometimes the sliding collector is carried on a large bow instead of a pantagraph, and in this case also an auxiliary bow for following small irregularities in the line conductor is frequently employed. Sometimes, in order to increase the capacity of the collector, two auxiliary bows are carried by the pantagraph or main bow. Sometimes, again, a number of contact strips are carried, side by side, fixed in a pan of sheet steel, borne by the pantagraph, and this type can be designed to collect any current that the trolley wire can be used to carry. The renewable contact strips are often made of aluminium, to which a small quantity of copper is added for the purpose of hardening it. The surface is usually lubricated by means of a mixture of grease and graphite, contained in a groove running the length of the strip. Lubrication not only reduces the wear of trolley wire and collector, but improves the collecting qualities, apparently by reducing the tendency of the rubbing surfaces to chatter ; it also seems to render sleet more easily detached from the trolley wire. In the pan type of collector, wearing strips of copper have been found preferable. Having a number of contact strips in multiple, the pan collector requires a much greater upward pressure than the bow collector : it in fact depends for continuity of contact on actually lifting a short section of trolley wire, whereas the bow collector is intended rather to follow the irregularities. The sliding collector, in all its forms, is kept in contact with the line conductor by means of springs, which are usually brought into action pneumatically, compressed air for raising the collector being taken from the main reservoir. Usually, on releasing the pressure, the collector falls by gravity, assisted by the springs ; but sometimes it is arranged to lock with extended springs and must be tripped to bring it down, a pneumatic device being used for this purpose. The whole collector is mounted on insulators carried on the roof of the locomotive.

Capacity.—A single contact bow collector having an aluminium wearing strip can collect up to about 150 amperes for short periods at low speeds, and up to 50 amperes at speeds

of 50 or 60 miles per hour. With copper wearing strip about double the above current can be collected. With multiple contacts the capacity is increased in proportion. The pan type collectors used on the Chicago, Milwaukee and St. Paul Railway, which have a double pan with copper wearing strips and work on a double trolley wire, have been proved capable of collecting up to about 4,000 amperes at 15 m.p.h. and up to 1,500 or 2,000 amperes at 60 m.p.h. without appreciable arking. This was probable under test conditions as no such current is required in service.

Zig-zagging of Trolley Wire.—A collector of sufficient delicacy to be used for taking current from a trolley wire at high speed requires to have its surface of contact continually varied, for as this surface approximates to a point, the local pressure is very high, as is the current density; and a short distance run over a fixed point in the collector will cut a groove in it, whatever the material of which it may be made. The rolling contact provides for this change of surface naturally, but the sliding contact requires suitable variation to be made in the location of the trolley wire to ensure it. Accordingly, it is necessary to zig-zag the trolley wire from one side to the other of the track centre when a sliding contact is employed. The amount of transverse motion needed has been determined by experience as about 1 per cent. of the train motion, so that the trolley wire moves one foot across the collector for every 100 feet travel of the train. The swaying of the locomotive also tends to prevent local wear of the collector.

There are certain differences between European and American practice in catenary construction which it may be well to note here. In general, American methods appear the simpler, from the point of view of the construction gang, but perhaps hardly take full advantage of the nature of the work to secure the best results from the point of view of the operator. European overhead work, on the other hand, generally gives the impression of having been very carefully thought out and designed to meet the conditions of railway work, but often introduces refinement which appears to exceed practical necessity.

American Overhead Work.—In America the messenger wire is usually installed in spans ranging up to 150 feet in

length. Sometimes this is supported from a second messenger of double the length, which is carried on the main supports, but usually the spacing of the main supports corresponds with the length of span of the primary messenger wire. The messenger cable is carried on petticoat insulators to each of which it is clamped. The trolley wire is suspended from the messenger cable by hangers of looped form, which are designed to be raised from the messenger cable under the pressure of the pantagraph. The hangers are generally spaced about 15 feet apart on each trolley wire, and are accordingly at a half this distance on the messenger when a double trolley wire is used. The trolley wire is anchored at intervals of about half a mile, but except on curves, is not systematically anchored against lateral movement, although occasional steady braces are introduced. In the less developed regions, wooden poles, having **T**-iron side brackets, are often used as main supports for the overhead work; sometimes the poles are located in pairs connected by span wires of steel cable, and with this construction the live conductors may be insulated by means of disc insulators. In the more settled regions, lattice poles with side arms, or light gantries are used. Typical American overhead work is shown in fig. 119.

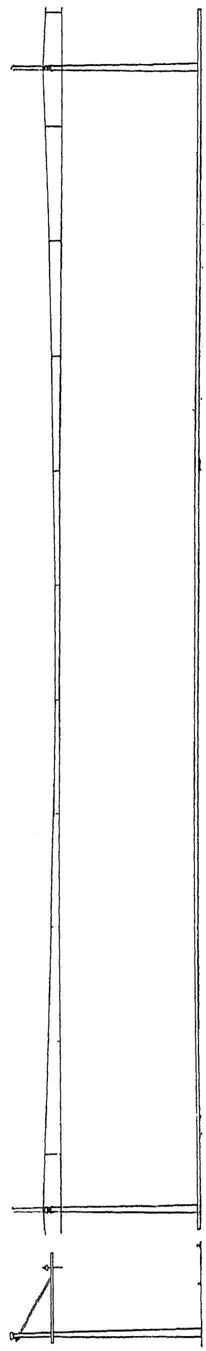


FIG. 119.—American Overhead Work.

European Overhead Work.—In Europe, the overhead work is generally installed in spans as long as the current collector will permit, sometimes reaching 100 metres (328 feet) on straight track. The trolley wire is anchored in the middle

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of each section of about a kilometre, and is kept under constant tension at all seasons by means of weights arranged to exert a pull on the free ends. It is also anchored against lateral movement at each catenary support, and by this means the lateral deflection is kept within bounds in spite of the use of long spans. In some forms of construction the messenger cable is clamped to the insulator at each support, in others it is allowed longitudinal movement with temperature variation, being kept under constant tension by means of weights acting on its ends.

In one form of construction, the trolley wire instead of being suspended directly from the messenger cable is supported by an auxiliary wire running the length of the line and suspended from the messenger cable by means of vertical hangers attached to both lines. The trolley wire is hung from the auxiliary wire by means of loops which allow the former to rise under the pressure of the collector, and also permits longitudinal motion to take place with change of temperature under the action of the tension devices. In this system, the messenger cable is attached to the insulators carried on the main supports; the variation in sag of the catenary accordingly produces a variation in height in the trolley wire at the centre of the span. Fig. 120 shows the arrangement in question. In another form of construction, both trolley wire and messenger cable are put under appropriate constant tension, so that there is no seasonal variation in the sag. The messenger cable is in this case carried on a freely turning bobbin insulator, on which it lies without being attached to it. This is the principle of the arrangement, but in the actual construction, as it is used on the Dessau-Bitterfeld line, and shown in fig. 121, a third wire is introduced, the so-called span wire which is attached to the messenger cable near its two ends. This is said to have the effect of ensuring uniform tightening in the messenger cable. The messenger cable is not actually carried over the insulator, but, together with the span wire, is allowed to hang below in a loop, to which the trolley wire is attached. A length of special suspension cable is employed to pass over the bobbin insulator, and disc strain insulators are introduced in this, giving double insulation from ground. By these means it is sought to ensure a uniform height of trolley wire at all seasons, but it would seem that this end is not completely attained. Near to the

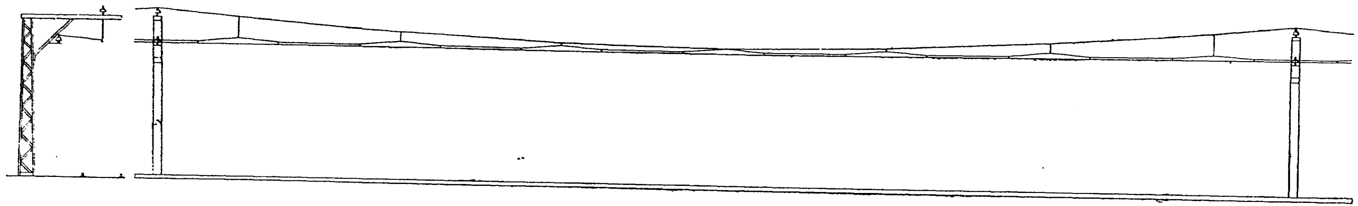


Fig. 120.—Continental Overhead Work. Siemens.

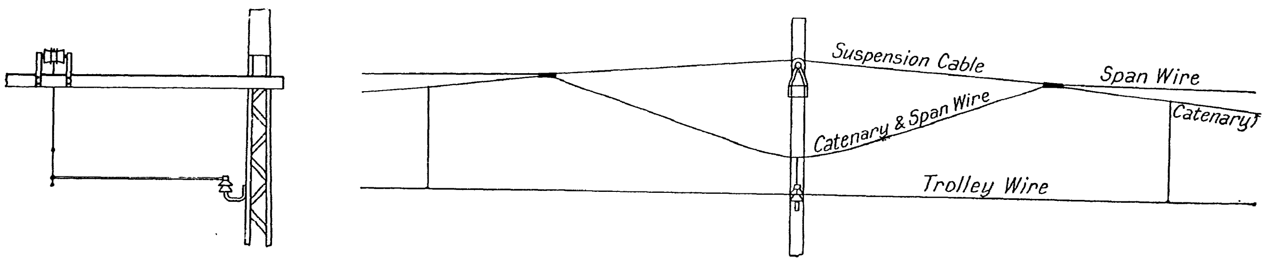


Fig. 121.—Continental Overhead Work. A.E.G.

free end, where the tension devices are placed, and which is 500 metres from the anchorage, the whole construction, trolley wire, hangers, messenger cable and span wire moves a foot or more longitudinally between summer and winter; and in so moving the hanger on one side of a main support is raised, whilst that on the other side is lowered, producing a local slope in the trolley wire quite comparable with that due to the sag in a catenary tied to its supports. In yet another form of construction, seen on the Riksgränsen line, the overhead work is carried on side brackets which are devised to swing about the poles. Both messengers and trolley wire are anchored in the middle of a section and kept under tension by means of appropriate weights applied to the free ends, the seasonal variation in length being taken up by the swing of the brackets.

Observations on Overhead Equipment.—In designing an overhead equipment for an electric railway, it makes for economy and conduces also to the clearer view of signals, to have the catenary spans on straight track as long as practicable; for the stresses in the parts are not the limiting features in economical design, but rather the lateral deflection of the trolley wire due to wind, and possibly the vertical deflection due to change of temperature, effects which vary as the square of the span. The former is reduced by lateral anchorage of the trolley wire at each catenary support; and by keeping the trolley wire under considerable tension in all seasons. Increase of tension in the messenger wire also tends to reduce the deflection due to wind, but this expedient leads to greater vertical variation due to temperature, so that the best compromise must be sought. The length of span, in its effect on lateral deflection, is limited by the design of the collector gear; and this may on the one hand be limited by the type as effecting practicable design, and on the other by existing tunnels and overhead bridges. The necessity for reducing vertical variation in the height of the trolley wire becomes the greater the higher the speed of operation and the greater the frictional resistance to deformation in the collector-gear. The inertia of the collector-gear may also have some effect, but this is probably negligible in the case of variations now under consideration. It is the rate of longitudinal variation of height that should everywhere be kept within bounds: the actual

height may vary between comparatively wide limits without detriment as long as the variation takes place sufficiently gradually. Tension in the trolley wire tends to reduce the variation in height due to the variation in sag of the messenger cable, relieving this cable of weight at the centre of the span in warm weather, and adding it towards the ends, thus tending to act as a compensating device.

Equation of Sag.—Let fig. 122 represent a span of cable or wire hanging freely between fixed points A and B with maximum sag, s. Let l be the length of the span, and l' that

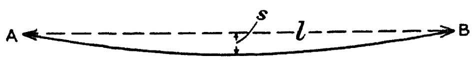


FIG. 122.—Diagram of Catenary.

of the wire itself. As long as the sag is small compared with the span, the form of the curve taken by the wire may be taken as a parabola of equation :—

$$x^2 = \frac{l^2}{4s} y \quad . \quad . \quad . \quad . \quad (1)$$

Accordingly :

$$l' = 2\int_o^{\frac{l}{2}} \sqrt{dx^2 + dy^2}$$

$$= 2\int_o^{\frac{l}{2}} [1 + \left(\frac{8s}{l^2}\right)^2 x^2]^{\frac{1}{2}} dx$$

$$= l\left[1 + \frac{8}{3}\frac{s^2}{l^2}\right] \text{ approximately} \quad . \quad . \quad . \quad (2)$$

Let T be the tension per unit area at the centre of the span, in gravitational units, A the area under tension, and w the density of the material. Then the weight of the half span, or the vertical component of the tension, is :

$$w\frac{\mathrm{A}l}{2} = \mathrm{TA}\left(\frac{dy}{dx}\right)_{x=\frac{l}{2}} = \mathrm{TA}\frac{4s}{l}$$

giving :

$$sT = w\frac{l^2}{8} \quad . \quad . \quad . \quad . \quad (3)$$

The tension is practically constant throughout the span. If in consequence of an increase of tension dT the length of the wire is increased by dl', then (E being Young's modulus of elasticity):

$$dT = E\frac{dl'}{l'} = E\frac{dl'}{l}$$

or:

$$dl' = \frac{l}{E}\,dT \quad . \quad . \quad . \quad . \quad (4)$$

If the temperature rises by dt, the length of wire increases by $aldt$, where a is the coefficient of linear expansion. The total change in length of the wire is therefore:

$$dl' = \frac{l}{E}\,dT + aldt \quad . \quad . \quad . \quad (5)$$

Hence if the suffix o indicates any initial condition:

$$l' - l'_o = \frac{l}{E}\,(T - T_o) + al(t - t_o) \quad . \quad . \quad (6)$$

Putting l' and T in terms of the sag by equations 2 and 3:

$$\frac{8}{3}\,l^2\left(\frac{s^2}{l^4} - \frac{s_o^{\,2}}{l^4}\right) + \frac{w}{8E}\left(\frac{l^2}{s_o} - \frac{l^2}{s}\right) = a(t - t_o) \quad . \quad . \quad (7)$$

This equation is fundamental, and gives the relation between the sag and temperature for any length of span. The effect of the elasticity of the wire, represented by the second set of terms on the left hand side, is considerable and cannot be neglected even as a first approximation. The minimum sag corresponds with the maximum permissible value of T, and is given by equation 3.

The above equations are applicable approximately to catenary construction with suitable modification in the values of the constants. E and a now apply to the material of the messenger cable, but w is a fictitious density, being equal to the weight per unit length of the whole span—messenger cable, hangers and trolley wire—divided by the area of cross sections of the messenger cable alone. As an example, take the case in which w = ·7 lbs. per cu. inch, corresponding to a trolley wire of about 30 per cent. greater cross section than

the messenger cable. Take a maximum working tension T_o of 10 tons per sq. inch. Equation 3 then gives :—

$$\frac{s_o}{l^2} = \frac{\cdot 7}{8 \times 22{,}400} = \frac{1}{256{,}000}$$

Taking E for stranded steel cable as 25×10^6 lbs. per sq. in. and a as $0{\cdot}65 \times 10^{-5}$ per deg. F., equation 7 can be written in the form :

$$t - t_o = 5{\cdot}91 \times 10^{-5} l^2 (x^2 - x_o^2) + \frac{538}{xx_o}(x - x_o) \quad . \quad (8)$$

where $x = 10^6 s/l^2 \qquad x_o = 10^6 s_o/l^2 = 3{\cdot}906$

Equation 8 gives the sag at any temperature and equation 3

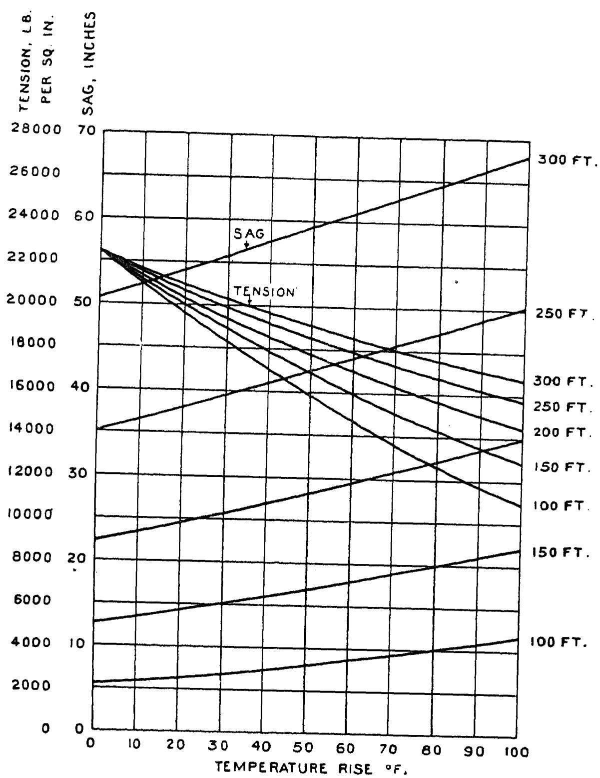


FIG. 123.—Curves of Sag and Tension.

the corresponding tension in the messenger cable. The results are plotted in fig. 123 for spans from 100 feet to 300 feet. It

will be seen from this figure that the variation in tension is smaller the greater the span; also that, with spans of different lengths between anchorage, the tension in the messenger varies from span to span and cannot be balanced at all temperatures. A residual horizontal pull has accordingly to be borne by the insulators to which the messenger cable is attached.

EFFECT OF TENSION OF TROLLEY WIRE. Inasmuch as the trolley wire is kept under tension, and that this tension bears a little of the weight when the wire sags, or adds to the weight when it bows upwards, the solution given above requires modification to make it agree with the fact. If the tension of the trolley wire is T', and s' is the sag of the messenger when the trolley wire is level, so that $s - s'$ is the sag of the trolley wire, the tension bears weight (see equation 3):

$$w' = \frac{8T'}{l^2}(s - s') \quad . \quad . \quad . \quad (9)$$

The messenger is relieved of this weight and A and A' being the areas of cross section of messenger and trolley wire respectively the tension in the messenger cable is given by:

$$sT = \left(w - \frac{w'A'}{A}\right)\frac{l^2}{8} = w\frac{l^2}{8} - \frac{A'}{A}T'(s - s') \quad . \quad (10)$$

Equation 7 accordingly becomes:

$$\frac{8}{3}l^2\left(\frac{s^2}{l^4} - \frac{s_0^2}{l^4}\right) = \frac{w}{8E}\left(\frac{l^2}{s} - \frac{l^2}{s_0}\right) - \frac{A'}{EA}\left[T'\left(1 - \frac{s'}{s}\right) - T_0'\left(1 - \frac{s'}{s_0}\right)\right]$$
$$+ a(t - t_0) \quad . \quad . \quad . \quad (11)$$

If the tension of the trolley wire is maintained constant equation 11 is simplified somewhat. If, on the other hand, the length of the trolley wire is kept constant, the tension T' is given by

$$T' = T_0' + E'\left[\frac{8}{3}\frac{(s - s_0)(s + s_0 - 2s')}{l^2} - a(t - t_0)\right] \quad (12)$$

in which, however, the first term within the brackets is generally of small importance.

Two additional quantities have therefore to be determined before the true sag can be computed, viz., the temperature, t', at which trolley wire is to become level, and the tension to be permitted in this wire. These quantities are at the disposal

of the engineer. As regards the former, it may be noted that the wire is lifted by the collector to a somewhat greater extent at the centre of the span than at its ends; thus a small sag at normal temperatures is not objectionable. As regards the latter, the material of the trolley wire should not approach the yield point under the most stringent working conditions. Fig. 124 shows the tension and sag of the messenger, and the

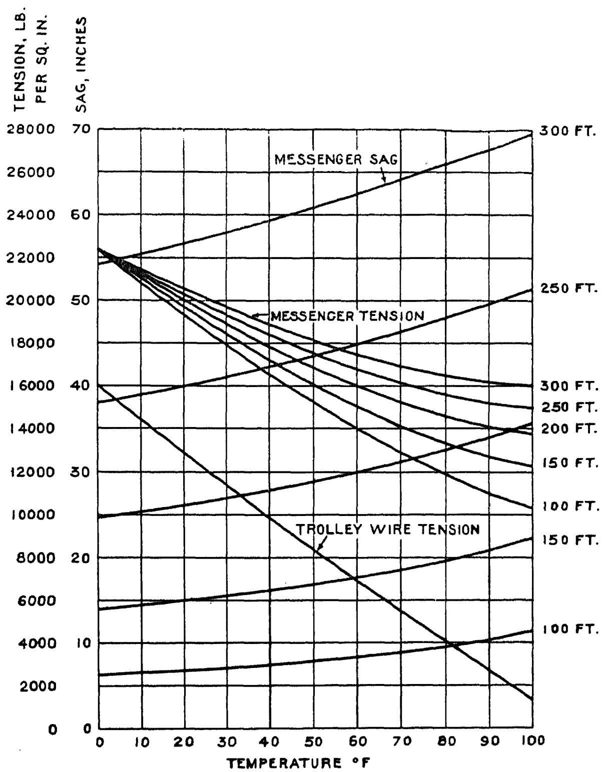


FIG. 124.—Curves of Sag and Tension allowing for Tension in Trolley-Wire.

tension of the trolley wire, as computed from equations 10, 11 and 12, from the data: $t_o = 0°$ F., $t' = 32°$ F., $T_o' = 16{,}000$ lbs. per sq. inch, $E' = 16 \times 10^6$ lbs. per sq. inch, $\alpha' = 0{\cdot}95 \times 10^{-5}$ per deg. F., $A' = 1{\cdot}3A$, with other data as before, viz., $w = 0{\cdot}7$ lbs. per cu. inch, $T = 22{,}400$ lbs. per sq. inch, $E = 25 \times 10^6$ lbs. per sq. inch, $\alpha = 0{\cdot}65 \times 10^{-5}$ per deg. F. The differences between figs. 123 and 124 are quite appreciable.

In the above example the maximum tension in the trolley wire has been taken at 16,000 lbs. per square inch, which is perhaps as near to the yield point as it is advisable to go. The wire nevertheless becomes slack at a little over 100° F., a temperature that might easily be reached under the combined influence of the sun and current. A little slackness does not interfere with good operation, being taken up to some extent in the yield of steady braces and curve pull-offs, but it may nevertheless be inferred that a copper trolley wire, under the conditions assumed, which are by no means abnormal, is employed to the limits of practicability. In American practice the tension imposed in installing the trolley wire is greater than is shown in fig. 124. The cold of the first winter, in fact, usually causes considerable yielding, and subsequent winters stress the wire to the yield point. The objection to this practice arises from the fact that the yielding of the whole section between anchorages is to a large extent concentrated at the weakest spot, whether the weakness be due to local wear or to defective material: eventual breakage of the trolley wire is therefore to be feared, and this under conditions unfavourable for repair work. The European practice of keeping approximately uniform tension on the trolley wire, although less simple than the other, is not only the more scientific but also the more reliable. On the other hand, there seems no sufficient reason for introducing the complication of a uniformly stressed messenger wire.

The Erector's Problem.—The curves of figs. 123 and 124, although of much interest to the engineer as showing how the quantities depend on one another, are not of great use to the erector of the overhead line: for since this is a compound structure it is not practicable to erect it in one operation. The problem as it confronts the erector is, first, that of putting up the messenger cable in such manner that, when subsequently weighted with the trolley wire, it may have the sag and tension appropriate to its temperature; and afterwards of putting up the trolley wire with the correct tension.

Let the suffix 1 distinguish quantities relating to the messenger cable when free of the trolley wire. Let λw be the weight of the messenger cable, w being as before that of the complete line, both referred to unit volume of the messenger. In hang-

ing the trolley wire the tension in the messenger is increased by (see equation 3):

$$\mathrm{T} - \mathrm{T}_1 = \frac{w}{8}\,\frac{l^2}{s} - \frac{\lambda w}{8}\,\frac{l^2}{s_1} \qquad . \qquad . \qquad (13)$$

and its proportional length by (see equation 2):

$$\frac{8}{3}\left(\frac{s^2}{l^2} - \frac{s_1{}^2}{l^2}\right) = \frac{w}{8\mathrm{E}}\left(\frac{l^2}{s} - \lambda\frac{l^2}{s_1}\right) \qquad . \qquad . \qquad (14)$$

From equations 7 and 14:

$$\frac{8}{3}\,l^2\left(\frac{s_1{}^2}{l^4} - \frac{s_0{}^2}{l^4}\right) + \frac{w}{8\mathrm{E}}\left(\frac{l^2}{s_0} - \frac{\lambda l^2}{s_1}\right) = a(t - t_0) \qquad . \qquad (15)$$

Since s_0 is given by equation 3, T_0 being supposed known, equation 15 determines the sag required. If it be desired to take account of the effect of tension in the trolley wire, the equation for s_1 takes the form (see equation 11):

$$\frac{8}{3}\,l^2\left(\frac{s_1{}^2}{l^4} - \frac{s_0{}^2}{l^4}\right) + \frac{w}{8\mathrm{E}}\left(\frac{l^2}{s_0} - \frac{\lambda l^2}{s_1}\right) + \frac{\mathrm{A}'\mathrm{T}'_0}{\mathrm{AE}}\left(\frac{s'}{s_0} - 1\right)$$
$$= a(t - t_0) \qquad (16)$$

Here s_1 and s_0 are obtained as before from equations 10 and 11 with $t = t'$. The problem is therefore readily solved in this case also. Fig. 125 shows the initial sag and tension of the messenger deduced from equations 3 and 16, and corresponding to the case of fig. 124, in which $\lambda = 0{\cdot}4$. Fig. 126 puts the same results in more workable form for the erector, the trolley wire being taken as 0·5 inches in diameter and its tension taken from fig. 124.

Although great elaboration in calculation or fineness in adjustment would be wasted in work of this nature, inasmuch as the settling of supports and acquisition of permanent strains destroy the initial adjustment, it is both expedient and desirable to carry out the work with knowledge of the effect of the season's changes. Such a set of curves as are given in fig. 126, computed from data appropriate to the circumstances, should be at the disposal of the erector if a great deal of subsequent adjustment is to be avoided.

Sag on Gradients.—The sag is naturally measured as the difference between the height of the support and the height of the wire at the centre of the span. If the supports differ

in height, the mean should be taken. If the span is on a uniform gradient the measurement of sag is unaffected and should be taken at the centre, it being a property of the parabola that the vertical through the centre of a chord cuts the curve at the point of contact of the parallel tangent. If, however, the gradient changes within the span a correction for the height of wire is needed. If there is length l_1 of

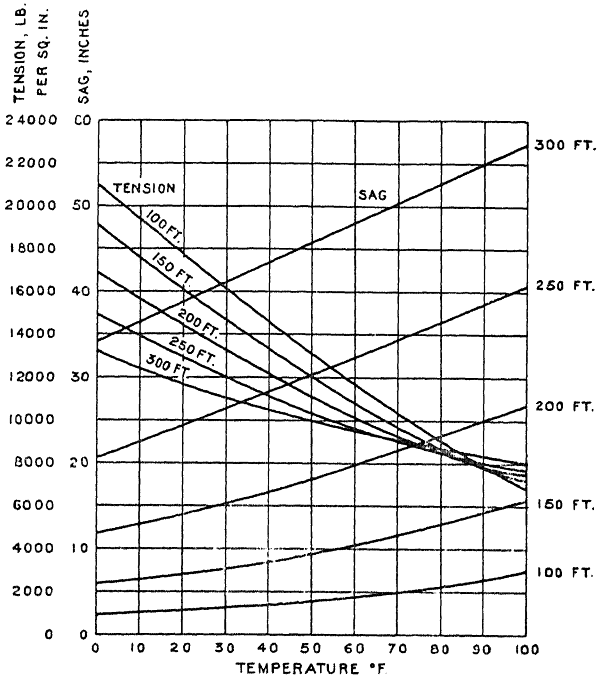


FIG. 125.—Curves of Initial Sag and Tension in Catenary.

gradient a_1, and length l_2 of gradient a_2 in the span, the level from which the height of the wire should be measured is $\frac{1}{2}(l_1a_1 + l_2a_2)$ above the first support. The actual measurement is taken from level $\frac{1}{2}(l_1 + l_2)a_1$, assuming the centre point to be on the first gradient. The apparent sag should therefore be increased by $\frac{1}{2} l_2(a_2 - a_1)$. The lengths of the trolley wire hangers are not affected by gradients and vary with y in equation 1, in which the sag should be taken as s', or that corresponding to straight trolley wire.

There is room for a considerable amount of rough calculation

in connection with the special work, which, however, is for the most part contained in the rule that a particular catenary (or a particular value of l^2/s') should be maintained throughout a section. If an overhead bridge intervenes, necessitating the making-off of the messenger, the attachment should be made where the normal catenary reaches the bridge. If, as is very common, it is necessary to lower the trolley wire at the

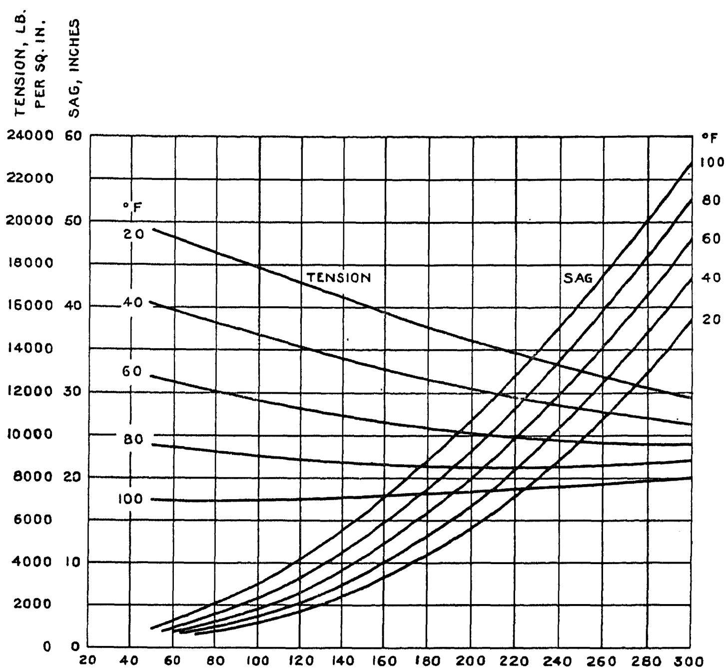


FIG. 126.—Curves of Initial Sag and Tension in Catenary plotted against support spacing.

bridge, the approach should be made at uniform gradient a, the more gentle the higher the speed of operation. At the same time the spacing of supports should be reduced by approximately $l^2a/4s'$, the height of the supports following the gradient of the trolley wire. Many of the curves on railways require no special work other than steady braces or side anchors at each support, and few require more than one intermediate pull-off.

Effects of Wind on Catenary.—If a wire hangs in a long

span, the wind is capable of deflecting it considerably from the vertical position, and the matter should be studied carefully in devising overhead line conductors. If v is the wind velocity, in miles per hour, and d the diameter, in inches, of smooth wire hung normal to the direction of the wind, the horizontal pressure on the wire is approximately $0{\cdot}002\ v^2d/144$ lbs. per inch length, or $0{\cdot}0025\ v^2/144d$ lbs. per cubic inch. These figures should be increased by 25 per cent. for stranded cable. For smooth wires, then, the span is deflected from the vertical by an angle, θ, whose tangent is $0{\cdot}0025\ v^2/144\ dw$, and the sag is increased to correspond with density $w \sec \theta$. If the tension on the wire is maintained constant, the vertical sag remains unchanged and the horizontal displacement at the centre is (see equation 3):

$$s \sin \theta = \frac{w \sec \theta}{8} \frac{l^2}{\mathrm{T}} \sin \theta = \frac{wl^2}{8\mathrm{T}} \tan \theta \qquad . \qquad . \quad (17)$$

Thus if $v = 60$ m.p.h., $d = {\cdot}5''$. $\mathrm{T} = 7{,}000$ lbs. per sq. inch, $l = 200$ feet, the horizontal displacement is about 13 inches; and if $w = {\cdot}32$ lbs. per cu. inch, $\theta = 21\frac{1}{3}$ degrees. If the catenary is fixed at the ends, the tension is increased by the wind pressure and the horizontal displacement is then less than is given by equation 17, and greater than $s \sin \theta$, where s is given by equation 7. The true value for s is indeed given by the equation 7, with $w \sec \theta$ written for w.

The deflection of a trolley wire carried by a messenger and allowed to swing freely, can be approximately determined by similar means. Using notation in the sense already indicated, with d' as diameter of trolley wire and d that of messenger, assumed stranded, and taking moments about the line of supports, since the centre of pressure of the catenary is one-third of the sag above the lowest point:

$$\tan \theta = \frac{0{\cdot}0025v^2}{144wd} \left\{ \frac{2{\cdot}5}{3} + \frac{d'}{d} \right\} / (1 - \tfrac{1}{3}\lambda) \qquad . \qquad . \quad (18)$$

Thus for the conditions of fig. 123 the horizontal displacement in an 80 m.p.h. gale is more than 80 per cent. of the sag of the messenger. It is clear, therefore, that a freely swinging trolley wire is impracticable where long spans are to be used. The dividing line between anchored and freely swinging trolley wires depends on the length of the current collector, but spans

of 200 feet and upwards are usually anchored against side movement at each support, whilst spans of 150 feet or less are generally free, except for occasional steady braces intended to prevent cumulative oscillations.

If the trolley wire is anchored at each support, the problem becomes more difficult, but an approximation to the extreme deflection is given by the following formula:

$$\tan\theta = \frac{0{\cdot}0025v^2}{144wd}\left(\frac{2{\cdot}5}{3} + \frac{d'}{d}\right)\Big/\left(\tfrac{2}{3} + \frac{\mathrm{T'A'}}{\mathrm{TA}}\right) \qquad (19)$$

The desirability of keeping tension on the trolley wire, particularly where long spans are used, is apparent from this equation.

In order that wind may have as little deleterious effect as possible the trolley wire should be hung so as to be in its central position at the centre of the span. Thus on curves the wire should be arranged as a series of tangents to the curve, rather than as a series of chords. If the wire is zig-zagged about a central position the displacement so produced must be added to that due to the wind. With the wire anchored laterally at the supports the maximum displacement from its central position may be greater than that due to the wind. If y_1 is the maximum deviation due to wind, and y_2 that due to zig-zagging, then, provided that y_2 is less than $2y_1$, the greatest deviation from the central track line is:

$$y_1\left\{1+\left(\frac{y_2}{2y_1}\right)^2\right\}$$

In the alternative, the maximum deviation is y_2.

Design and Methods of Construction of Overhead Work.—The general considerations entering into the design of overhead line work having been discussed, little need be said here of details of construction, of which there is great variety. The main insulation is usually of porcelain, although in the United States strain insulators of treated wood are used for pressures below 4,000 volts. Double insulation from ground is frequently provided. The main supports may consist of lattice-poles with cross girders, lattice or wood poles with cross spans, or for single lines, poles with side brackets. Cross girder construction has been hitherto most extensively used for main line work, with more than one track, but cross spans of galvanized steel cable appear to be gaining in favour,

principally on account of their being less costly, and obstructing the view of signals less than other forms of construction. In cross span work, moreover, the insulation can be effected entirely by means of porcelain strain insulators of the disc type, and these form very satisfactory insulation in that the stresses in them are purely compressive, and breakages after installation are likely to be few. Section insulators are very easily provided, being formed by carrying the two sections of trolley wire side by side for a short distance with an air space between them, the sections being bridged by passing collectors. The trolley wire is led off at junctions in a similar manner. Slight curvature in the line does not usually require any special work other than careful attention to the lateral anchorage of the trolley wire at the supporting poles, but greater curvature may necessitate pull-off arrangements of type similar in principle to those in common use on tramways.

Bridge Work.—The installation of the trolley wire under existing bridges or in tunnels is frequently a matter of great difficulty on account of the small clearances available between maximum rolling loads and permanent structures, in the design of which the possibility of interposed high voltage conductors was not contemplated. The problems presented by these overhead structures require individual consideration, and often tax the engineer's faculty for compromise to the utmost. In some cases by careful study of the clearances, it is found possible to introduce the conductor and design a collector to work with it. In others the collector is run through on two wires, in order to define its clearance more exactly. In others again it is found necessary to lower the tracks in order to provide greater headroom. In yet others, the clearances are so small and the difficulties of lowering the track so great that a dead section has to be introduced beneath the structure, through which the trains coast without taking power. Practically all steam railways in this country would, if electrified and employing overhead line conductors, be restricted with respect to the length or position of collecting devices by existing structures, and costly as the restriction would prove, on account of the limitations it imposes on the overhead work, it would usually be found less costly, if less satisfactory, than the change in the structures necessary to remove it.

The Trolley Wire.—The trolley wire is generally com-

posed of hard drawn copper of grooved section. Where large currents have to be collected, as on the Chicago, Milwaukee and St. Paul Railway, a double trolley wire is employed, and the two wires may be suspended from a single messenger cable. In one case, viz. that of the New York, New Haven and Hartford Railroad, a steel wire has been installed which takes the wear of the current collectors, the original copper wire being used simply to carry the current. This arrangement was however primarily intended to add flexibility to the overhead line. The messenger cable is usually composed of stranded galvanized steel wire, although cables of stranded bronze wire are sometimes used, these being less liable to corrosion and having smaller electrical resistance than the steel wires. However the mechanical characteristics of steel wire usually give it the advantage for the purpose, even when the use of bronze wire would save the expense of extra feeders.

SECTIONIZING.—Like the conductor rail the trolley wire is divided into sections for isolating purposes ; in fact where the trolley wire is kept under definite tension, it is necessary, as already explained, to break its continuity at intervals of about a mile, and advantage is taken of this circumstance to insert isolating and interconnecting switches at the points of section. Single line working is the rule on high voltage railways ; at certain places however the lines may be connected in multiple for the purpose of introducing track boosters, but even this can be avoided by the use of separate boosters or separate windings for the several lines.

THREE-PHASE SYSTEMS.—For a three-phase system of distribution two overhead wires are necessary, and since these must be suspended so as not only to be clear of structures and of each other everywhere, but also so that each may always clear the current collectors of the other phase, whilst all current collectors must always clear the permanent structures, the difficulties of installation are much greater than when a single overhead line only is necessary, and the limits of practicable deviation correspondingly reduced. The spans must accordingly be short ; and the trolley wire anchored frequently against lateral displacement, and pulled-off duly on curves. The special work at turnouts and the like is complicated by the crossing of wires of different phases, requiring many insulated sections. The difficulties of installation indeed make a

formidable obstacle to the use of a polyphase distribution system in the neighbourhood of large towns where much special work exists, and confine its sphere of usefulness to simple and isolated railways.

INTERFERENCE OF DISTRIBUTION SYSTEM WITH NEIGHBOURING SYSTEMS

The distribution system of an electric railway affects neighbouring electric systems by induction, both electrostatic and electromagnetic, whilst leakage currents from the track or line conductors may enter directly into other systems which are inefficiently insulated or which make use of the earth for conduction. In particular electric railways in some cases interfere greatly with the operation of telegraph and telephone systems which parallel them, even to the extent of preventing service entirely. Telegraph systems are usually worked with grounded return, and although requiring in general an appreciable working current, are liable to interference from stray earth currents. Telephone systems, on the other hand, generally use an insulated network, but so sensitive is the telephone receiver and human ear as a detector of alternating currents, that interference due to induction or leakage is extremely difficult to eliminate.

Continuous Current Railways.—Interference with weak current systems is not serious in the case of railways worked by continuous currents. In the early days a certain amount of trouble was experienced with telegraph instruments due to the earth currents; this was effectively eliminated by inserting resistance in circuit with the instruments, and increasing the battery power for working the telegraphs. At present the instruments used have themselves a high resistance and the trouble does not in general arise. Inductive interference with telephones is small on account of the feebleness of the high frequency ripples which accompany the continuous current. The most noticeable ripples are usually those due to the magnetization of the teeth of synchronous converters in the substations; but as the tooth-frequency is definite these ripples are readily short-circuited out by means of resonant shunts tuned to synchronize with this frequency. The ripples due to the train motors are not often heard; and the reason of this is probably that the motors generally have four poles

and an odd number of slots, so that there are four ripples set up by each motor at any frequency; these being in quarter-phase relation, yield no appreciable resultant. Frequent transposition of the telephone wires tends to reduce all inductive troubles. The commonest cause of noise in telephones is leakage, due to insufficient insulation of the telephone lines. Where the wires come in contact with trees, or where the insulators are exposed to salt water spray, there is always the possibility of leakage currents entering the circuit and giving rise to trouble which can only be eliminated by removing the cause.

Alternating Current Railways.—If the interference of continuous current railways with communication systems is of small moment and readily dealt with, it is far otherwise in the case of alternating current railways, in which the problems presented have resisted satisfactory solution in most cases, except by the drastic expedient of removing all the weak current circuits to a considerable distance from the railway. The same palliative measures can of course be applied as have been found effective with continuous current railways, but the predisposing causes of interference are so enormously intensified that these measures avail only to mitigate the trouble and rarely furnish an effective cure.

Primary Cause of Trouble.—In seeking the cause of the interference phenomena, the tendency of magnetic induction to increase the area of the circuit of a current should be taken into account. An effect of this is that the return current is driven from the rails and is carried for the most part by the earth. Experiments made on the Midi Railway * showed that, except for short distances near the feeding point and near the train, the current carried by the rails did not exceed 5 per cent. of the whole. The effective position of the return path was, from the observations, estimated to be at about 1·9 km. from the route. This was moreover for the fundamental frequency; the higher harmonies were doubtless even more dissipated. In its electromagnetic effects, therefore, the actual circuit differs greatly from the ostensible circuit. Its great area renders it effective for inductive interference at great

* See *La Lumière Electrique*, vol. 32 (1916), pp. 217, 241, 265, 289; vol. 33, p. 3.

distances, and the wide distribution of the earth current affects grounded or imperfectly insulated systems which might be supposed to be beyond the region of its influence. The electrostatic effects on the other hand are practically due to the trolley wire and its reversed image in the ground, and the radius of their influence is small.

Example of N.Y., N.H. and H. Ry.—The seriousness of the trouble may be judged from an experience recorded of the New York, New Haven and Hartford Railroad, that, with the original distribution system, a parallel telegraph line, situated at the distance of four to six miles from the road, could not be operated when the railway was working. The original distribution system had the simplicity usually associated with alternating current working, the line being fed directly from one point only, at 11,000 volts. As modified, the feeding points have been brought close together, an outdoor substation being located about every third mile along the route. The expense of the change was very great but by no means exceptional in this connection, and the measures adopted do not by any means appear to have eliminated all interference.

Example of Midi Railway.—As another instance may be cited the experience of the Midi Railway as recorded by M. Devaux-Charbonnel in the article referred to above. "The question of the protection of telegraph and telephone lines paralleling the route proved a matter of grave concern to them. The electromagnetic field created by the traction current is intense and was found very troublesome from the outset. Its effects were sensible at great distances, even up to several kilometres. The neighbouring conductors were transformed into veritable energy lines; in fact it was found possible to light a 110 volt 25 c.p. lamp on a telegraph line running between Perpignan and Prades. Under such circumstances telegraph service was rendered impossible; fuses were frequently blown and when they held the receiving apparatus was subjected to uninterrupted vibration. The telephone circuits, although having no earthed return, and moreover being regularly transposed, were nevertheless traversed by varying currents sufficiently strong to render communication impossible." The article from which this extract was taken discusses also possible means for reducing the interference.

Example of L.B. and S.C. Ry.—As a further example, the

experience of the London, Brighton and South Coast Railway may be mentioned, as cited in a paper by S. C. Bartholomew, read before the Institute of Post Office Electrical Engineers: * "This railway is of great interest, in that a serious attempt has been made in the lay-out to prevent leakage and electromagnetic induction, and although the result has not been successful, the steps taken have undoubtedly minimized enormously the injurious effects arising from those causes. The contact wires are sectionalized, and are fed from feeder or distribution cables laid alongside the track. This reduces the electromagnetic disturbance to some extent. The principal steps have been taken, however, with the return of the current, two conductors being employed to assist the return of the current apart from the rails and earth. One of the return conductors, the 'distributor outer,' is connected to the rails by a copper bond. With the original installation between London Bridge and Victoria, each rail was connected to this distributor outer, but on the extension this has been modified, the rails being bonded together and a connection made to the distributor at intervals only. The other return conductor, called a 'booster' cable, is connected to the distributor outer, and so to the rails, at a few points only. Both these cables run beside the track, and if the whole of the current returned in them it would have a considerable neutralizing effect in counteracting the electromagnetic induction from the trolley wires and the distributors. In spite of these arrangements, however, interference with telegraphs and telephones has not been eliminated, and a number of circuits had to be diverted from the railway owing to interference with the working."

The interference of alternating current railways with communication lines accordingly necessitates the introduction of a number of special and expensive features; and even when these have been installed, usually requires telephone lines to be carried in sheathed cables, and other expensive modifications of the communication circuits and instruments to be made. Far from being a secondary matter, accordingly, it has sufficient influence in itself to render alternating current operation of doubtful expediency in settled regions.

* See *Electrician,* vol. 74, p. 565.

CHAPTER VI

POWER EQUIPMENT

The generation and supply of electrical energy for railway purposes possesses few features to distinguish these functions from generation and supply for other industrial purposes; and such differences as occur are for the most part in degree rather than in nature. The energy may be, and in many cases is, bought from an electric power supply company, whose chief concern lies in other directions, and to whom the railway load is incidental. Important as are matters of generation and supply therefore to economical operation, they are not necessarily part of the business of a railway company at all, and it is the author's purpose to discuss them in general rather than in detail, dealing briefly with preferred methods rather than attempting a comprehensive treatment.

Certain characteristics are indispensable for the power equipment of an electric railway, although perhaps not more so than in some other classes of industrial work. First among these may be placed reliability; it is not good practice to neglect any practicable safeguard against interruption of service. Although a power equipment is necessarily a complex contrivance, the simplest is usually the preferred practice, as most conducive to reliability. Still with the view of preventing the dislocation due to breakdown, adequate standby plant should be maintained in reserve ready for service. The power plant should have considerable overload capacity, for great and sudden variation in the load is unavoidable in railway work. With these requirements satisfied, it is only necessary that the equipment should be capable of doing its work of supplying power in form and amount suitable for use by the trains.

The Generating Plant.—The generating plant may be

divided into three groups of apparatus: viz. the prime movers, the electricity generating plant, and the electricity controlling gear. Each group includes also a quantity of apparatus auxiliary to its main functions. Five classes of prime movers may be recognized: viz. reciprocating steam engines, steam turbines, gas engines, oil engines, and hydraulic turbines. Of these however only the second and fifth are of much importance in the present connection. The reciprocating steam engine is practically superseded by the steam turbine for electricity-generating purposes, and would not even be considered for a new installation of any size. The gas engine, in units of large capacity, is an expensive machine, and although its thermal efficiency is high, it is surpassed by that of the steam turbine when used in large units, particularly at light loads. The gas engine can be designed to utilize the gas formed as a by-product in the operation of blast furnaces, but since this is not railway business, the energy if generated in this manner would usually have to be purchased. The oil engine is not yet available in units of sufficiently large capacity for the ordinary requirements of railway service, and in any case is so expensive that where natural oil is the available fuel of a locality it has hitherto proved preferable to use it for steam raising rather than to employ the internal combustion engine. Apart, however, from these considerations, the characteristics of both gas and oil engines are such that the natural rating allows little overload capacity; and duty in which frequent large peaks of load are normal features requires either that the engines should be much under-rated, or that the rated capacity of the station should be greater than that of a steam turbine station designed to perform the same work. These alternatives amount to about the same thing, involving greater investment and lower efficiency of operation than in stations of equal output designed for steadier loads. The internal combustion engine is therefore not in general attractive as prime mover for electric railway use, but there are certain local systems of small extent, where it can be used with advantage, in combination with a floating battery to steady its load.

The Steam Turbine.—As a means of utilizing the intrinsic energy of steam, no prime mover surpasses the steam turbine in economy. It is able to make use of the steam at as high a

pressure as it is practicable to generate it, and to take advantage of a high degree of superheat and of vacuum. It accordingly has a high thermal efficiency, many modern designs surpassing the gas engine in this respect. Having an unvarying gradient of temperature, internal condensation is absent, and as moreover there are no reverse motions to cause shock, the mechanical efficiency is high, and well maintained over a wide range of variation of load. It can readily be designed with good overload capacity. It is simple in construction, and not liable to derangement: it is in fact by no means unusual for a steam turbine to run for weeks and even months without intermission. It is easily and preferably designed in large units; but nevertheless runs without undue waste at light loads. It is moreover convenient for purposes of electrical generation by means of directly coupled generators, its economical speed not being unduly restrictive in most cases. It is true that the economical speed is high, making generation at the higher frequencies preferable, but these frequencies are quite suitable for industrial purposes, including most classes of railway work. Up to a unit capacity of about 10,000 k.w. a speed of not less than 3,000 r.p.m. or a frequency of 50 cycles, is preferable in that it makes better use of the materials of construction; and the capacity at which a speed of 1,500 r.p.m. becomes the upper limit appears to be in the neighbourhood of 40,000 k.w. It may be concluded therefore that the European standard supply frequency of 50 cycles suits generation by steam turbines better than any lower frequency.

THE HYDRAULIC TURBINE.—The hydraulic turbine imposes no such natural restriction on the frequency of supply as the steam turbine, being by comparison a low speed machine. Although not limited in this respect, it is most commonly employed for generation at 50 cycles for the reason that this suits general industrial supply. The investment in hydraulic works is usually great, much more so than in a steam station of equal capacity. The cost of operation being small, the interest on the investment comprises almost the whole of the cost of generation. A high load factor is accordingly particularly desirable for a water power plant, in order to reduce the cost of supply. In a steam plant on the other hand a considerable fraction of the cost of generation is practically proportional

to the load, and the high load factor is of lesser importance. The railway load factor, unless artificially kept up, is usually poor, and it is expedient to combine the load with an industrial load, in order to make the best of the investment by raising the load factor. This is generally the case, but, for the above reasons, is of greater importance with hydraulic than with steam supply.

Electricity-Generating Plant.—Electric power is usually generated three-phase, and with the generating plant may be associated transformers for raising the voltage to a value suitable for transmission, switchgear for controlling the power and suitable measuring instruments. These things however present no features especially characteristic of railway work. The transmission line may be of cable or overhead wires, but, except for the fact that it usually follows the route of the railway where practicable, it also has no distinguishing characteristics.

Substations.—Of the power equipment, the substations alone possess features which, apart from the nature of the supply, distinguish them from lighting and industrial substations, but the differences even here are neither great nor fundamental. Substations are groups of apparatus by means of which electrical energy is changed from the kind and voltage suitable for generation and transmission to the kind and voltage suitable for use by the trains. They are moreover often made to serve as sectionizing stations for the transmission line system, and always form the main sectionizing, controlling and metering stations for the distribution system. The substation plant therefore divides itself naturally into three groups of apparatus: (1) the incoming line equipment, by means of which the energy is brought in, and the transmission line sectionized; (2) the converting plant, which changes the form of the energy to suit the requirements of the trains; (3) the feeder equipment, by means of which the energy from the converters is metered and passed to the distribution system. The first group extends from the transmission lines to the primary busbars, the second from the primary to the secondary busbars, and the third from the secondary busbars to the track feeders. Since the chief function of a substation is to act as an adapting link, its features are in large measure determined by the forms in which energy enters and leaves.

Characteristics of Railway Substations. As mentioned above, there is little in substation plant that can be considered peculiar to railway installations, and it is accordingly unnecessary to describe the apparatus in detail or to enter into questions relating to its design. The characteristic which most markedly distinguishes the railway substation from the

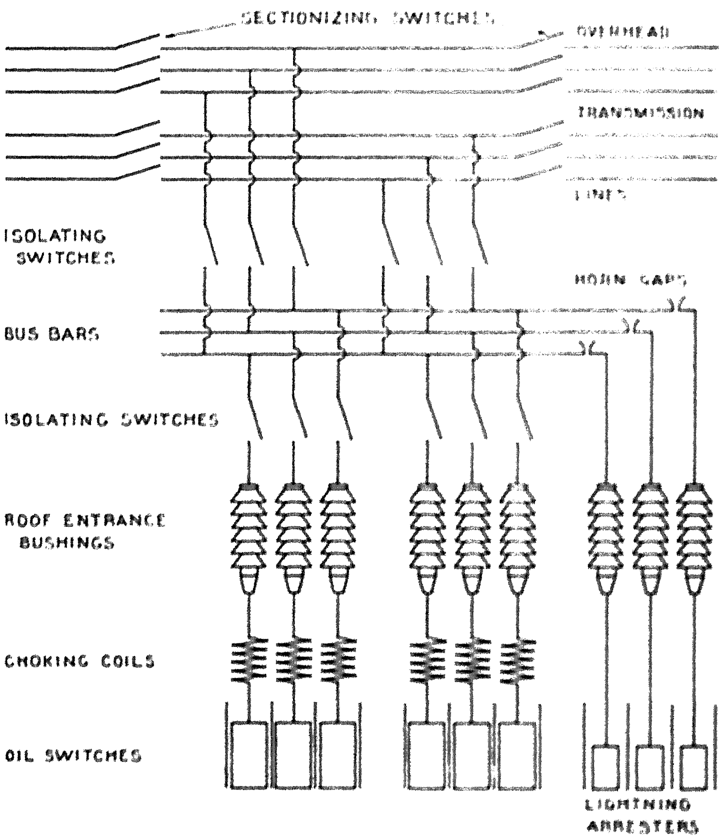


FIG. 127. General Arrangement of Incoming Lines of Substation.

lighting or industrial substation arises from the necessity it is under of withstanding sudden and heavy overloads without detriment. The momentary overload capacity of the units of plant is rarely made less than 100 per cent., even for rotating machinery supplying a dense service of trains, whilst modern good practice usually requires such machinery to stand three

times its rated load for short periods without deleterious consequences. Substation machinery is accordingly designed especially for railway work, so that the mechanical and electrical limitations on its operation are reached at much greater load than the thermal limitation.

INCOMING-LINE EQUIPMENT.—The incoming-line equipment calls for little remark. Its general nature depends on whether the transmission lines are carried overhead or underground, and its details on the voltage and circumstances. Disconnecting switches, of the air-break type, are used for isolating any transmission line or unit of apparatus from the busbars, the general arrangement being as shown in fig. 127, for two incoming-lines and two units of plant. Overhead transmission lines are usually anchored to the substation building, being brought inside through suitable entrance bushings. In American practice, when the transmission voltage exceeds about 15,000 volts, the busbars are often carried on the roof of the substation building. The plant is protected from lightning discharge by means of arresters, with horn gaps, connected to the busbars, and choking coils connected in circuit on the line side of each unit of plant. With underground transmission, the same features of isolation are observed, but the protection against lightning is no longer required.

The Converting Plant.—The converting plant is an aggregate of units each comprising in general four elements: viz. (1) switchgear for connecting the units of plant to the high tension busbars; (2) transformers for reducing the voltage; (3) converters for changing kind or frequency of electrical energy supply; (4) switchgear for connecting to the railway busbars. Some of the elements may however be absent in particular installations, depending on the nature of the conversion. On the other hand, starting gear and other auxiliaries may be required in connection with the plant.

STATIC CONVERTERS FOR ALTERNATING CURRENTS.—The simplest form of converting plant is used in the case of alternating current railways in which the frequency of alternation required by the trains is the same as that of the supply. In this case it is only necessary to reduce the pressure from that of the supply to that of the distribution system by means of static transformers. For large attended substations, oil-

immersed water-cooled transformers are most economical where the necessary cooling water is available; but for unattended or outdoor substations, oil-immersed self-cooling transformers, such as are used on the New Haven line, are more appropriate. Actually on the New Haven line auto-transformers or compensators are employed, the centre of the winding being connected to the track, and the terminals to the transmission lines, one of which however is formed by the trolley lines as shown diagrammatically in fig. 128. Any other desired ratio between the windings on the two sides of the grounded point might have been used, and the ratio of equality was chosen in

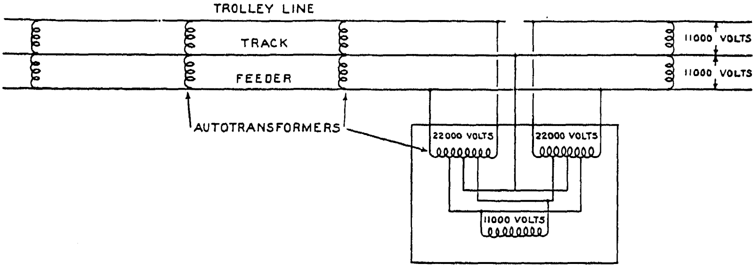


Fig. 128.—New York, New Haven and Hartford Distribution System.

order to reduce inductive interference with neighbouring communication circuits.

Rotating Converters.—Where it is necessary to change the kind or frequency of electrical energy, rotating machinery in some form has generally to be used. The mercury arc rectifier, of which more will be said later, is however a possible alternative for certain transformations. The most general apparatus for changing the nature of electrical energy is the motor-generator, a combination unit consisting of a motor, suitable for taking power from the incoming lines, directly coupled to a generator, suitable for supplying power to the distribution system. Such a unit can be constructed to meet any conditions that may be imposed by the methods of transmission and operation. Where the transmission voltage is not too high, the motor can be wound to take power from the supply, without the intervention of transformers. The limiting voltage at which this is desirable is of the order of 10,000 volts; and, since this is low as a transmission voltage, the inclusion of step-down transformers with the units is the more

usual course. With a polyphase supply the synchronous motor is generally used.

FREQUENCY CHANGERS.—For changing the frequency of an alternating supply, a unit consisting of a synchronous motor direct coupled to an alternator is employed, the number of

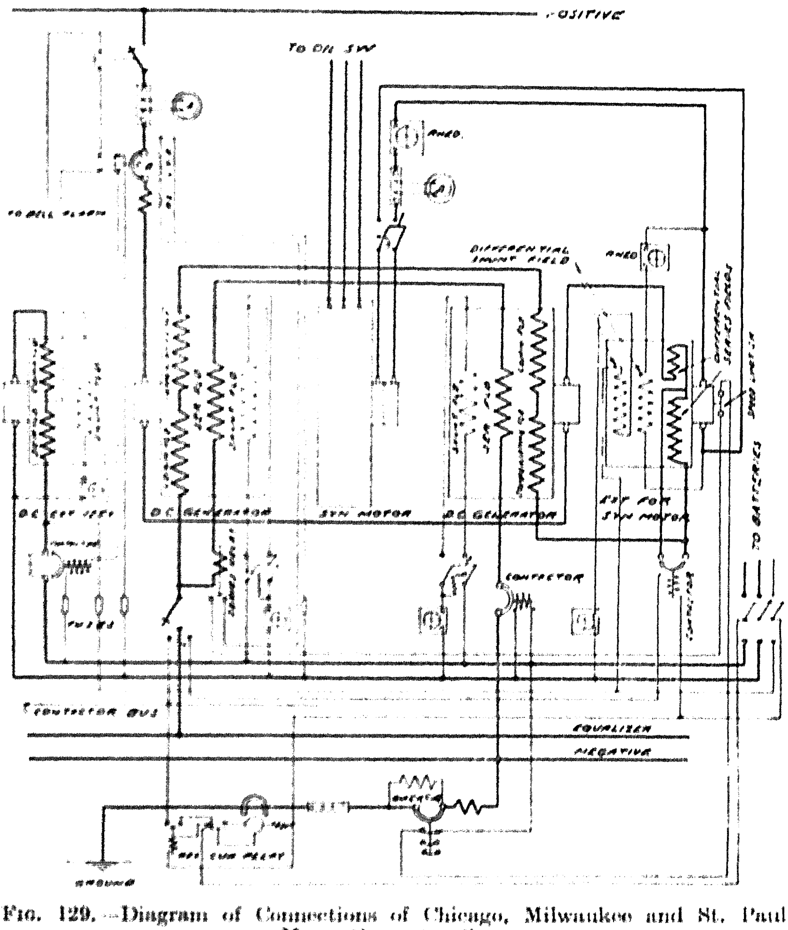


FIG. 129.—Diagram of Connections of Chicago, Milwaukee and St. Paul Motor-Generator Sets.

poles on the two machines being in proportion to their frequencies. Where several such units are run in parallel on a system, it is necessary that they should synchronize in phase on both transmission and distribution frequencies. It is the manufacturer's part to see that this is possible, and the

operator's to see that it is done. Next to the static transformer the frequency changer is perhaps the simplest and most robust of converting units.

Motor-Generators.—Motor-generators are used in the substations of the Chicago, Milwaukee and St. Paul Railway. The supply is taken at 100,000 volts, 60 cycles, three-phase, and the distribution is by means of continuous current at 3,000 volts. The motor-generator sets are of two sizes, viz. 1,500 k.w. running at 600 r.p.m. and 2,000 k.w. at 514 r.p.m. Each set consists of five machines, aligned on one shaft, and carried on a bedplate. In the centre is the synchronous motor, wound for 2,300 volts. On each side of this is a 1,500 volt, compound wound, compensated, continuous current generator, the two being connected in series. At one end of the shaft is an exciter for the field of the generators, and at the other an exciter for the field of the synchronous motor (fig. 129). The sets are designed for either direct or reversed operation, regenerative control being a feature of this installation. The 1,500 k.w. set weighs about 45 tons and covers a floor space of approximately 200 square feet. Somewhat similar units are used in the substations of the Butte, Anaconda and Pacific Railway, and of the Canadian Northern Railway (Mount Royal Tunnel).

The Rotary Converter.—Although the motor-generator may be regarded as the universal machine for interconnecting two electric systems, the required transformation may, within a limited range of conditions, often be more advantageously effected by means of machines of special type. Chief among these is the rotary converter for use in connecting a polyphase supply and continuous current distribution system. The rotary converter may be regarded as a synchronous motor-generator, in which conditions on the alternating current side have been so adapted to those on the continuous current side that it is possible to use the same armature and the same field structure for both input and output. The adaptation is made with respect to voltage; and the machine is designed to suit the frequency of supply.

The limitations of design imposed by the advisable volts per commutator bar, the safe peripheral speed of the commutator, and the minimum practicable width of bar, restrict the

voltage of a rotary converter for a given frequency. Thus with the commutator speed limited to 6,000 feet per minute, for a 50-cycle machine the distance between brush studs of opposite polarity measured along the commutator is about a foot: if the minimum width of bar and mica be taken as a fifth of an inch, there are at most 60 bars between points of opposite polarity, and if the average volts per bar be taken as not more than 13, the voltage of the machine is limited to about 780 volts. The limiting voltage practically varies inversely as the frequency of supply, and for machines of ordinary construction the limitation is approximately in accordance with the above figures.* For higher voltages, however, rotaries may be run in series. Thus for the Shildon-Newport installation, where the supply is taken at 40 cycles, and the distribution is made at 1,500 volts, continuous current, the rotaries are run two in series.

LOSSES AND THEIR DISTRIBUTION IN A ROTARY ARMATURE.—A three-phase rotary when excited to give unity power factor, has only about 57 per cent. of the armature copper loss of a continuous current generator of the same output and armature resistance, since the alternating current partly neutralizes the continuous current in the armature and thereby reduces the loss. For the six-phase rotary the corresponding figure is 27 per cent. and for the twelve-phase rotary 21 per cent. Thus a clear gain in capacity results from increasing the number of phases. The armature of a rotary is not however uniformly heated, for the loss in the coils is greater the nearer they are to the alternating current taps. In a three-phase rotary the loss in the coils adjoining the taps is about 114 per cent. greater than the mean loss; in a six-phase rotary the excess is reduced to 57 per cent.; and in a twelve-phase rotary to 18 per cent. The inequality thus decreases with increase in the number of phases, and provides an additional reason for such increase. There is some objection to a twelve-phase rotary on account of the large number of slip rings required, but it would appear that the advantages may outweigh the objections. The twelve-phase rotary can be run from a three-phase supply by providing the transformers with two secondary windings per phase, and connecting them between points

* See, however, F. P. Whitaker, *Jour. Inst. E.E.*, vol. 60.

150° apart on the armature, the chords of connection being parallel, as shown diagrammatically in fig. 130. The usual

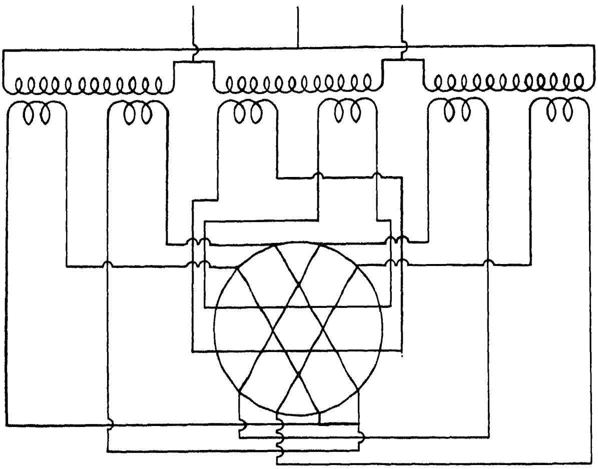


FIG. 130.—Connection Diagram of Twelve-phase Rotary.

practice where the supply is three-phase is however to use six-phase rotaries. If the supply is quarter-phase, and quarter-phase rotaries are used, the mean armature copper loss is 38 per cent. of the continuous current loss and the maximum coil loss 92 per cent. in excess of the mean. With eight-phase rotaries which can be run from a quarter-phase supply by the use of suitable transformer windings, the mean loss is approximately 24 per cent. of the continuous current loss, and the maximum 36 per cent. in excess of the mean.

VOLTAGE AND CURRENT.—The voltage between the brushes of a rotary converter is principally governed by the alternating voltage supplied at the slip rings. The brushes are so placed as to obtain the maximum voltage generated by the armature, and when two tappings from the windings to the slip-rings can coincide simultaneously with the positive and negative brushes, the maximum alternating voltage between these is equal to the continuous voltage. Thus the virtual alternating voltage is in this case about 70·7 per cent. of the continuous voltage. The ratio however varies slightly with wave form. The distribution of voltage round the armature may be represented by a circle, the difference of potential between any two points varying as the length of the chord joining them. Thus

in the three-phase rotary, fig. 131, the ratio of alternating to continuous voltages is cos $30°/\sqrt{2}$, or 61·2 per cent.; in the twelve-phase rotary of fig. 130, the ratio is cos $15°/\sqrt{2}$, or 68·3 per cent. The ratio between alternating and continuous currents may be obtained approximately by equating input and output, and more accurately by making allowance for efficiency.

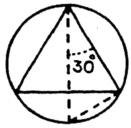


FIG. 131.—Diagram. Three-phase Rotary.

COMPOUNDING ROTARIES.—Since the continuous voltage is governed by the alternating voltage some form of regulation is necessary if it is desired to compensate for voltage drop in transmission and distribution lines. This is effected in the case of traction rotaries, by a series field and reactance coil. The mere increase in field strength does not affect the voltage appreciably but, as in all synchronous machines, tends to make the current lead the e.m.f. In order to take advantage of this for compounding purposes, inductive impedance is put in series with the rotary on the alternating current side. Thus, in fig. 132, if OC represents the phase of the current, leading its voltage OV, and OR is the reactance voltage, leading the current by a right angle, then the line voltage is represented by OV¹, which is clearly smaller than OV. The effect of the inductance accordingly is to raise the voltage at the slip rings when the current leads the voltage, and by the same rule to reduce it when the current lags. The necessary inductance may be provided by a separate piece of apparatus, but it is now common practice to provide for it by means of magnetic leakage in the transformers. The latter is the cheaper, but it is open to the objection that, unless great care is exercised in the design of the transformers, the leakage flux is apt to cause parasitic losses in the heavy secondary windings. Where there is more than one rotary in a substation the series fields are connected in parallel by means of an equalizing bus, in order that the machines may tend to divide the load equally.

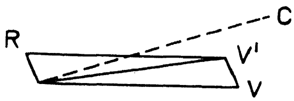


FIG. 132.—Vector Diagram of Compound-wound Rotary; Current Leading.

Starting-up the Rotary.—Rotary converters may be started up from the continuous current side, being then

synchronized in on the alternating side. This is not a usual practice in railway service. They may be started from the alternating side as induction motors, starting torque being provided by current, generated by induction, in the damping bars. The alternating voltage is applied to the slip rings from special low voltage taps on the transformer, the field circuit of the rotary being for the time open. The machine then runs into synchronism without excitation. This is standard American practice, and although it avoids the process of

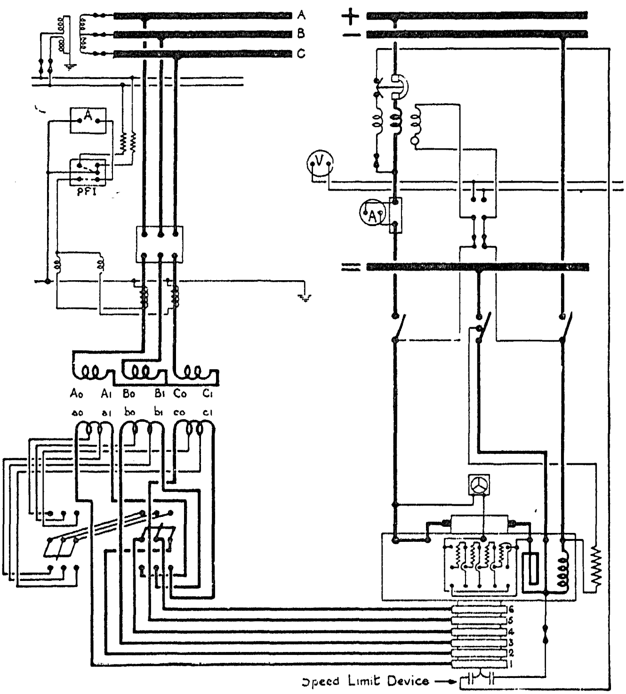


FIG. 133.—Connection Diagram for Starting by means of Taps on Transformer.

synchronizing, it is open to the objections that it draws a large wattless current at starting, that it becomes necessary to raise the brushes from the commutator during the starting, and that synchronism is often reached with wrong polarity, necessitating the jumping of a pole, or reversal of the field. Fig. 133 is a connection diagram for a six-phase rotary started in this manner. British practice favours the provision of a separate starting motor on the end of the shaft of each rotary. The

starting motor is of the induction type, having a high resistance squirrel-cage rotor (sometimes, in fact, consisting of a cylinder of solid iron), and wound with a pair fewer poles than the rotary, in order to take it through synchronous speed. Fig. 134 is a connection diagram for a six-phase rotary started by means of an induction motor, and subsequently synchronized. By a recent improvement, the winding of the induction motor

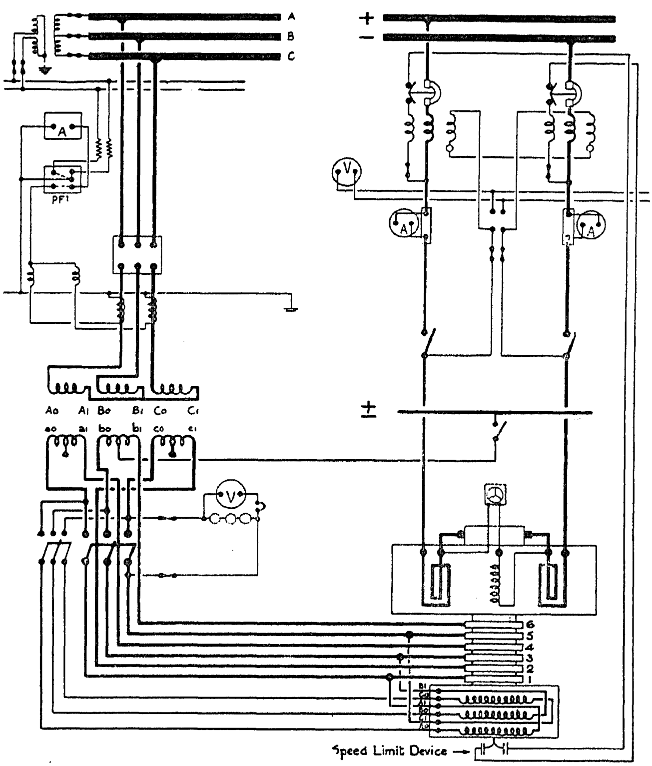


FIG. 134.—Connection Diagram for Starting by means of Induction Motor (Separate Synchronizing).

is connected in series with the alternating current winding of the rotary (see fig. 135). With this arrangement the rotary rises in speed to synchronism and no higher, the voltage on the induction motor winding disappearing largely when this speed has been attained. The induction motor winding is then short-circuited, and the rotary is ready to supply current; for the polarity is governed by the remanent magnetism, and

is accordingly not reversed in starting. By this means a rotary can be put into operation in very short time, machines of large capacity being ready for service within a minute, without taking more than a half of full load current during the starting period.

SOURCES OF TROUBLE WITH ROTARY CONVERTERS.—The rotary converter at its best is one of the most satisfactory

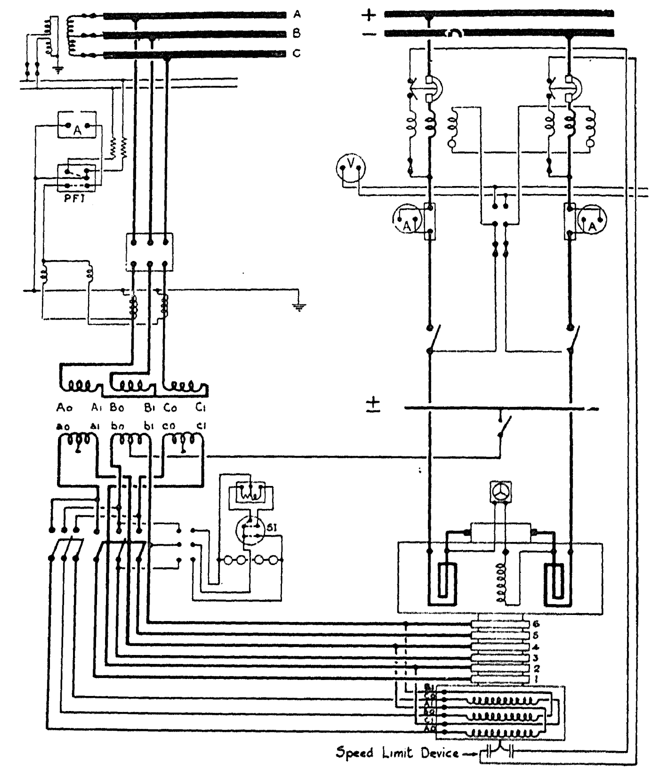


FIG. 135.—Connection Diagram for Starting by means of Induction Motor (Self Synchronizing).

pieces of rotating apparatus that have been produced, occupying little space, taking heavy and sudden overloads without harm, requiring little attention, giving little trouble, having high efficiency, and compared with satisfactory substitutes, being inexpensive withal. In common with all synchronous machinery, the rotary converter may under certain conditions develop a tendency to hunt or oscillate about the state of steady

running. The frequency of the oscillation is a natural constant of the machine, and hunting may be the result of defective design, requiring no external stimulus to produce it. However, the conditions which favour the defect are well understood, and substantial dampening bridges of copper are always provided, imbedded in and surrounding the pole faces. The trouble is therefore non-existent in well-designed modern machines. Another possible source of trouble, with which the designer of rotaries has to contend, is that of flashing at the commutator. In the rotary converter, the armature reaction due to the continuous current is, to a large extent, neutralized by that due to the alternating current, the residual being a pulsating reaction, the mean value of which is in modern machines usually annulled by means of commutating poles. The amplitude of the pulsation is the smaller the greater the number of phases. The commutation is therefore usually satisfactory, but the other factors tending to cause flashing are often more difficult to deal with, the commutator speed and the volts per bar being in many cases higher than the most conservative practice would approve. The conditions are worse the higher the frequency and voltage of the machine; but the limitations of design are well understood and, within the limits, the flashing trouble is not of serious importance. In difficult cases, the parts between which an arc might be formed, are protected by insulating screens, and barriers of insulating material are sometimes employed to remove the layers of ionized air from the commutator. Blow-out devices, magnetic or air, are sometimes used to suppress any arc that may be formed. As most cases of arcing over the commutating machines result from external short circuit, high-speed circuit breakers are now sometimes used, which throw a current-limiting resistance into the circuit before the current has had time to attain a dangerously high value.

THE TRANSFORMERS.—The transformers which link the rotary with the transmission system may consist of two or three single-phase units, according to whether the supply is two-phase or three-phase; but it is now common practice to employ a polyphase transformer rather than an assemblage of single-phase transformers. The use of single-phase transformers has the advantage, particularly valuable in railway work, that a defective transformer can be cut out, and the unit

still operated at reduced capacity. Where water is available at the substation, the transformers may be of the oil-immersed water-cooled type: otherwise they may be cooled by a forced draught of air, or by radiation of heat from the outer case, being, in the latter case, of the oil-immersed type, with tanks specially designed to give a large radiation of heat. The last is the most common practice. Where rotaries are run in series, a single group of transformers, having multiple secondary windings, may be used with each unit group of rotaries. The transformers are usually provided with a few taps on the primary winding, by means of which a certain adjustment of secondary voltage can be made; thus permitting compensation to be made for unequal transmission line drop and more effectively paralleling the substations.

The Motor-Converter.—The motor-converter of La Cour may be looked upon as a combination of a motor-generator

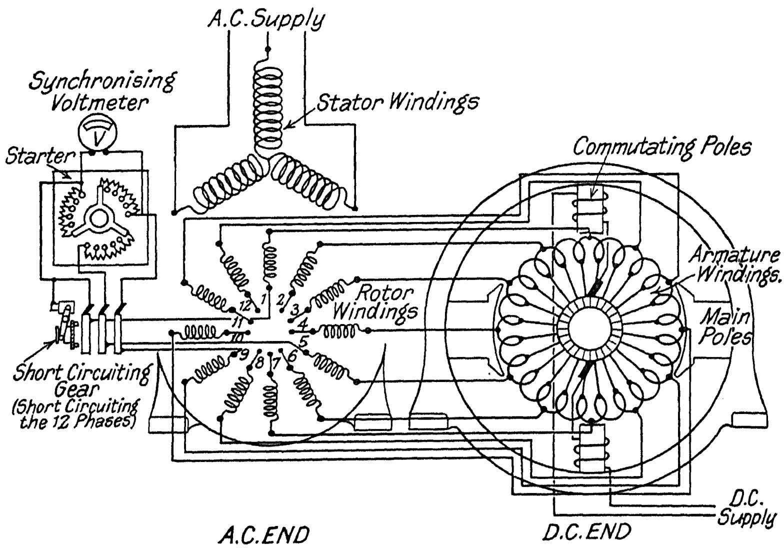


FIG. 136.—La Cour Converter.

with a frequency changer and rotary converter. It consists of an induction motor having a wound rotor, mounted on the same shaft with a continuous current machine, which operates both as a rotary and a generator. The latter machine is fed as a rotary from the rotor of the induction motor, which is

designed to give a suitable voltage for the purpose, and is also driven directly as a generator. Fig. 136 shows diagrammatically the connections of the motor-converter.

The power input of the motor is in part transmitted mechanically to the continuous current machine, operating as a generator, and in part given out electrically to this machine operating as a rotary converter of appropriately reduced frequency, viz. that of the slip of the induction motor. If f is the supply frequency, $2n$ the number of poles in the motor, and $2n'$ the number in the generator, the speed of rotation of

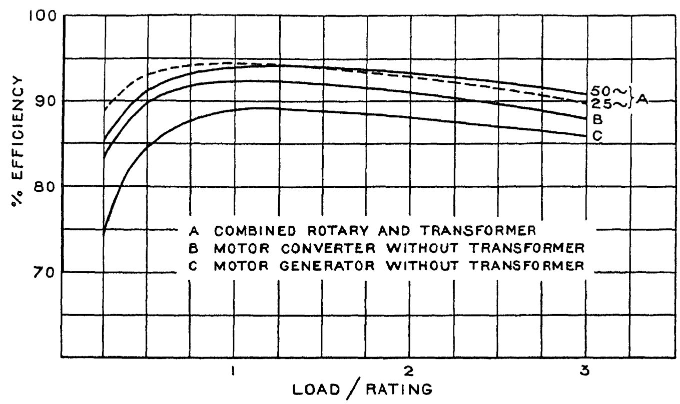


FIG. 137.—Comparative Efficiencies of Rotary Converter, Motor-Converter and Motor-Generator Units.

the unit is $f/(n+n')$ and the frequency of the continuous current machine $n'f/(n+n')$: thus the frequency of this machine is reduced by increase of n/n'.

The motor-converter is capable of much the same work as the rotary converter, and, where the transmission voltage is sufficiently low, avoids the interposition of transformers. Even in this case, however, the complete unit is more costly than the rotary converter unit, whilst the efficiency of conversion is lower by some 2 per cent. In fact, the true sphere of the motor-converter would appear to be as a substitute for the motor-generator in cases where the conditions of frequency and continuous voltage are too high to permit of a satisfactory design of rotary. Fig. 137 gives average curves comparing efficiencies of rotary, motor-converter and motor-generator, as functions of the load.

The Mercury Vapour Rectifier.—A different form of converter entirely, and one which gives promise of great possibilities, is the mercury vapour rectifier. This, in the form of a somewhat delicate apparatus constructed largely of glass, has been employed for some time for the purpose of obtaining continuous current in small quantities, from an alternating current supply. It has however now been developed in form and capacity suitable for the requirements of railway supply, and although it has hardly yet attained the condition of reliability needed in such work, its development has disclosed no insuperable defect. It should therefore be watched, as a development which may prove to have considerable influence on the future of railway electrification.

The mercury vapour rectifier depends for its action on the property of an arc between certain dissimilar electrodes in opposing greater resistance to the passage of current in one direction than in the other. The arc is formed between mercury and iron electrodes in an attenuated atmosphere of mercury vapour, and whilst an electromotive force of from 15 to 30 volts is sufficient to cause a current to flow from iron to mercury, an exceedingly high electromotive force, depending on the length of the arc, is required to overcome the resistance when applied from mercury to iron. When an alternating voltage is applied between the electrodes, the rectifier allows a current to pass only when the iron and mercury electrodes stand in the relation of anode and cathode respectively. The application of such a voltage accordingly results in a number of unidirectional pulses of current at the supply frequency, separated by intervals of zero current. In order to obtain a more continuous output, several anodes are provided discharging to a single cathode and connected to different phases of the supply. Fig. 138 shows the arrangement diagrammatically in the case of a single-phase supply. Here R is the rectifier, having anodes A A and cathode C : T the transformer which supplies it with power. The centre point of the secondary winding of T forms the negative terminal for the unidirectional current, the positive terminal being the cathode. The output voltage of

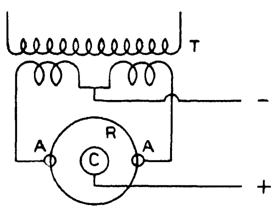


FIG. 138.—Single-phase Mercury Vapour Rectifier.

the arrangement is determined by the mean voltage of the half-secondary winding taken over the half wave. Since the current flows in each half winding for only half the time, the total secondary copper loss is the product of the resistance of a half winding into the mean square of the current; and is therefore the same as that in an ordinary alternating current transformer of equal output, but having half the secondary copper. The electromotive force of a single-phase rectifier is pulsating, its value dropping to zero every half period. In the case of a polyphase rectifier as illustrated diagrammatically in fig. 139, the load at any instant is on the phase whose voltage

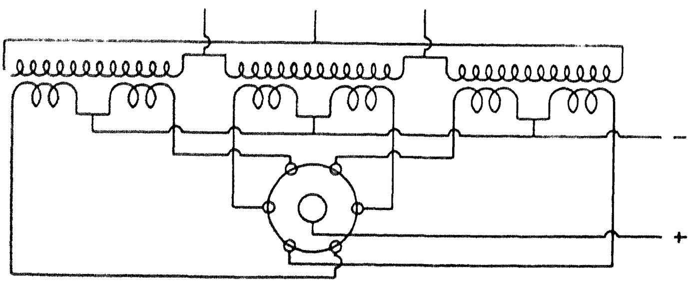


FIG. 139. Six-phase Mercury Vapour Rectifier.

exceeds that of the others at the time. The unidirectional voltage is therefore nearly constant, following the peaks of the separate waves as shown, for a six-phase rectifier, in fig. 140, where the mean value of the voltage is approximately 1·35 of the alternating voltage per phase or 95 per cent. of that at the peak of the wave.

ELECTRICAL AUXILIARIES.—The action of the mercury vapour rectifier is however greatly modified by the use of certain auxiliary apparatus, shown diagrammatically in fig. 141. Here the six secondary windings of the transformers are divided into two three-phase star-connected groups; and the neutral points of these are connected through opposing windings of a reactance C. The effect of this is to cause the continuous current to be taken in equal amounts from the two three-phase windings, thus maintaining two arcs simultaneously at all times. For one-sixth of the time, arcs 1 and 2 are in action, for one-sixth arcs 2 and 3, and so on. Each of the six transformer coils accordingly, and each anode, carries current for one-third of the total time. This is a fundamental modification

which not only uses the transformer and rectifier to greater advantage, but also results in a great improvement in the

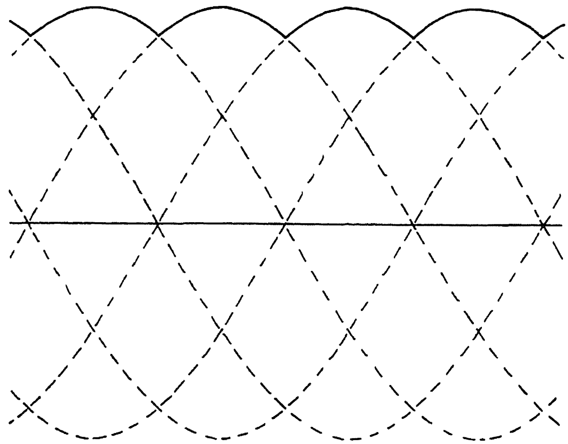


FIG. 140.—Voltage Fluctuation, Six-phase Rectifier.

power factor of the unit. For a simple three-phase rectifier the power factor, assuming a sine wave of voltage, is $3/\pi\sqrt{2}$, or 67·5 per cent. For a simple six-phase rectifier, as shown in fig. 139, the figure becomes $\sqrt{6}/\pi$, or 78 per cent. For a six-

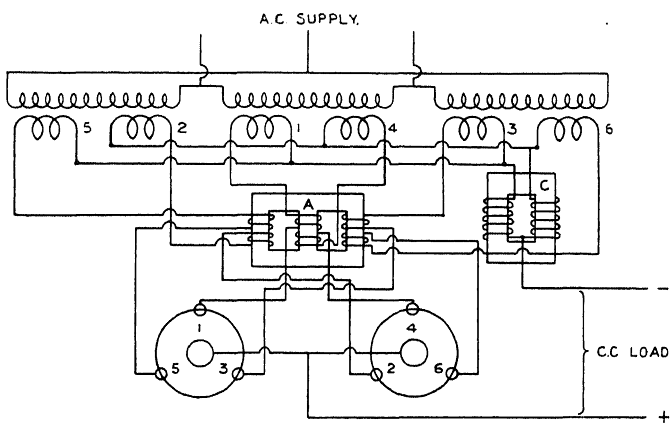


FIG. 141.—Six-phase Mercury Vapour Rectifier with Reactors.

phase rectifier with reactance, as shown in fig. 141, the theoretical power factor is $3/\pi$, or 95·5 per cent. The rectifier is shown in fig. 141 as consisting of two three-phase units; but this,

although a practicable arrangement, is not necessary, and the six-phase rectifier, shown in fig. 139, might equally have been shown here. A second reactance, A, having a coil in the circuit of each anode, is shown in fig. 141. This is highly saturated and assists in steadying the arcs and transferring them from anode to anode. The general form of the current and voltage curves is shown in fig. 142.

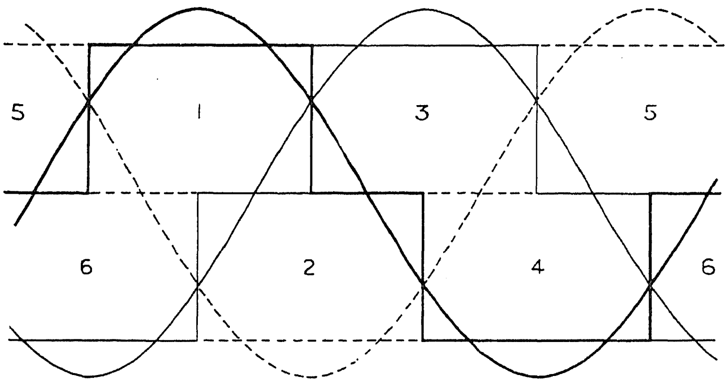


FIG. 142.—Current and Voltage Curves of Mercury Vapour Rectifier.

CONSTRUCTION AND OPERATION.—The mercury vapour rectifier, as used in railway substations, consists of a steel chamber into which the electrodes are introduced through gas-tight insulating bushings. The cathode is located in the centre of the base, and the anodes are inserted through short side tubes which join the main chamber near the base. The upper part of the chamber forms a condenser for the mercury vapour evaporated from the cathode, and is the means of transferring the heat developed to outside space. In small rectifiers the condenser dissipates its heat by radiation, but in larger sizes arrangements are generally made for water cooling. It is necessary however that the anodes do not form condensing surfaces for mercury globules; otherwise reverse arcing may take place and the rectifying effect be destroyed. In order to prevent this, the anodes are sometimes made hollow, and the cavities filled with mercury, which is maintained at the boiling point by means of independent heating coils immersed in the mercury, the vapour formed being condensed in chambers connected with the cavities.

In this manner the temperature of the anodes is maintained above that at which the rarefied mercury vapour of the rectifier can condense. If, as is usual railway practice, the negative side of the circuit is earthed, the whole rectifier is at the positive voltage (see figs. 138, 139 and 141); it must accordingly be properly isolated and insulated. The rectifier is started by the aid of auxiliary anodes which are brought into operation by external means. An efficient vacuum pump is an indispensable accessory to a rectifier installation, for it is of the greatest importance to maintain a high vacuum.

THE TRANSFORMER.—The transformer, as used with the mercury-arc rectifiers, is special in design. In fig. 139 each secondary winding carries the output current for one-sixth of the total time. In fig. 141 each carries a half of the output current for one-third of the total time. Comparing the transformer with an ordinary six-phase alternating current transformer, having the same output and secondary copper loss, let r be the resistance per phase of the secondary of the latter and R that of the rectifier transformer. Then, in the case represented in fig. 141, c and C being the secondary currents:

$$6\ c = 2 \times 1{\cdot}35\mathrm{C} = 2{\cdot}7\mathrm{C}$$
$$6\ rc^2 = 2\mathrm{RC}^2$$

Hence :—

$$\frac{1}{6}r = \frac{2}{2{\cdot}7^2}\,\mathrm{R}$$

or

$$r = 1{\cdot}64\,\mathrm{R}.$$

Thus more than 60 per cent. more copper is required in the secondary windings of the rectifier transformer; but the amount would have been doubled if the arrangement shown in fig. 139 had been adopted.

EFFICIENCY.—The mercury vapour rectifier has good efficiency, even at light loads, for the voltage drop in the apparatus does not vary greatly with load. The transformers however are less efficient than those used in ordinary alternating current supply, and the auxiliary apparatus further reduces the efficiency. The efficiency of the complete unit is comparable with that of a rotary unit, being generally slightly inferior at average loads but somewhat superior at light loads. The overload capacity of the rectifier as ordin-

arily rated is not equal to that of a modern rotary, and the installed capacity is accordingly greater. The cost of the complete installation is not usually advantageous compared with that of a rotary substation designed for the same service.

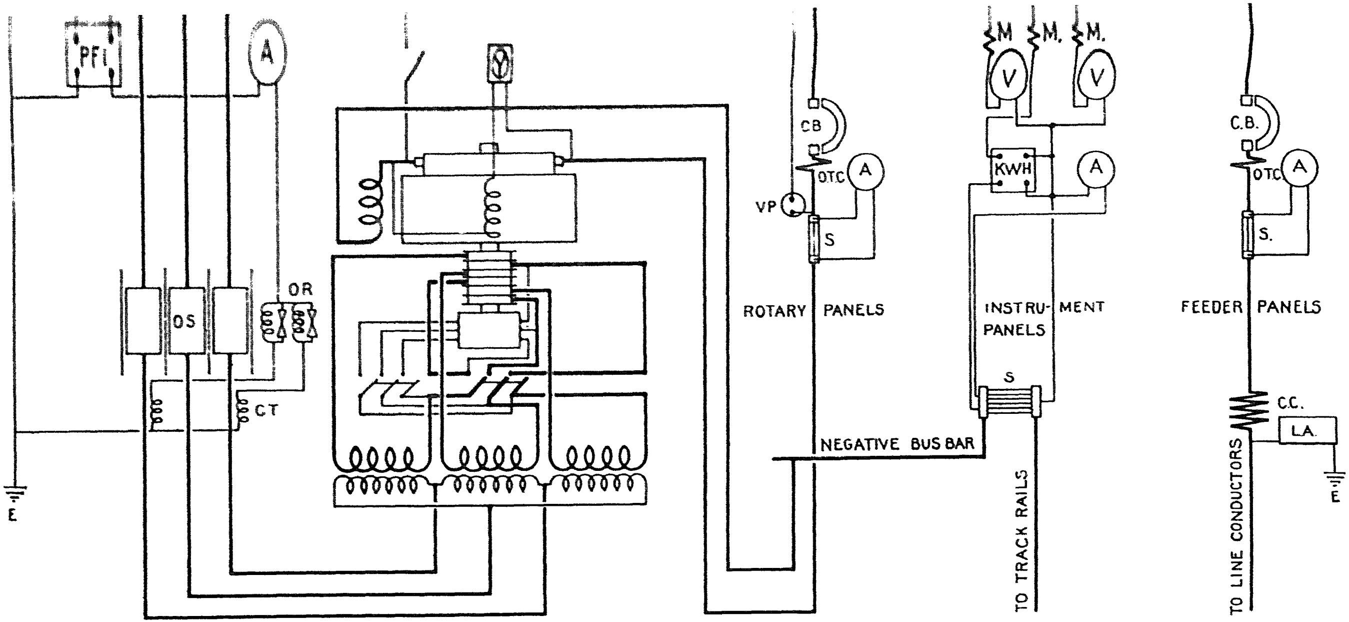
The mercury vapour rectifier does not impose a limit on the frequency of supply, nor on the voltage of the output. It accordingly appears a fitting development to meet the needs of high voltage continuous current railways taking power from industrial supply networks, although further experience is necessary before it could be recommended without reserve for such work. The simplicity of operation of the plant, and its good efficiency at all loads, gives promise of the development of substations requiring no constant supervision. It is quite within the bounds of possibility that the future may disclose the continuous current system of operation, with unattended rectifier substations, as the normal preferred practice in railway working. It must be said however that this development is not yet in sight, for the rectifier requires more skilled attention than the rotary. A result to be expected from the sudden changes in line voltage, produced when the arc shifts from anode to anode, is that this form of converting plant will cause much greater interference with neighbouring communication circuits than the usual forms with rotating machinery.

Storage Batteries in Substations.—The poor load factor with which the railway substation usually operates, and which is the justification and reason for using machinery of great overload capacity, has led some engineers to instal storage batteries in connection with the substations. These float on the line, assisting the machinery at times of heavy load and absorbing charge from it at times of light load. The load on the rotating plant is thus equalized, and smaller ultimate capacity is required in it. The battery however is an expensive item, and its upkeep costly. Most operators therefore prefer to invest in overload capacity, and avoid the use of the battery altogether.

Distribution Switchgear.—With each unit of converting plant is provided a switchboard panel containing circuit

breakers designed to open on excessive overload, and switches necessary to connect the set to the busbars, together with instruments for measuring its output. Between the machine panels and distribution panels is usually located a totalizing panel at which the whole energy output of the substation is metered. The distribution or feeder panels contain the circuit breakers, switches and instruments required in feeding the line and isolating any section that may be desired. The line conductor of each track is connected to the busbars through a separate panel, and each direction is treated as an independent track for this purpose, section insulators being inserted in each line conductor at the substation. Thus, with a two-track road, there will be at least four feeder panels, unless the substation is situated at a terminus. Sometimes, where there is possibility of current collectors on the same or different coaches bridging the insulator gap and putting voltage on a line on which men may be working, a sufficient length of line conductor on each track is isolated near the substation and fed from a separate panel. The distribution lines, being grounded on one side, the switchgear on the distribution side of the substation is of the single pole type. Fig. 143 gives a typical diagram of substation wiring, in which some of these features are shown.

Unattended Substations. Until recently it has been regarded as a necessity that valuable and powerful rotating machinery should be constantly under supervision. The machinery, controlling, and protective apparatus used in railway substations however have reached such a state of reliability that serious mishap is a remote contingency, and the work devolving on the attendants is usually light, and almost entirely of a routine character. It is true that in substations feeding important and busy lines there is plenty of work to keep the attendants occupied, but in the case of substations feeding the remoter sections of inter-urban railways, the attendant's duties are tedious and, from an economic point of view, hardly worth while. The pressure of circumstances is accordingly tending to produce a type of substation in which continual supervision is unnecessary, and particularly is this the case in America where the shortage of suitable labour is very keenly felt.



A = AMMETER
CB = CIRCUIT BREAKER
CC = CHOKING COIL

CT = CURRENT TRANSFORMER
E = EARTH CONNECTION
F = FUSE
KWH = ENERGY METER

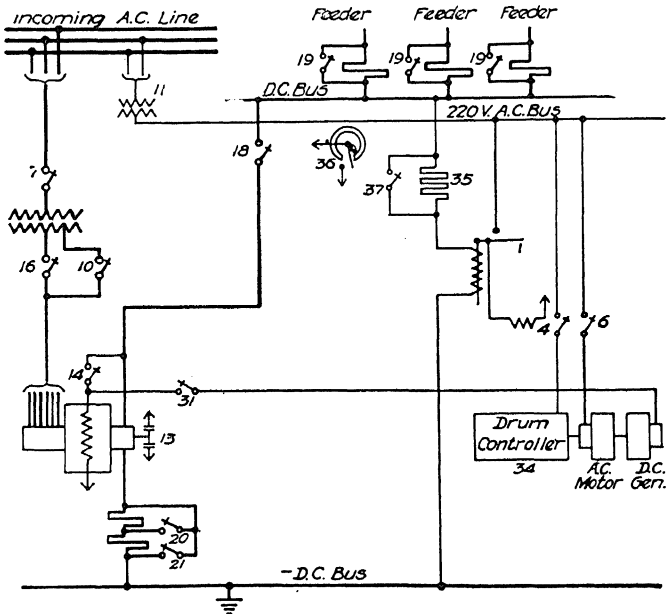
LA = LIGHTNING ARRESTER
M = MULTIPLIER
OR = OVERLOAD RELAY
OS = OIL SWITCH

OTC = OVERLOAD TRIP COIL
PFI = POWER FACTOR INDICATOR
PT = POTENTIAL TRANSFORMER

S = SHUNT
V = VOLTMETER
VP = VOLTMETER PLUG

FIG. 143.—Substation Wiring.

The unattended substation may take one of several forms. It may be put into service, and taken off, by a travelling attendant, who visits it in due season for the purpose, being then provided with automatic devices for cutting off power in



	SEQUENCE	DEVICE NUMBER																	REMARKS
		1	4	6	7	10	13	14	16	18	19 A	19 B	19 C	20	21	31	36	37	
1	Shut Down										●	●	●					●	Controller "Off"
2	Low D.C. Voltage	●	●	●							●	●	●					●	Controller starts
3	1st Controller Position	●	●	●	●						●	●	●					●	Oil Switch closes
4	2nd " "	●	●	●	●	●					●	●	●					●	Starting Tap
5	3rd " "	●	●	●	●	●	●				●	●	●					●	Synchronous Speed
6	4th " "	●	●	●	●	●	●				●	●	●			●	●	●	Polarity Fixed
7	5th " "	●	●	●	●		●	●	●		●	●	●				●	●	Full volts - Self Excited
8	6th " "	●	●	●	●		●	●	●	●	●	●	●				●		All Resistance in
9	7th " "	●	●	●	●		●	●	●	●	●	●	●	●			●		Part Resistance out
10	8th " "	●	●	●	●		●	●	●	●	●	●	●	●	●		●		All Resistance out
11	9th " "	●	●		●		●	●	●	●	●	●	●	●	●		●		Controller Stops "Run"
12	Light overload Feeder "A"	●	●		●		●	●	●	●		●	●	●	●		●		19 B or 19 C open on
13	Medium overload Feeder "A"	●	●		●		●	●	●	●		●	●	●			●		Feeders "B" or "C"
14	Heavy overload Feeder "A"	●	●		●		●	●	●	●		●	●				●		Correspondingly
15	Underload			●	●		●				●	●	●				●	●	Controller Starts
16	Shut Down										●	●	●					●	" Stops "Off"

Fig. 144.—Diagram of Automatic Substation Wiring.

case of accident. It may be started and stopped at certain times by means of clockwork. It may be controlled electrically from a convenient distant point. Finally it may come into service automatically in response to a demand for power, and fall out automatically as the demand abates.

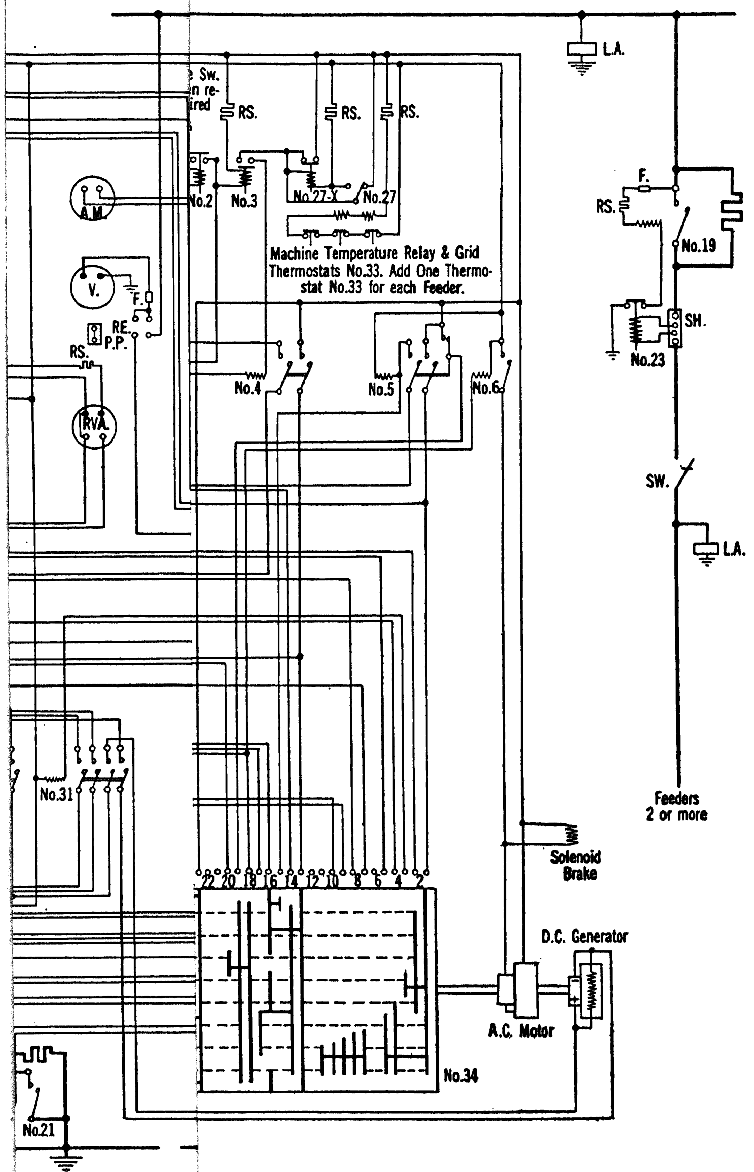
Fig. 145 gives a typical wiring diagram, and list of the control apparatus, for a substation of the last kind, in which the power plant consists of a rotary converter unit.* Fig. 144 is a simplified wiring diagram of the substation, with a table showing the sequence of the principal operations. The devices are similarly numbered in the two figures. The power apparatus possesses no special features, and it is only in the controlling gear and protective devices that the substation differs from an ordinary attended one. The sequence of operations necessary to bring the substation into action, and the conditions that must be satisfied by the machinery before the successive operations are permitted to occur, can be traced in detail in fig. 145. It should be noted that the 220 volt A. c. bus line from which the controlling devices are operated, is continually excited from the incoming line, through a small transformer provided for the purpose.

SEQUENCE OF OPERATIONS FOR STARTING THE MACHINERY.—The device which is ultimately responsible for starting the machinery is, in this case, the contact-making voltmeter (No. 1), which is connected permanently between the positive line-conductor and ground. As a train approaches the substation, and recedes from the adjacent substation, the drop in the line conductor increases, until a certain prescribed minimum value of the voltage is reached, at which the voltmeter (No. 1) opens the operating circuit of the relay (No. 2), which had previously to this been energized from the control busbars. This relay in dropping closes the circuit of relay (No. 3), causing it to pick up and energize the contactor (No. 4), provided the hand reset switch and the contacts of the low alternating voltage relays (No. 27) are closed. Relay (No. 2) is provided with a dashpot intended to prevent the starting of the machine in consequence of short spells of low voltage. The sequence of subsequent events is governed by the motor-driven drum controller (No. 34), which in the " off " position, provided the brushes are raised from the commutator of the rotary, completes a circuit from the control bus, through contactor (No. 4), the fingers 13 and 16 of the drum-controller, and the operating coil of contactor (No. 6). The closing of (No. 6) completes the circuit which supplies power to the motor of the drum-controller and starts this rotating. After

* *Trans. A.I.E.E.*, vol. 39, p. 659.

a brief interval segment 15 makes contact with its finger, causing contactor (No. 5) to close, which in turn energizes the operating mechanism of the oil circuit breaker (No. 7), which connects the main transformers to the line. The operating coil connection of contactor (No. 5) is then transferred from segment 15 to segment 14, its circuit passing through auxiliary contacts on the oil circuit breaker, which ensure the return of all devices to their normal zero positions should the breaker open for any reason. Segment 2 then makes contact, causing the contactor (No. 10), which connects the rotary to the low voltage taps of the transformer, to close, provided the three-phase supply is available, as determined by relay (No. 32). The rotary then starts up, and meanwhile the drum controller pauses at the gap in segment 16 until it has attained approximately synchronous speed. When the rotary has reached this speed, the speed control switch (No. 13) closes, bridging the gap by the aid of segment 20 and causing the drum controller to restart and continue the process of connecting the rotary to the line.

TAKING LOAD.—Segment 3 now makes contact, causing contactor (No. 31) to close and connect the rotary fields to a 250-volt supply obtained from a small generator on the drum controller motor shaft. This is to ensure correct polarity. Contactor (No. 31) is then opened, and contactor (No. 14), which connects the rotary field between the brushes, is shortly afterwards closed by the agency of segment 4. Very soon segment 5 causes the running contactor (No. 16) to close, contactor (No. 10) being simultaneously opened. The starting and running contactors (Nos. 10 and 16) are interlocked both mechanically and electrically, in order to prevent the accidental short-circuiting of a portion of the transformer winding. The rotary being now connected with the supply on the alternating current side, segment 26 starts the motor-operated brush gear, causing the d.c. brushes to be lowered on to the commutator, and completing the operation of preparing the machine for connections to the busbars. Segment 7 is now connected to the positive terminal of the rotary and almost immediately afterwards segment 8 comes in contact, completing the operating circuit of the d.c. line contactor (No. 18), through field relay (No. 30), polarized relay (No. 36), and auxiliary switches on running contactor (No. 16) and control contactor (No. 4).



Fi

ıg relay. (3) Time de|rs. (23) Feeder load limiting relays. (24) Converter load limit-
ntactor for making co|(27) A-c. low-voltage relay. (27x) Auxiliary relay for No. 27.
e-phase oil circuit-brea|nt relay. (30) Converter field relay. (31) Converter field con-
mer, 220 volts. (12) S|otective relays. (33) Thermostat relays. (34) Motor operated
field contactor shunt |irect-current underload relay.
d limiting contactors.

These provisions ensure that before the line contactor is closed, the machine has correct polarity, excited field, and full alternating voltage connections. As soon as the line contactor closes, the machine delivers current through load limiting rheostats, which however are almost immediately short-circuited by means of contactors (Nos. 20 and 21) operated by segments 9 and 10 of the controller. The drum is then stopped by the break in segment 17.

Current Relay.—When the line contactor closes, the flow of current to the line causes relay (No. 37) to close, and as long as this is closed, relay (No. 3) remains closed, regardless of the contact-making voltmeter (No. 1), whose action started the operations. Thus the control of the station is now dependent on the contacts of (No. 37), which remains closed as long as the current supplied by the rotary exceeds a prescribed value. Should the current fall below this value, relay (No. 37) opens and causes relay (No. 3) to open, after a certain interval of time determined by a dashpot, and the station then shuts down. The dashpot is provided in order that momentary or brief low values of the current may not shut down the station, although causing relay (No. 37) to open.

Shutting Down.—When the station shuts down, relay (No. 3) opens, causing contactor (No. 4) to drop out, and in doing so to open the operating circuit of line contactor (No. 18) and control contactor (No. 5). The opening of the last causes running contactor (No. 16) to open, and at the same time establishes a circuit which closes contactor (No. 6), thereby starting the controller (No. 34) and running it to its "off" position. During this operation segment 24 trips out the oil circuit breaker, and segment 25 causes the commutator brushes to be raised, thus leaving the apparatus in readiness for the next demand for load.

Protective Devices.—The preceding description deals briefly with the operation of the apparatus under normal conditions, but there remains the equally important functions of protecting the machinery if something untoward occurs whether on the railway, in the supply, or in the apparatus itself. Some of the protective devices have already been referred to in the foregoing description. In the event of a heavy overload relay (No. 24) opens, causing contactor (No. 20) to open and insert resistance in the machine circuit. A further

increase of load opens relay (No. 25) and inserts still more resistance. Individual feeder loads are limited in like manner by means of an overload relay (No. 23) and contactor (No. 19), which short-circuits a resistance. A continued overload raises temperature relay (No. 33) and shuts down the station. The heating of the machine bearings in like manner opens relay (No. 38), causing the main circuit breaker to open and shut down the station. Relay (No. 29) provides against reversal of the continuous current, and speed limit switch (No. 12A) against over-speed in the rotary. Should the line-contactor (No. 18) freeze in, the machine would motor from the d-c. end when the A-c. supply is cut off, and the excessive speed resulting trips the hand-reset circuit breaker (No. 15) by the operation of speed switch (No. 12). In case of a short circuit on the A-c. side of the equipment, the definite time-limit overload-relay (No. 28) trips out the main oil circuit breaker, shutting down the station, and at the same time opening the hand-reset switch. In either case the visit of an inspector to investigate the cause of the trouble is necessary before the station can be returned to operation.

Where there are two units in the substation, the first is brought into operation by low voltage and the second by continued high current. However, the elimination of the attendant removes the chief incentive to the use of a large aggregate of machinery in the substation, and it becomes practicable with little extra expense to dispose the plant in single units, at short intervals along the route. In this way the track drop and line drop are reduced, and the line conductors may be made smaller. Indeed, the automatic substation introduces a new feature into railway operation. If full use is made of its advantages, the whole layout is affected. It is not merely a question of adding automatic devices to existing substations, or of locating automatic substations where attended substations might have been, but of studying the whole matter anew, having regard to the fact that it is practicable to distribute the feeding points with much greater freedom than heretofore. However, the time has not yet arrived when the attended substation can be entirely dispensed with, but the unattended station nevertheless appears a promising development, particularly for lines on which the traffic is infrequent.

Portable Substations.—On certain sections of line, the traffic is normally light : but on a few days in the year, as on the occasion of race meetings, it is exceedingly heavy. To instal substation machinery sufficient to deal with the heaviest traffic would clearly be uneconomical under these circumstances. The case can however often be met satisfactorily by using a portable substation, consisting of one or more box-cars containing the necessary machinery, and capable of being moved on the rails to any place where an undue load is expected. It is then only necessary to make permanent provision of transmission lines and suitable sidings, and these will in many cases be already available. The portable substation may be automatic or attended according to the use to which it is to be put. It has hitherto been practically confined to America.

CHAPTER VII

SYSTEMS OF ELECTRIFICATION

In the foregoing chapters, some account has been given of the plant and apparatus employed in the electrical working of railways; and although the circumstances under which devices are used have generally been indicated, it appears desirable, even at the risk of repetition, or of dealing with trite matters, to devote a chapter to the consideration of electrical operation as a whole, discussing therein the methods that experience has developed and approved in connection with the various systems of electrification. At the time of writing, the art as applied to main-line railways is in a comparatively early stage of development, and it is natural that the methods that have been employed are not always such as would have been used if the experience gained in their use could have been foreseen. It is however not usually practicable to make changes in essentials after a system has been put into operation, for such changes are very expensive, and clearly uneconomical unless the certain benefit from their adoption is sufficient to warrant the large outlay involved. There is accordingly greater diversity in existing installations than technical and economic considerations warrant, but the tendency is now towards a few clearly distinguished systems and substantially uniform methods of working. There are three primary systems of operation, distinguished by the form of supply of power to the trains: (1) the continuous current system, (2) the single-phase system, (3) the polyphase system. In addition to these, two subsidiary systems may be recognized: (1) the split-phase system, in which the distribution is single-phase and the locomotive motors of the polyphase type; (2) the rectifier-locomotive system, in which the distribution is single-phase, and the locomotive motors of the continuous current type.

THE CONTINUOUS CURRENT SYSTEM

The continuous current system is the oldest and, in some applications, the most nearly standardized system of railway working. As applied particularly to urban and suburban passenger service, it is a natural development of the electric tramway, and the success of the system in such service arises in great measure from the same causes as have led to its success in tramway work, being chiefly due to the very suitable characteristics of the continuous current series motor for train service in which stops are frequent. The practicability and economy of employing a multiple unit system of train operation have also contributed to its success in dealing with urban traffic. The continuous current system has however been adapted, with complete success, to the heaviest classes of main line service, and there is now no railway traffic which could not be satisfactorily worked by the system.

General Description of Plant.—Energy is generated for this system in polyphase form, usually three-phase. The voltage of generation is immaterial, but the frequency is a matter which may affect the cost and nature of the substation plant. For efficient generation, by means of steam power plant, the higher frequencies are preferable; but for the most satisfying conversion, particularly when the distribution voltage is high, the lower frequencies have the advantage. When a generating station is erected for the sole purpose of dealing with a railway load on this system, the frequency chosen is usually 25 cycles. At the same time it is becoming recognized that, in the interests of economy and efficiency, it is desirable to combine the railway load with other industrial load, which in this country is more usually at a frequency of 50 cycles; and this in spite of the fact that the higher frequency is less suitable from the point of view of conversion to continuous current. The transmission of the power to the substations may be either by cable or by overhead lines. The usual, and in general the most satisfactory and economical substation plant for the system, consists of transformers and rotary converters, with appropriate switchgear and accessories. Where the voltage of distribution, having regard to the fre-

quency of supply, is too high to make this a practicable solution, the motor converter or motor generator may replace the rotary converter. The mercury vapour rectifier is another possible alternative. If the voltage is sufficiently low, the distribution of energy to the trains may be effected, on the positive side, by means of insulated conductor-rails laid beside the track; or at higher voltages, by means of overhead trolley lines, the track rails being usually employed as return conductors in either case. Most of the London Electric Railways however use an insulated negative conductor rail, whilst on the Lancashire and Yorkshire Railway electrified lines, an uninsulated negative rail helps to carry the current collected by the track rails.

The train equipments are, for urban passenger lines, carried on special motor coaches, any desired number of which can be included with trailing coaches in a train, and controlled from a driver's compartment at the head of the train. Motor coaches are of two general kinds, those having two-motor equipments and those having four-motor equipments. Both arrangements are used in urban passenger service at low voltage, but the four-motor equipment is the usual practice at the higher voltages. There is a tendency in this country to concentrate much power in a few motor coaches, often equipped with four motors each; and to make up the trains with many trailers. In America, on the other hand, it is common to compose trains entirely of motor coaches, each carrying a two-motor equipment. The British practice makes for initial cheapness; but, in so far as it has sought to use more powerful motors than the bogie of a motor-coach naturally accommodates, it compels the sacrifice of desirable features in the design of the motors, leading in the long run to greater expense of upkeep. American practice, on the other hand, makes for flexibility in operation; and, in that it imposes fewer restrictions on the design of the motors, is economical in maintenance. It is also usually preferable from the point of view of control, since the adhesion of the driving wheels is farther removed from the limit. For the heavier classes of main line service the driving equipments are carried on special locomotives designed for the purpose. The controlling gear is arranged so that the motors can be grouped in different ways for starting the train and for varying the speed of running.

Distribution Voltage.—It is usual to distinguish the continuous current system according to the voltage of distribution; but, although there are considerable differences between extremes, there is no real line of demarkation between the high voltage and low voltage continuous current systems. The greater part of the work that has been carried out, and indeed all that of importance before the advent of the commutating pole railway motor, has been at a motor voltage of 550 to 650 volts, this marking about the practicable limit of successful operation with non-commutating pole motors. Since the development of the commutating pole motor, however, there has been a growing tendency towards the use of higher voltages, a practice which has the advantage of bringing larger areas within the range of economical operation. Whether the use of the distinguishing adjectives has reached stability, or whether indeed stability is possible in the case, time alone can show; but at present the terms are used practically in their historical sense, the low voltage system working in the neighbourhood of 600 volts, and the high voltage system at substantially higher voltage.

Low Voltage System.—It is a mistake to look upon the low voltage system as in any sense obsolete, or as being tolerated on account of the expense of proceeding to higher voltage. Where the traffic is dense and difficult, requiring frequent trains, which start quickly and stop at short intervals, the balance of advantage is still with this system. If the question of the electrification of the London Underground Railways were under discussion to-day, it is probable that an unbiased consideration of the circumstances would indicate, as the most suitable and economical methods of operation, substantially those in actual use on the railway. Doubtless commutating pole motors cooled by induced draught would be used throughout, and other late improvements introduced; but it is unlikely that a radical change of methods would be rendered desirable in view of our present-day knowledge.

The great strength of the low voltage continuous current system for service in which trains have to be started rapidly and often, lies in the peculiar suitability of the driving motor for such work, in its mechanical ruggedness, in the relation between its torque and speed, and in its comparatively small and well-distributed internal loss, which can be dissipated

without undue local rise of temperature. No other type of motor has yet been produced which, taken in relation to its size and service capacity, approaches the continuous current motor in the latter respect; and its reliability in service and low maintenance cost are the direct results. No other type of equipment moreover approaches the continuous current in lightness when applied to the service under consideration; and lightness is a most desirable feature in the equipment for this class of service, in which the energy consumption is nearly proportional to the weight of the train.

THE HIGHER VOLTAGES. As the voltage is increased the difficulty of designing the motors increases, especially such motors as can be carried on the coach-trucks. This is only slightly on account of the increased insulation and creepage surfaces needed, but largely on account of the greater peripheral commutator space necessitated by the high voltage between brushes. A point is soon reached where it is no longer practicable to design a good motor for the line voltage; and the motors, although insulated for the full line voltage, are wound for a half of this, and operated at least two in series. This is the preferred practice for multiple unit trains using the high voltage continuous current system. It has the disadvantage of making four motors the control unit, so that four-motor equipments are needed, and if a motor is disabled at least two and sometimes four must be cut out. As the voltage is raised still further, the multiple unit train becomes less and less desirable on account of the difficulties of motor-design, and at the highest voltages the separate locomotive is the only practicable method of working the traffic efficiently.

Although only two varieties of continuous current operation have been distinguished, the foregoing discussion indicates that three might more logically be recognized. There is a low voltage system, extending perhaps to about 800 volts, but particularly distinguished by the employment of train-motors wound for the line voltage. This is suitable for the heaviest urban service. There is a medium voltage system, extending to perhaps double the above figure, in which the motors are preferably wound for half line voltage, and in which multiple unit trains composed of motor and trailer coaches are still practicable. This is suitable for main line work in well-populated countries. There is a high voltage system in which

the motors are wound for a fraction of line voltage, but in which locomotive operation alone is practicable. This is suitable for main line work in sparsely populated countries, and for lines exclusively concerned with goods service.

Method of Distribution.—In the low voltage system the current is large and power is usually distributed to the trains by means of conductor rails. The preferred substation unit is the rotary converter, with transformers. The possibility of using this form of plant is indeed one of the reasons for preferring the system of operation wherever traffic is sufficiently dense. The trains used in service for which this system of operation is suitable are almost exclusively of the multiple unit type.

In the medium voltage system, the power may be distributed to the trains either by conductor rails, as in the case of the Manchester–Bury line, or by overhead trolley lines, as in the case of the Victorian Railways. The substation plant may be based on rotary converters, operated in independent units if the frequency is sufficiently low, or, at higher frequency, operated in pairs connected in series. The motor converter, motor generator, or mercury vapour rectifier may also be used. The trains may be hauled by locomotives or by motor coaches in multiple unit.

In the high voltage system, the power is distributed by means of overhead lines. The substation plant may be based on the motor generator, or the mercury vapour rectifier, although to the present the first only of these has been employed for the work. The trains in this system are locomotive-hauled, for not only is it inadvisable to cramp the design of the motors, but the control equipment occupies much greater space than is the case when lower voltage is used, and the motor coach is not a suitable vehicle for carrying it.

Regeneration.—In mountainous districts, where gradients are long and steep, there is great advantage in keeping control of the trains on falling gradients by causing the motors to regenerate, or to act as generators driven by the descending train itself. In this manner the speed is restrained without the need of running with brakes applied, a practice which wears both brake-blocks and tyres rapidly, and sometimes even loosens the wheel-rims on their centres. Incidentally, energy

is fed back to the line for use of other trains. In the continuous current system, regeneration is effected by exciting the fields of the motors with greater or less intensity, from a separate source, which may consist of a motor generator, an axle-driven generator, or even one of the driving motors themselves. Regeneration necessitates the carrying of a considerable amount of extra gear on the locomotive, and on this account is in general hardly worth the expense merely for the purpose of securing a return of energy in stopping the trains. Its practical use is therefore for this system of operation confined to certain sections where gradients are suitable. Since the energy returned to the line may sometimes pass through the sub-station, it is necessary that the power converting units should be reversible in their nature. The mercury vapour rectifier is accordingly ruled out, as a converter for use when regeneration is employed.

THE SINGLE-PHASE SYSTEM

In the single-phase system of operation, the power is preferably generated single-phase, at frequency suitable for use by the trains; and is fed directly to the distribution system at high voltage. The distribution pressure may be anything that it is practicable to deal with on a locomotive; and voltages up to 20,000 have been tried experimentally, whilst 15,000 volts is actually in use on the Swiss railways and elsewhere. A low frequency is very desirable in the interest of satisfactory operation of the train motors, although less suitable for purposes of generation and transformation; and frequency of 15 to $16\frac{2}{3}$ cycles per second is generally accepted as about the best compromise. The trains are preferably locomotive-hauled, for the single-phase motor is not satisfactory if cramped for room. The system has many modifications which are used as the circumstances of particular cases require.

A low frequency single-phase supply is not well suited for generation by means of steam plant, which for such purpose is unduly expensive and inefficient. The load-factor of a railway moreover being in general very low, generation for the sole purpose of railway supply is apt, for this reason also, to be expensive, whether the prime movers are worked by steam or water-power. The single-phase railway supply is

however useless for any other purpose on account of its low frequency, so that the load cannot be directly combined with the industrial load of a district and the load factor thereby improved. The generation is therefore on all counts handicapped by the system of operation. The situation may be accepted as it stands, or a compromise adopted. In England and America, the compromise has taken the form of employing the higher frequency of 25 cycles. This is perhaps the least satisfactory way of evading the difficulty; for it throws the stress on a much weaker element in the chain of appliances, viz. the train-driving motor. A preferable compromise is to take the railway supply through a motor-generator acting also as a frequency changer. The generation may then be performed in the most economical manner, and the railway load combined with the local industrial load. This arrangement, although detracting from the severe simplicity of single-phase generation, is usually less expensive in spite of the introduction of the converting machinery.

The chief attraction of the single-phase system, however, lies in the simplicity of its distribution arrangements. Even before a suitable motor for train-driving had been developed, it was predicted that the system would prove the only practicable one for main line railway working, principally on account of the fact that power at high voltage could be taken from generation station to train by means of an overhead trolley wire, and there reduced to a suitable value for use by means of a simple transformer. The prediction has not been justified by events, and the distribution has proved less simple than was expected; but it nevertheless remains the strongest feature of the system, and that on which all its claims to superiority are based.

Interference with Communication Circuits.—Where the railway is an extensive one, the project of feeding the whole of it directly from the power station has to be abandoned; for the distribution voltage, although as high as can be dealt with on a locomotive, is not high for purposes of simple transmission. This however does not detract seriously from the merit of the system; for the provision of transmission lines and a few transformer substations is a minor matter in connection with such a railway. Much more serious is the effect

of a distribution system grounded on one side and fed by alternating current at a few points in producing disturbance in neighbouring telegraph and telephone systems, an effect which may extend to several miles from the railway. On account of this, the notion of feeding the railway in this simple manner becomes quite impracticable. It is necessary to parallel the railway throughout with insulated feeders, and booster lines, connected to the distribution lines through transformers at very frequent intervals. The purpose of these devices is to reduce to the smallest practicable amount the current straying from rails to ground, for it is these currents which are chiefly concerned in producing the disturbances in the weak current circuits. Although the transformers are often placed out of doors and need little attention, their cost, with that of the additional lines, increases considerably the cost of the distribution system. The result moreover is amelioration, and not cure; and it is still necessary to remove the weak current circuits from the immediate vicinity of the railway. It is not an over-statement to say that this interference with neighbouring communication circuits is one of the chief objections to the use of the single-phase system of operation in populated regions.

The Locomotive Motors.—The train motors are now generally of the compensated series type, sometimes however being started as repulsion motors. The motor-voltage is low, and auto-transformers are used for stepping down from the distribution voltage,—taps being provided for purposes of starting and speed variation. The limitations of motor design make it the preferable practice to concentrate the driving power in a few large motors rather than to drive the locomotive axles individually. The preferred locomotives are accordingly of the coupled wheel types, in which the power of the motor is transmitted to the wheels through a jack shaft and a pair of horizontal side rods.

The dynamical characteristics of the single-phase railway motor resemble those of an unsaturated continuous current series motor, and for the purposes of general railway work are quite as suitable as this machine. Indeed for most classes of service it is decidedly superior to the ordinary continuous current motor, for the variation in power taken by the train in

running is smaller; and accordingly, if due advantage is taken of this characteristic of the motor, a smaller reserve of power is required in the generating plant. For heavy urban passenger service however the need of rapid acceleration governs the train control, and the high peak of power at starting is inevitable. The dynamical characteristics of the single-phase motor, in so far as they differ from those of the continuous current motor, are more suitable for main-line service, and less so for service which requires frequent and rapid starting of the trains.

Energy Loss and Heating.—The loss of energy in the interior of the single-phase motor is much greater, in proportion to the input, than that of the continuous current motor, particularly when used in service which calls for frequent starting of the trains; and more efficient cooling arrangements have accordingly to be adopted in order to get rid of the heat developed. Forced draught is used wherever practicable for cooling the motors and transformers. For motor-coach work, and particularly for multiple unit trains, in which the motors are likely to get no attention for long periods, the use of the external blower is inadvisable, besides being inconvenient. In such work, if the service is heavy, it is usual to allow the motors to run hotter than would, but for the necessity, be considered good practice.

Weight of Electric Equipment.—The single-phase motor, working as it does at low saturation, is inherently much heavier than the continuous current motor of equal dynamical capacity; and the difference is accentuated by the fact that the frame of the motor cannot be used as magnetic material. On the basis of equal service capacity, with similar methods of cooling, the difference of weight is even more marked, on account of the great internal losses in the single-phase motor, particularly at starting. The transformer is also a heavy item. In average urban passenger service the equipment weight per unit weight of train is some 75 per cent. greater in the single-phase system than in the continuous current system, even when allowance has been made for the motors being permitted to run hotter in the former system. Since the equipment itself accounts for a considerable fraction of the train weight, the total increase in its weight exceeds the above figure; and

the more exacting the service the greater is the disparity. Thus if, in the continuous current system, the equipment weight is only 10 per cent. of the train weight, the single-phase equipment works out about 91 per cent. heavier, and the train itself about 9 per cent. heavier. If however the equipment weight is 20 per cent. of the whole in the continuous current system, the single-phase equipment would be about 115 per cent. heavier and its train about 23 per cent. heavier. Even these increases are conservative, for they take no account of increase of train weight required to carry the heavier equipments—the possible increase in the number of motor-bogies or in stiffness of underframes. It indeed generally understates the facts to assert that the single-phase equipment is twice as heavy as the equivalent continuous current equipment for urban passenger service by multiple unit trains.

The great comparative weight of the single-phase equipment implies a cost approximately in proportion; and the increase in weight of the train leads in itself to a corresponding increase in the energy consumption. In multiple unit train operation however the train equipments, even in the continuous current system, usually form by far the biggest item in the expenditure on plant; and a system which doubles this item without improving traffic facilities, and at the same time increases the energy consumption, not to mention the maintenance cost, whilst interfering seriously with neighbouring communication systems, has no future for the class of work under consideration.

For normal railway service, in which trains are, of necessity, locomotive-hauled, and the distance between stations is great, some of the disabilities of the single-phase system, as above enumerated, disappear. With the motive power concentrated in a few large motors, which are kept cool by means of external blowers, and supplied at suitable low frequency, the locomotive, although in general more costly than the equivalent continuous current locomotive, is not disproportionately so; and the saving in the substation and distribution systems is substantial. The chief objections to the single-phase system as compared with the other for such service, are: (1) the more costly generating plant for the low frequency; (2) the greater quantity of generating plant due to the fact that it cannot combine its load with the local industrial load; (3) the inter-

ference of the distribution system with neighbouring communication circuits; (4) the more costly locomotives; (5) the less satisfactory type of locomotive, which is costly in maintenance. Whether these outweigh the saving in the substation and distribution systems depends on the circumstances of the case; and a general statement would in the present state of development be inappropriate.

THE POLYPHASE SYSTEM

In the polyphase system, power is generated in form and frequency as required by the trains. This may be fed directly to the distributing system, or transmitted at high voltage to distant feeding points, and there reduced in pressure. The term "polyphase" in this connection practically means three-phase, and two insulated line conductors are necessary, the third connection being made through the track. The train motors impose no natural restriction on the frequency of the supply. The Italian State Railways, which furnish the chief examples of polyphase operation, use frequencies from 15 to $16\frac{2}{3}$ cycles per second, the low value being used presumably in order to enable the motors to be run without gearing, and at the same time to have a good power-factor although made with comparatively large air gap. The Great Northern Railway (Cascade Tunnel) uses a frequency of 25 cycles, the motors being geared to the wheels.

There is some difficulty in locating two overhead conductors and two sets of collector gear in such manner that under no conditions can the one encroach on space belonging to the other. At special work particularly the overhead system becomes complicated. In consequence of this, the voltage used for distribution is not so high as is the usual practice in the single-phase system. The Italian railways are run at a pressure of about 3,300 volts, using sliding bow collectors. The Cascade Tunnel electrification is run at 6,600 volts, but the current is in this case collected by means of trolley wheels carried by poles, the speed being limited to about 15 m.p.h. The distribution lines, like those of the single-phase system, affect neighbouring communication systems deleteriously; but the trouble is perhaps less marked, on account of the less pronounced harmonics arising from the motor-slots.

U

Locomotive Motors.—The train motors are of the induction type, and for mechanical merit no better selection could be made. Having no commutator one of the principal sources of trouble in the other systems is avoided. The construction is rugged, and, but for the small air-gap, the motor has no essentially weak features. The small air-gap is desirable in order to obtain good power-factor and efficiency ; and, although a larger gap is usually permitted in traction motors than in stationary induction motors, this feature has to be continually looked after as the bearings wear. The motors may be wound for line voltage, as on the Italian locomotives, or transformers for stepping down the voltage may be carried, as on the Cascade Tunnel locomotives, whose motors are wound for 500 volts only. The rotors are wound, in order that a good starting torque may be obtained by the insertion of resistance. The motors are sometimes controlled in pairs, being then connected initially in concatenation or cascade, and afterwards in parallel. With similar motors connected in cascade the locomotive runs at a half of full speed. In some cases the motors are designed so that the number of poles can be changed by change in the connections, and other speeds are then obtainable. This is common in the Italian State Railway locomotives, such running speeds as 37·5, 50, 75 and 100 km. per hour being obtained on some of the locomotives by the use of a cascade and pole-changing control system.

The dynamical characteristics of the induction motor are not very suitable for railway work. It is essentially a constant speed motor, and a small variation in its running speed causes a large variation in its tractive effort. With separately driven axles, even a small difference in the size of the wheels of a locomotive throws an unduly great part of the load on the axle which drives the larger wheels. It is therefore sound practice to couple the driving axles, as is done in the Italian locomotives. Even then the locomotives have usually to be run singly ; for if two locomotives, having wheels of slightly different diameter, were hauling a train, one would take the greater part of the load. An attempt is sometimes made to rectify this by running the faster locomotive with a certain amount of outside resistance permanently connected in the rotor circuits of the motors. The remedy however is in general only partial, and leaves to the driver an adjustment

which should depend on considerations of weight of train, gradient, wind and other circumstances, but which he can only judge by indirect observation. The constant speed characteristic of the induction motor makes the polyphase system quite unsuitable for multiple-unit train working. Another consequence arising from the same cause is that the power taken is practically proportional to the tractive effort, and the peaks due to gradients and the like are much more pronounced than in systems in which the speed falls with increase of tractive effort.

Regeneration.—The induction motor possesses a feature of great value in certain classes of railway work, arising from its dynamical characteristics. This is the property of regeneration, whereby without any special apparatus, and in fact without necessary cognizance of the driver, it returns energy to the line whenever the conditions exist to provide the energy. As long as the locomotive is hauling, the motors run at slightly less than synchronous speed, taking energy from the line; but when the train reaches such a down gradient that it tends on its own account to increase its speed, the motors, then running somewhat above synchronous speed, begin to regenerate. This is a very valuable property, particularly where the gradients are long and steep; not only, or even principally, on account of the saving of energy, but particularly on account of the simpler and more reliable control of the train which is thereby rendered less dependent on the judgment of the driver. The extensive use of the polyphase system in Northern Italy is probably to be ascribed in large measure to the existence of conditions favouring regeneration. The return of energy to the line when there is no load to absorb it is an embarrassment; and special provision of artificial load has usually to be provided in the generating stations to meet the emergency. As has been mentioned, regeneration is practised with other systems of operation, being used regularly, for instance, on the Chicago, Milwaukee and St. Paul Railway. It is however not in these cases a natural characteristic of the motor used, but requires a considerable amount of external apparatus, and is dependent on the driver to make use of it at the appropriate time and in the appropriate manner.

THE SPLIT-PHASE SYSTEM

The split-phase system was due to Alexanderson, who proposed the use of an auxiliary machine, of induction-motor type, to be used as a phase converter for taking power from a single-phase line, and giving it out, in appropriate phase relation to the windings of polyphase motors. The system accordingly has the advantage of single-phase distribution, but avoids the use of the single-phase motor. It is essentially a system for locomotive operation, both on account of its use of polyphase driving motors, and because it requires phase converters which could not be accommodated on motor coaches. The main connections of the locomotive wiring, as used on the Norfolk and Western line, are shown in fig. 70. Having polyphase motors, the system shows to greatest advantage when used for working sections of heavy traffic on which gradients are long and steep, and on which accordingly full advantage can be taken of the regenerative qualities of the motors. There are two such sections, using the split-phase system, in the United States. The Norfolk and Western Railway has about 30 miles of electrified route, including about 3·8 miles of 2 per cent. gradient, against which heavy trailing loads, sometimes attaining 3,000 tons, are hauled. The Pennsylvania Railroad in its Altoona–Johnstown section uses the system for heavy mineral traffic; this line has about 12 miles of 2 per cent. gradient and 24 miles of 1 per cent. gradient. The split-phase system is therefore one suitable for meeting local conditions, but unsuitable for working the general traffic of railways. The use of the phase converter is a disadvantage against which must be balanced the advantages of single-phase over polyphase distribution. As compared with the ordinary single-phase system, the system has the advantage of not requiring an unduly low frequency.

The Rectifier Locomotive System.—The rectifier locomotive is the result of an attempt to make use of the strongest features of both single-phase and continuous current systems. It has the advantage of single-phase distribution at high voltage, but is able to use the continuous current motor. In order to accomplish the object, a single-phase

mercury vapour rectifier, suitably fed from a transformer, is carried on the locomotive for the purpose of supplying the motor current. Such a locomotive was tried some years ago on the New York, New Haven and Hartford Railway, but apparently did not pass the experimental stage.

CHAPTER VIII

PRELIMINARY MECHANICS

Expenditure of Energy by Train-driving Equipment.—The energy supplied to the driving motors of a train is expended partly in giving velocity to the mass of the train; partly in doing work against the aggregate of resisting forces known collectively as tractive resistance; partly, where the track is inclined, in raising the train against gravity; and partly in inevitable losses in the train equipment itself. In rapid suburban passenger service, a large proportion of the energy goes in giving kinetic energy to the train and is ultimately dissipated in the brakes. In service, however, in which the distance between stops is considerable, the greater part of the energy supplied is in general expended in doing work against the tractive resistance. In the latter class of service accordingly it is necessary, if an accurate estimate of energy consumption is to be made, that this resistance should be estimated correctly; in the former class it is more important to know the mass of the train and the moment of inertia of the rotating masses. The effect of gradients on energy consumption is often, in the aggregate, small; for with trains passing in both directions the average is, in the greater number of cases, but little different from the level track consumption, though usually somewhat greater. This is not the case, however, where gradients are so long and steep that speed restriction is necessary in descending, unless the restriction is effected by causing the motors to act as generators and return energy to the line. Where stopping places are located in hollows the gradients cause increase in energy consumption, and where they are located on summits, decrease. Some railways, such as the Central London Railway, are purposely laid out so that the stations are all on summits and the conse-

quent saving in energy is notable. What has been said of energy consumption as affected by gradients may also be said of motor heating, except that regeneration is in this case no alleviation, but quite the reverse; for return of energy to the line requires greater service capacity in the motors than its dissipation in brake blocks. It is in fact the increased cost of the equipment, with its greater weight and higher maintenance cost, that offset the saving of energy due to regeneration, and make it, on examination, unattractive unless gradients are suitable.

Tractive Resistance.—A great amount of labour has been expended with the view of determining the tractive resistance of trains, by which is meant the frictional resistances to their motion in still air on straight and level track; but so many are the factors entering into the determination that it is rare indeed to find two sets of experiments yielding results substantially in agreement. This is particularly the case at high speeds, for few experimenters have resisted the temptation of presenting their results in the form of a formula which has frequently been applied irrespective of the range of the tests. If it were practicable to divide the observed tractive resistance of a train into its constituent elements, to determine the factors upon which each element depended and to express them in terms of the given conditions, it might be possible to predetermine a train resistance by computation: but the number of elements is large and cannot be effectively segregated, the influence of the several factors on which they depend is frequently conjectural, whilst the best numerical expressions that can be found for them are usually based on arbitrary assumptions, the matter being altogether too complex to admit of rational treatment.

Mr. C. O. Mailloux enumerates the following elements of the tractive resistance of a train:—

A.—Sliding Friction

i. Lubricated Sliding-Friction:—

(1) Rotational friction of axle or journal.

ii. Unlubricated Sliding-Friction:—

(2) Slipping or skidding-friction.

(3) Wheel-flange friction.

B.—Rolling Friction

(1) Friction due to mangling or crushing effects.
(2) Friction due to non-yielding inequalities of surface.
(3) Track hysteresis.

A.B.—Composite Friction

(Combining sliding and rolling-friction).

(1) Effects of oscillation and concussion.
(2) Effects of curves.

C.—Fluid Friction

(Including two varieties, both involved in train-resistance.)

(*a*) Semi-fluid Friction :—

1. Friction of ties, ballast, embankment, earthwork, etc.

(*b*) True Fluid Friction :—

2. Air friction at head of train.
3. ,, ,, rear ,,
4. ,, ,, sides ,,
5. Wind-friction.

When it is realized that not one of the above elements can be separated out with certainty and that the actual tractive resistance in any particular case can only be obtained as the average of a large number of more or less discordant results, it will be appreciated how nearly impossible it is to estimate it with accuracy in the case of trains differing substantially from those on which test results are available. Unfortunately the number of satisfactory tests on which estimates can be founded is very few, although many casual readings of tractive resistance are on record.

Lancashire and Yorkshire Tests.—Perhaps the most complete tests of tractive resistance of modern English passenger stock are those of Sir J. A. F. Aspinall,* taken on trains of bogie coaches on the Lancashire and Yorkshire Railway. Fig. 146 shows the type and general dimensions of the coaches. The curve for a train of five coaches and a dynamometer car, of which the weight was 115·2 English tons (258,100 lbs.), and length 284·8 feet, and on which upwards of 200 readings were

* *Minutes of Proceedings Inst. C.E.*, vol. 147, p. 155.

taken, is given in fig. 147, the results occurring in each division of five miles having been averaged and the curve plotted in accordance with these collected results. The number of tests represented by each point is given beside the point. It will be seen that except for the last three points a smooth curve can be drawn in close agreement with the tests. On the same figure is marked a point at about 50 m.p.h., showing the mean of thirteen tests on a train consisting of ten coaches and a dynamometer car, of total weight 218·7 English tons and length 542·2 feet. The starting resistance of the five-coach train was found to be about 17 lbs. per ton, dropping however to a minimum of about 3 lbs. per ton before a speed of 5 m.p.h. had been attained. This feature is characteristic of train resistance curves and does not merely represent a discontinuity between static and moving friction or the like; for if the train is

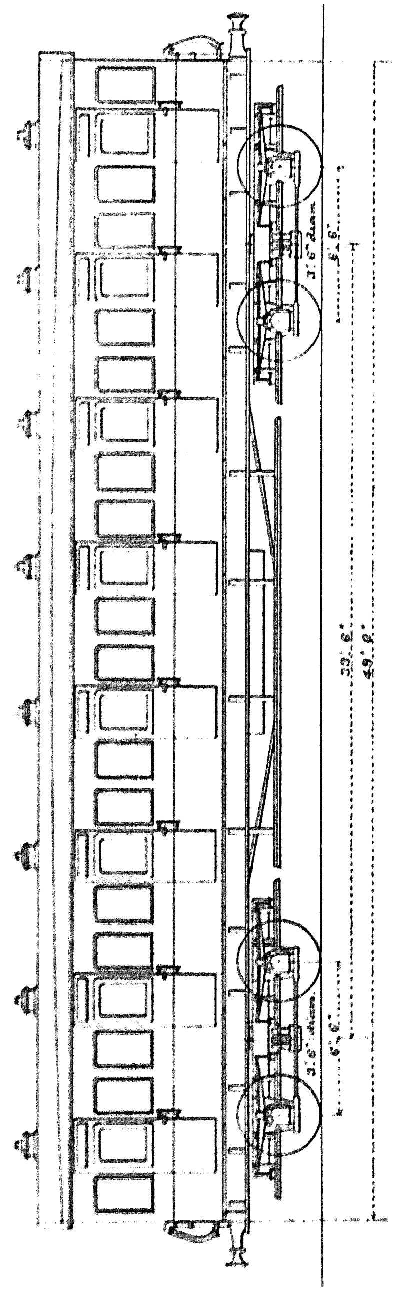


FIG. 146.—Bogie Coach. Lancashire and Yorkshire Railway Tests.

allowed to coast to rest the resistance at low speeds is found to rise with falling speed after the manner shown in the figure, and the whole curve can be traced. However, except when starting a heavy goods train on a gradient, with couplings extended, this portion of the curve is of small importance from its limited extent. Mr. Aspinall's tests were made by measurement of draw-bar-pull behind a locomotive, and therefore omit

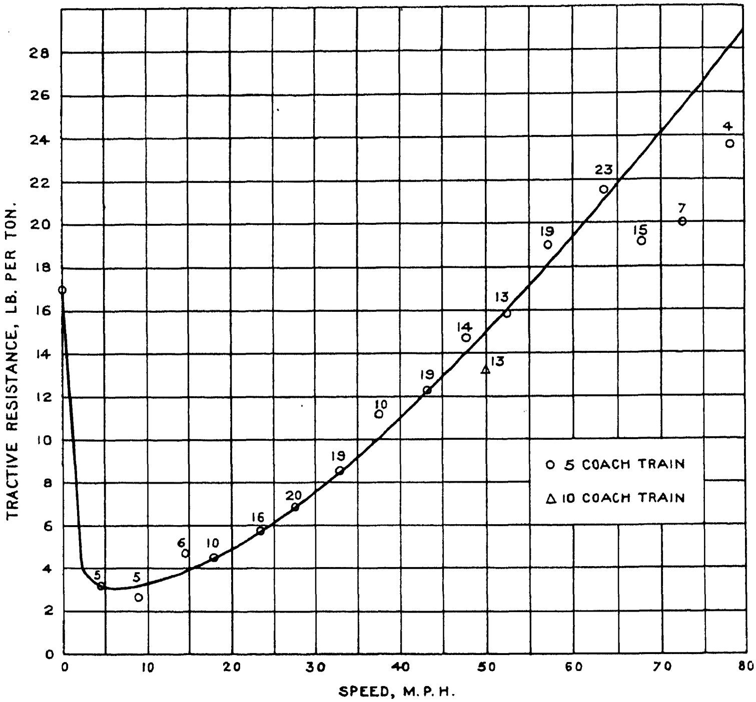


FIG. 147.—Tractive Resistance. L. & Y. Tests.

the head resistance to the motion of the train, or the tractive resistance directly chargeable to the locomotive.

New York Central Tests.*—A large and valuable series of tests with trains of various lengths were made by the General Electric Company at Schenectady, N.Y., in connection with the experimental runs on the first electric locomotive of the New York Central and Hudson River Railroad. In these tests the resistance of the locomotive, which was of a type having practically no mechanical resistance other than is properly

* See *Minutes of Proc. Inst. C.E.*, vol. 201, p. 243.

chargeable to train resistance, was included. The results of the tests are given in figs. 148, 149 and 150, and represent months of systematic work, some hundreds of runs having

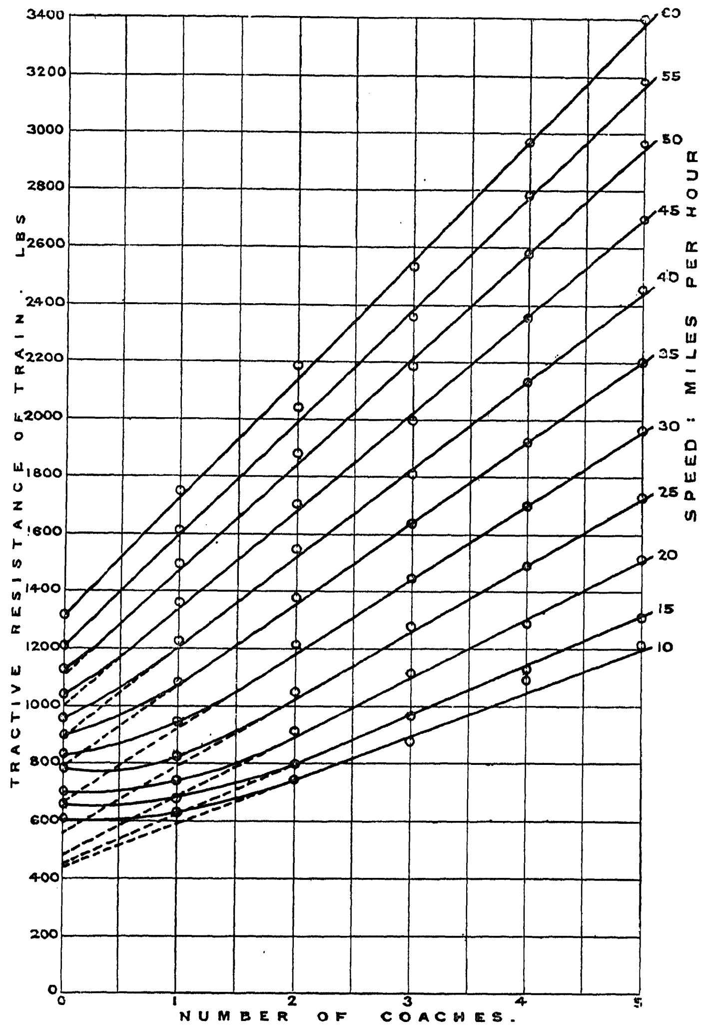


FIG. 148.—Tractive Resistance. New York Central Tests.

been made and readings taken with each length of train. Their importance is therefore of a high order, and their value is further enhanced by the fact that, as will be explained later,

they throw general light on the subject of tractive resistance of trains such as is given by no other series within the author's cognizance. Two types of coach were used; and, fortunately for the purpose of effective deduction, these were not mixed in the tests. Both were of the standard American saloon type

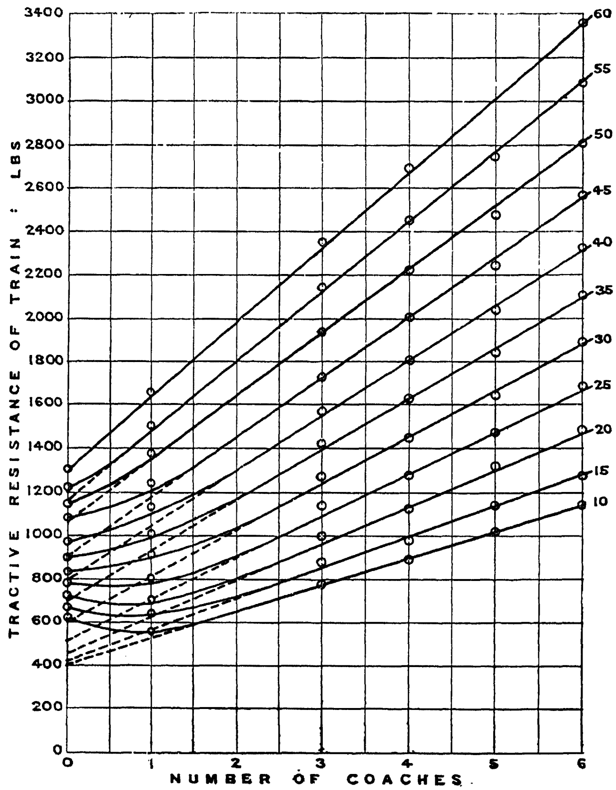


FIG. 149.—Tractive Resistance. New York Central Tests.

known as day-coaches. One was carried on six-wheel bogies, and the other on four-wheel bogies. The cross section of both types of coach, including trucks, was approximately 115 sq. feet. The coaches were, for some reason, loaded with sand to a weight considerably more than their normal passenger capacity. Outlines of the two types of coach are shown in figs. 151 and 152; and the leading dimensions and average weights are given in table 9.

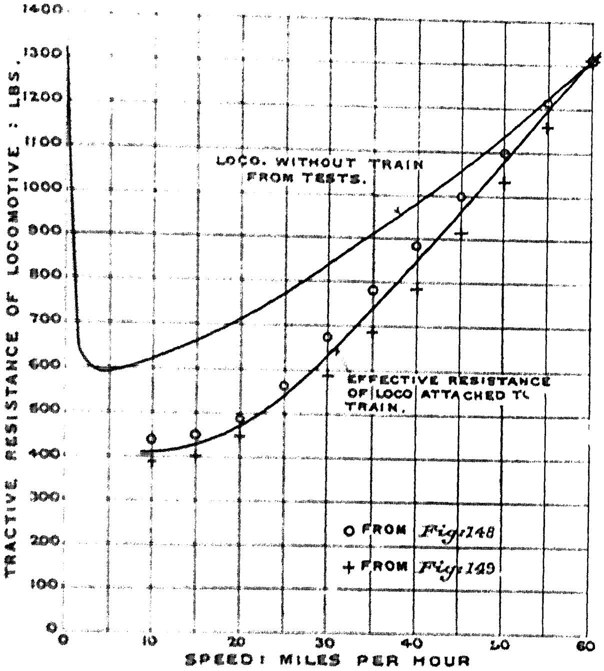


FIG. 150. Locomotive Resistance. New York Central Tests.

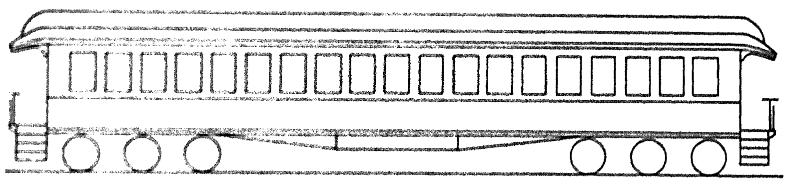


FIG. 151. New York Central 6-wheeled Bogie Coach.

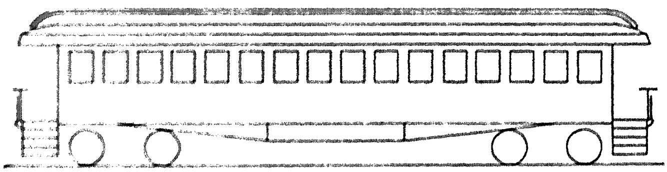


FIG. 152. New York Central 4-wheeled Bogie Coach.

TABLE 9

	Six-wheel bogies.		Four-wheel bogies.	
	ft.	ins.	ft.	ins.
Overall length	66	0	55	0
,, width	9	$11\frac{1}{2}$	9	$11\frac{1}{2}$
,, height above rail	13	$8\frac{1}{2}$	13	$6\frac{1}{2}$
Length, body	60	2	47	6
Distance between bolster centres	43	$8\frac{1}{2}$	36	$5\frac{1}{2}$
Rigid wheel base	10	6	6	6
Diameter of wheel	3	0	3	0
Weight empty	36 tons		21·7 tons	
,, loaded	45·3 tons		26·2 tons	

Berlin-Zossen Tests.—The Berlin-Zossen high-speed railway experiments * made in 1902 and 1903 included tests of tractive resistance of single coaches to speeds exceeding 100 m.p.h. These tests were carried out with great care and skill, and the results showed greater consistency than is usual in this subject. Two approximately similar coaches were used, and the results obtained on one of them are given in fig. 153. The coach weighed 92 tons (206,500 lbs.) and had a length of 72 feet over buffer beams and 76 feet over buffers; its cross section down to rails was approximately 110 sq. feet. It was fitted with 6-wheel bogies, the outer axles of which carried the gearless motors which drove the coach. The form of the ends of the coach is shown in the figure. Tests were also made of the air pressure on the front of the coach, and it was found practically to agree with the formula—

$$P = 0{\cdot}00275V^2 \quad . \quad . \quad . \quad . \quad (1)$$

where P is the pressure in lbs. per sq. foot and V the velocity of the coach in miles per hour. This pressure is of course only fully effective over the flat portion of the end which impinges directly on the air, and the difficulty in making use of the formula arises entirely from the impracticability of determining the effective area of the coach end.

* *Berlin-Zossen Electric Railway Tests,* McGraw Pub. Co., New York.

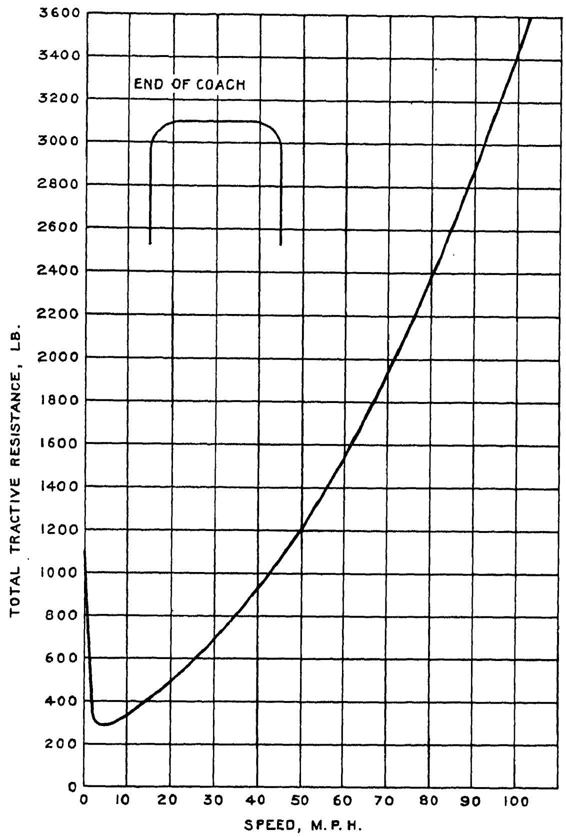


FIG. 153.—Tractive Resistance. Berlin-Zossen Tests.

Louisiana Purchase Exposition Tests.—Important tests of air resistance were made in 1905 under the auspices of the Electric Railway Test Commission, appointed in connection with the Louisiana Purchase Exposition. The tests were carried out on a specially constructed car of the interurban type, in which the body and front and rear vestibules were separately and freely movable in the direction of the length of the car and were kept in position by balancing arrangements devised to indicate the forces tending to displace them. In this manner the pressure on the front vestibule and the suction on the rear vestibule were found. The friction on the sides of the car was determined by subtracting the forces on the vestibules from the total, but as the measured quantities were

large and the differences in question small, the results were, in this regard, very indefinite and of little value. However, the tests determining the forces on the vestibules were interesting and valuable, though not sufficiently numerous to make the results conclusive. The projected area of the vestibules appears to have been about 72 sq. feet, and this is the area on which the forces were measured. In the report of the Com-

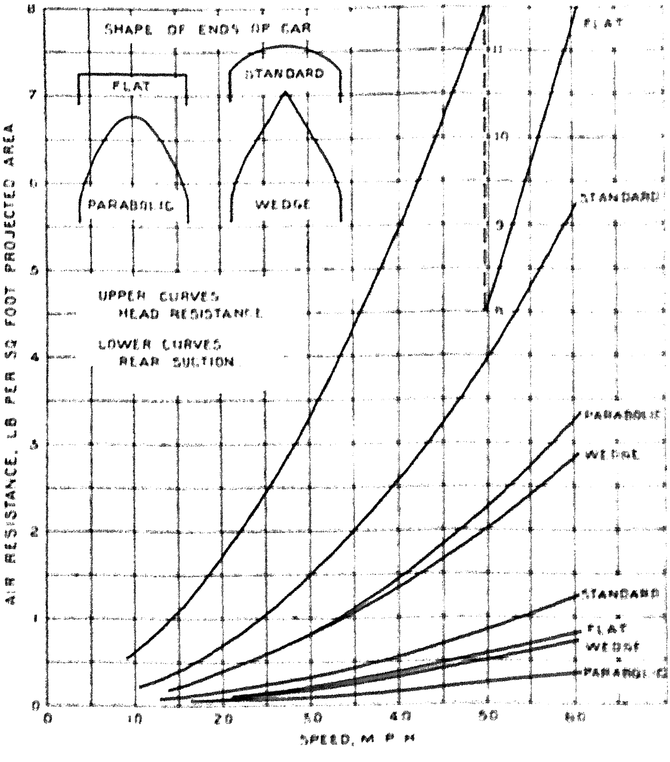


Fig. 154. Air Resistance, Louisiana Tests.

mission, however, the total forces were divided by an area of 96 sq. feet, which is presumably the cross section of the car with the under part filled in down to rail level, this area being conventionally and reasonably usual in estimating head resistance. It would seem however to be a mistake to take the larger area in the present instance, for the force on the lower parts was not included in the measurements and is therefore additional to the force measured. Accordingly, the

curves in fig. 154, which represent the results of these tests, have been computed from the records on the basis of a projected area of 72 sq. feet. Probably the best assumption that can be made with regard to the under parts of the car is that the head resistance and rear suction per square foot is the same as in the case of the flat-ended vestibules ; as the effect of this on the train resistance is comparatively small, any error introduced by the assumption cannot be great. The result for a flat-ended car, though greater than given by the formula of equation 1, is not necessarily inconsistent with it, as the eddying of the air at the sharp edges introduces an element not contemplated in the formula.

University of Illinois Tests.—The University of Illinois have recently carried out a series of tests of tractive resistance on a typical American Interurban car having a weight of 25 English tons (56,000 lbs.), an overall length of 45 feet, and an area of cross section, taken down to rail level, of about 90 sq. feet. The car was carried on a pair of 4-wheel bogie trucks, and was driven by a 4-motor equipment. The plan and cross section of its body are shown in fig. 155. The results of the tests, which contain evidence of care and skill, are shown in fig. 156.

Author's Method of Presentation of Results.—It is usual to express tractive resistance in pounds per ton, thereby implying that it is principally governed by the weight of the train. Since however this is not by any means the chief factor in the resistance, at any rate in the case of passenger trains running at high speeds, it is desirable, if intelligent estimates are to be made, to present the results in a form more in accordance with the physical facts. Some years ago the author suggested a method of treatment having this end in view ; * and on reconsideration of the whole subject in the light of more extended experience, and particularly in view of the confirmation furnished by the New York Central tests, is of the opinion that it merits reference, as being simple and derived directly from test, whilst giving results as reliable as the complex nature and our present imperfect knowledge of the subject permits. The underlying idea of the method is

* *Proceedings of Rugby Engineering Society*, vol. 2, p. 59.

that the total train resistance at any speed consists of the sum of two principal parts, one depending on the weight of the train and the other on its size and shape; and that the latter of these is again divisible into two parts, one depending principally on the length of the train and the other on the cross section.

THE EFFECT OF LENGTH OF TRAIN. Reverting to the New York Central tests already mentioned, it may be said that the results were originally published in the usual form, the train resistance in lbs. per ton being plotted against speed for each train employed. From these the curves of figs. 148 and 149 have been deduced, in which the total tractive resistance is plotted against number of coaches for several constant speeds, all within the range of the tests. A significant feature in both sets is that every curve ultimately, and very soon, becomes a straight line at its upper end; that is to say, after the first few coaches, every additional similar coach adds the same amount to the train resistance at a given speed. This important result was assumed in the author's former treatment of the subject. It is physically obvious in the case of a train running in a tunnel; but required the proof furnished by these tests for trains in the open. If now curves be plotted between speed and additional resistance per additional coach, as determined from the slope of the straight portions of the curves of figs. 148 and 149, these, in both sets of tests, will be found to be sensibly straight lines (fig. 157). The additional resistance in the case of the 45-ton coaches of figs. 148 and 151 is, in lbs., $100 + 5{\cdot}22 \times$ speed in m.p.h., whilst that in the 26·3-ton coaches of figs. 149 and 152 is $84 + 4{\cdot}28 \times$ speed in m.p.h. This result was not anticipated by the author, who would have expected the additional resistance to contain terms dependent on higher powers of the speed. The agreement between the two sets of tests in establishing the result is however sufficiently remarkable to render its general truth probable, at any rate within the range of speed considered; for it may be noted that even at the highest speed attained there is no indication of any deviation from the straight line law. The additional resistance per added coach is partly due to solid friction, and proportional to the weight, though dependent on the type of truck used, and partly to air friction on the sides and proportional to the length, though dependent on the nature of the

outer surfaces. It is probably not strictly correct to identify the two factors respectively with the two terms determining the straight line, but in the absence of information it is convenient to do so. It must not be supposed that the error introduced by this assumption produces a corresponding error in the estimation of train resistance, since the same assumption

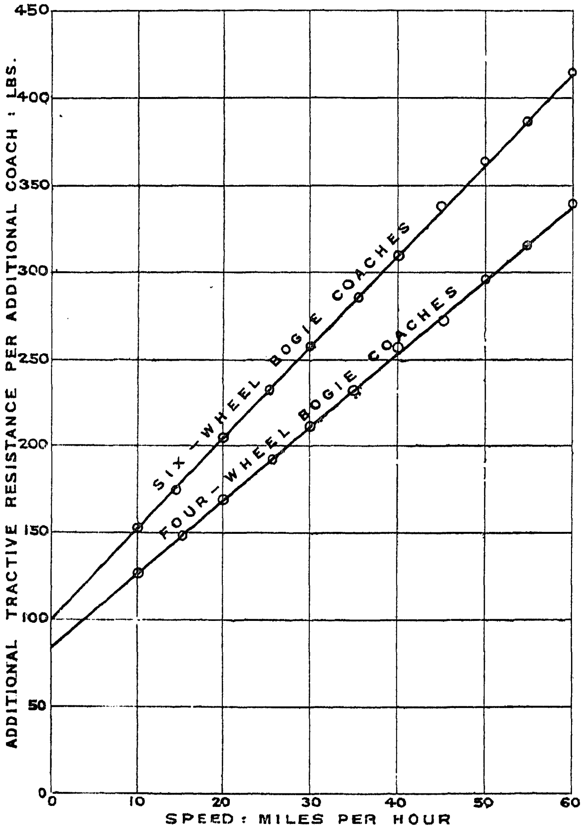


FIG. 157.—New York Central Tests. Additional Tractive Resistance per Additional Coach.

is made in obtaining the data as in using them. Thus the error introduced from imperfect knowledge of the effect of weight is largely restored, as the effect of length, particularly if the weight per foot length is substantially the same in the train considered as in the test train from which the data was derived.

In the case of the coaches of fig. 151, the additional resistance may be written : 2·22 lbs. per English ton + 0·0791 × (speed in m.p.h.) lbs. per foot length of coach, whilst that for the coaches of fig. 152 may be written : 3·18 lbs. per English ton + 0·0754 × (speed in m.p.h.) lbs. per foot length of coach. In this form the data obtained may be applied to other coaches of similar type without risk of great error. There is room for speculation concerning the causes of the differences in the constants, but with so limited a variety of circumstances such speculation could hardly lead to reliable conclusions.

Effect of Ends of Train.—The effect of length of train on the tractive resistance having been thus determined, there remains the effect of the ends, in which is included the locomotive friction, the head resistance, the rear suction, and all other special resistances distinguishing front and rear from intermediate coaches. In the present instance, actual tests were made of tractive resistance of the locomotive without the train, and the results are given in the upper curve of fig. 150. This curve, although of interest, is not of great value, since it is not a normal condition to run the locomotive without the train. Inspection of figs. 148 and 149, and particularly of the latter, shows that at low speeds the total tractive resistance of the locomotive with one coach is actually smaller than that of the locomotive alone ; the natural inference from this observation is that at low speed the first effect of the train is to steady the locomotive, causing it to run more smoothly and thus diminishing its tractive resistance. As the speed rises the effect gets smaller, until at 60 m.p.h. it appears to be inappreciable. This behaviour may be a characteristic of the type of locomotive used, a drawing of which is shown in fig. 5. If the straight portions of the curves of figs. 148 and 149 be extended, as shown in broken lines, to cut the axis of no coaches, the points of intersection with this axis determine an effective locomotive resistance, which is more useful than the actual locomotive resistance as found by test ; for this effective locomotive resistance, used in conjunction with the coach resistance of fig. 157, suffices to give the total tractive resistance of a train of any number of coaches. The effective locomotive resistance is shown, plotted against speed, in fig. 150. It will be seen that figs. 148 and 149 yield slightly different values for

this resistance, one being approximately a constant amount greater than the other throughout. This may possibly represent an actual change in locomotive resistance, due to improvement in track and journals as the tests proceeded, or it may have its origin in the difference in the coaches behind the locomotive. The mean resistance is however probably near enough to the facts for practical purposes. The value of effective locomotive resistance at low speed, amounting to $3\frac{1}{2}$ to 4 lbs. per ton, indicates that the amount chargeable to weight is greater for the locomotive than for the coaches, although the greater concentration of the weight in the locomotive might have been expected to result in an opposite effect, just as the 45-ton coaches show a proportionally lower resistance than the 26·3 ton coaches. This may be due to depression of the roadbed by the locomotive, to which the coaches do not sensibly contribute. Such action may also partly account for the high tractive resistance at low speeds usually found in tests on single coaches. There is of course much room for further tests with different types or weights of locomotive and coach, but the method of treatment here described would seem to be a fruitful one.

British Trains.—In applying these results to British conditions, an estimated allowance must be made for differences in the rolling stock, which is of necessity smaller than is used on American railways, since the normal loading gauge is smaller. In the case of trains of bogie coaches such as were employed in Mr. Aspinall's tests, some guidance may be obtained from the results found for 5-coach and 10-coach trains. The 5-coach train of weight 115·2 tons and length 284·8 feet shows a resistance of 15 lbs. per ton or 1,728 lbs. total at 50 m.p.h.; the 10-coach train of weight 218·7 tons and length 542·2 feet shows a tractive resistance of 13·2 lbs. per ton or 2,887 lbs. in all, at the same speed. Thus the addition of 5 coaches, of weight 103·5 tons and length 257·4 feet, adds 1,159 lbs. to the train resistance. The cross section of the coaches is about 85 sq. feet against 115 sq. feet for the American coaches. The term in the tractive resistance depending on the length of the coach may be assumed as roughly proportional to its perimeter, or to the square root of its cross section; and taking the effect of weight as the same in the case of English as in American coaches, the additional

tractive resistance for added coaches becomes approximately, for the English coaches :—

3·2 lbs. per ton + (0·065 × speed m.p.h.) lbs. per foot length.

Thus the additional tractive resistance corresponding to the addition of the five coaches becomes at 50 m.p.h.

$$3{\cdot}2 \times 103{\cdot}5 + 0{\cdot}065 \times 50 \times 257{\cdot}4 = 331 + 839 = 1{,}170 \text{ lbs.}$$

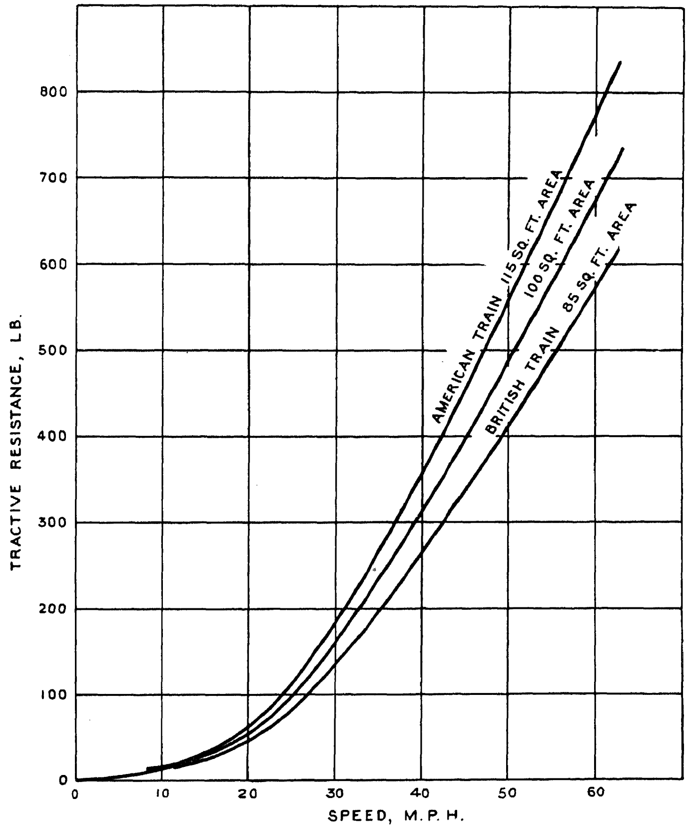


FIG. 158.—Head Resistance, Train Hauled by Locomotive.

The close agreement between this figure and that deduced from Mr. Aspinall's tests may be an accident, but it is an encouraging one ; and in the absence of further experimental confirmation justifies the adoption of the above formula to give the additional tractive resistance for added coaches of normal English 4-wheel bogie stock.

LOCOMOTIVE RESISTANCE.—The tractive resistance due to the locomotive must, in the absence of other information, be estimated from the lower curve of fig. 150, the part persisting at low speed being charged to the weight and expressed in lbs. per ton, and the remainder divided between the length of the locomotive and cross section of the train. This method would give for the effective resistance of a British locomotive about 4 lbs. per ton + (0·065 × speed in m.p.h.) lbs. per foot length + the resistance given by fig. 158, a result which may stand until direct tests have furnished a more suitable formula.

Multiple Unit Trains.—The multiple unit train may be treated in exactly the same manner as the locomotive drawn train. The additional tractive resistance per added coach

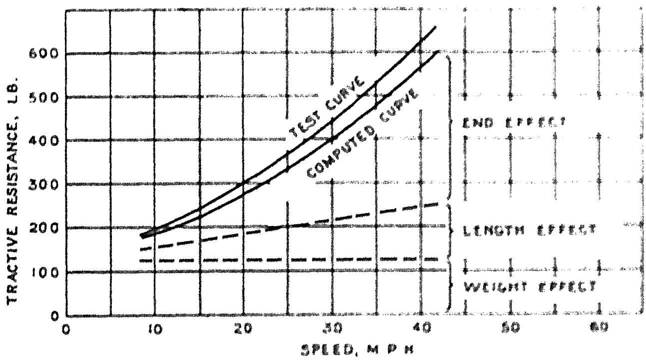


FIG. 159.—Computed Curve of Tractive Resistance.

having been determined, this amount should be charged to each coach in the train, all that remains of the measured tractive resistance being then charged to end effect. There are at present no available records suitable for analysis of this kind, but it seems reasonably accurate to compute the end effect by (1) increasing the weight effect of the first coach somewhat, and (2) using the results of the Louisiana tests (fig. 154) to give the effect of the air resistance, the truck portion of the cross section being treated as flat surface. Thus the 25-ton coach of fig. 155, which has a total cross sectional area of approximately 90 sq. feet, of which about 75 sq. feet is in the vestibule, yields the computed curve of fig. 159, the weight effect having been taken at 5 lbs. per ton (for American interurban cars are for some reason high in weight effect as

compared with railway coaches), the length effect at (·067 × speed) lbs. per foot length, and the air resistance from fig. 154 by taking five-sixths of the cross section as standard end and one-sixth as flat end. Similarly fig. 160 gives computed tractive resistance of the Berlin-Zossen coach, the weight effect having been taken at 3 lbs. per ton, the length effect at (0·077 × speed) lbs. per foot length and the air resistance effect from fig. 154, curves for flat ends and for five-sixths standard and one-sixth flat both being shown, in addition to that derived directly from test. The computed curves are in reasonable agreement with the test results, and where the effect of added

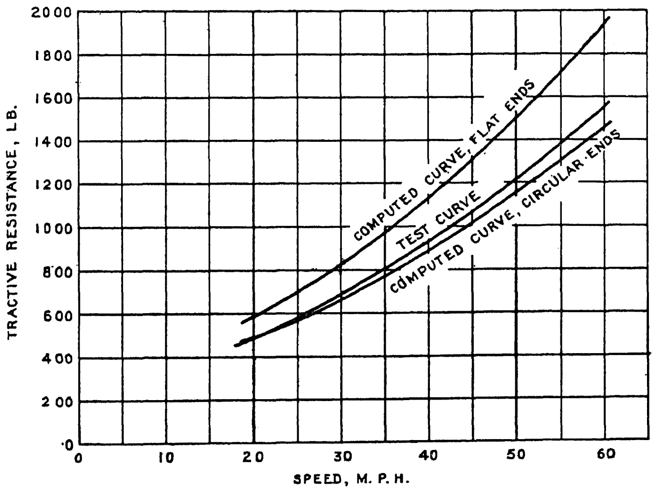


FIG. 160.—Computed Curve of Tractive Resistance.

coaches can be estimated with fair accuracy the total tractive resistance of a train can be computed without risk of great error. Until better data is available therefore the method indicated is acceptable and is much less likely to lead to serious error than the usual applications of results expressed in lbs. per ton, derived from tests on particular trains, but used without reference to the weight and conformation of the trains under consideration.

GOODS TRAINS.—The resistance of a goods train behind its locomotive appears to vary but little with the speed, particularly when loaded. Fig. 161 gives the results of tests by Mr. A. C. Dennis and shows little change in resistance after

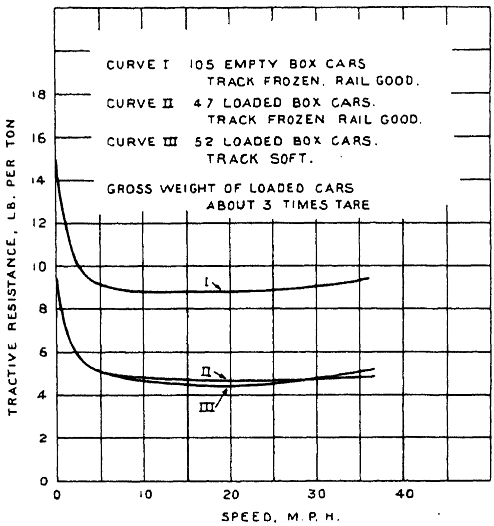


FIG. 161.—Tractive Resistance of Goods Train.

the initial fall, whilst the following table gives train resistance of loaded cars of various weights and is stated to be substantially true between 5 and 30 m.p.h.

TABLE 10

Weight of Loaded Car.		Train Resistance.	
English tons.	American tons.	Lbs. per English ton.	Lbs. per American ton.
17·8	20	8·80	7·84
22·3	25	7·42	6·62
26·8	30	6·48	5·78
35·6	40	5·21	4·65
44·6	50	4·42	3·94
53·5	60	3·85	3·44
62·5	70	3·43	3·06
64·3	72	3·36	3·00

It may be inferred then that the variation of tractive resistance with speed is not important within the usual limits of speed of goods trains, it being always understood however that the starting resistance is of the order of 20 lbs. per ton.

Resistance tests on English 4-wheel goods wagons were carried out in 1893 on the London and North-Western Railway between Rugby and Willesden. The train consisted of fifty-seven 10-ton coal wagons and three brake vans, the average tare weight of each wagon being 5·4 tons and its load 7·44 tons, making the total train weight 772 tons. The mean speed was 16·5 miles per hour and the mean train resistance found was 6 lbs. per ton. Similar tests conducted on the New York, Ontario and Western Railway on a train of twenty-four 8-wheel bogie goods wagons, each weighing on the average 10·7 tons tare and carrying a load of 23·4 tons, gave a mean train resistance of 3·5 lbs. per ton at a mean speed of 19·0 miles per hour. These tests were taken behind a locomotive and dynamometer car, so that the resistance of the locomotive should be added in any use made of them for electric railway work.

Tractive Resistance of an Electric Train when Coasting.—The tractive resistance discussed above does not include that due to friction in the motors and mechanical transmission gear between motors and wheels. This portion of the resistance is always, and properly, charged to the motors; for it is clearly not a function of speed alone, but also of the power that is being transmitted to the wheels. During the time that the train is taking power therefore the tractive resistance discussed above is appropriate, the other frictional elements being allowed for in the motor characteristic curves. When however the train is coasting the motors are being driven, and their frictional resistance then becomes an addition to the tractive resistance of the train. The amount of this additional resistance is easily determined by driving the motor light from the wheel axle or the equivalent shaft of the testing stand. Fig. 162 shows the amount of this resistance in the case of a 250 H.P. motor having 70/22 gear and 42-inch wheels. There are therefore two resistances which come into the discussion of an electric railway problem, the true tractive resistance which should be used while the motors are taking power, and a greater figure, which should be used when the train is coasting, and which is composed of the true resistance with an addition to provide for the friction of the driven motors. The difference between them is, however, practically negligible when axle-borne, gearless motors are used.

Determination of Tractive Resistance.—There are two methods by which tractive resistance of electric trains can be determined: one by measuring the input to the motors at any speed and taking the corresponding tractive effort from their characteristic curves, as obtained by shop tests; and the other by allowing the train to coast, and determining the resistance from the rate of retardation. In the former method it is desirable to have the train under-motored for two reasons. In the first place, the tractive effort is the more definitely known the greater the load; for at light load the gear and friction

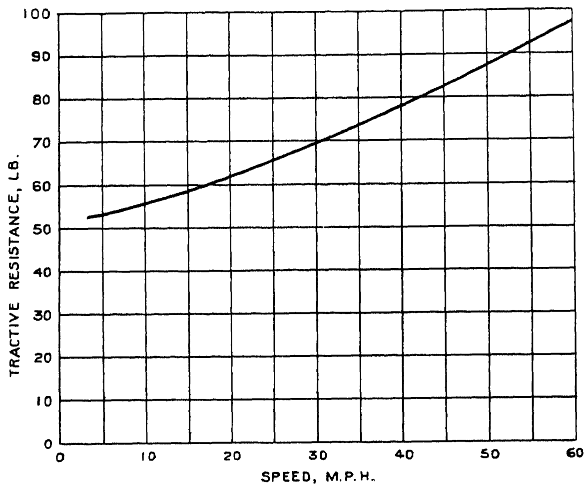


FIG. 162.—Tractive Resistance due to 250 H.P. motor with train coasting.

losses are proportionately great and very uncertain, varying considerably from motor to motor. In the second place, uniformity of speed is more quickly reached when the load is large; for with motors as ordinarily geared a long distance must usually be traversed before the speed becomes sensibly uniform, and accuracy in the results requires that it shall not be necessary to make a large correction for acceleration, since this is difficult to measure with certainty. The best profile on which to make this type of test is a level stretch, as long as practicable, with a rising gradient at each end on which speed can be gained in either direction. The readings required are the volts and current for each motor, the speed, the rate of acceleration and the gradient, if any. Characteristic curves

should be taken on each motor, preferably with its own gears, at about the correct voltage, although, as the variation of tractive effort at a given current is small, and in fact usually negligible, exactness in respect of the voltage is not of primary importance. The motor speed should be taken as a check on the observed train speed, the characteristic curves being plotted for the correct gear reduction and size of wheel. The total tractive effort so obtained, diminished by that required to produce the acceleration and by that required to overcome gradient, is the tractive resistance of the train under the conditions of the test.

The commoner method of determining tractive resistance is however that of bringing the train to speed and then allowing it to coast without power, measuring the rate of fall of speed in order to determine the resistance to the motion. If v is the velocity of the train and M its total mass, the kinetic energy of translation is $\frac{1}{2}Mv^2$. If r is the radius of a wheel, its angular velocity is v/r; and if I is the moment of inertia of a pair of wheels and axle, the kinetic energy of their rotation is $\frac{1}{2}I\left(\frac{v}{r}\right)^2$, whilst if a gear of moment of inertia i is carried on the axle its kinetic energy of rotation is $\frac{1}{2}i\left(\frac{v}{r}\right)^2$. If γ is the ratio of gear reduction and I' the moment of inertia of an armature about its shaft, the kinetic energy of rotation of the armature is $\frac{1}{2}I'\left(\gamma\frac{v}{r}\right)^2$. Hence the total kinetic energy of the train may be written:

$$T = \frac{1}{2}\left(M + \Sigma\frac{I}{r^2} + \Sigma\frac{i}{r^2} + \Sigma\frac{\gamma^2 I'}{r^2}\right)v^2 \qquad . \quad (2)$$

the summations being taken over all wheels, driving and trailing, all gears, and all armatures. The quantity within brackets is known as the effective mass of the train. Calling this M', equation 2 may be written:

$$T = \tfrac{1}{2}M'v^2 \qquad . \quad . \quad . \quad . \quad (3)$$

The decrease of kinetic energy per unit distance coasted is the value of the total opposing force; this is therefore $M'a$ where a is the rate of retardation. Accordingly, if M' is determined for the train, and a is measured at any particular speed, the total

resistance to motion can be obtained. In order to determine tractive resistance, correction must be made for any gradient (and this correction is proportional to M, not to M′), and also for the frictional resistance due to the motors, which must be determined by separate test on the motors as explained above. Instruments for measuring acceleration or detardation directly, necessarily include the effect of gradient with the acceleration. If the gradient is 1 in $1/\beta$, β being positive when opposing the train, the total frictional resistance to motion is:

$$\begin{aligned} F &= M'\alpha - M\beta \\ &= M'(\alpha - \beta) + (M' - M)\beta \qquad . \qquad . \qquad (4) \end{aligned}$$

When the instrument is so adjusted as to read zero with the coach standing on level track, its indication is: $\delta = \alpha - \beta$. If δ is read as the effective gradient, the masses being expressed in English tons, the total tractive resistance, including that due to gradient and that required to drive the motors from the wheel axles, is, in lbs.:

$$F = 2{,}240[M'\delta + (M' - M)\beta] \qquad . \qquad . \qquad (5)$$

If however the instrument reads acceleration directly, in miles per hour per second, the formula for the tractive resistance becomes:

$$F = 102\,M'\delta + 2{,}240(M' - M)\beta \qquad . \qquad . \qquad (6)$$

THE MEAN TRACTIVE RESISTANCE.—In rough calculations of energy consumption, it is often desirable to form an approximate estimate of the mean tractive resistance of a train in order to avoid the labour of detailed calculation from a train resistance curve. In this connection the following theorem is of importance:

"The mean tractive resistance of a train is greater than the tractive resistance at the mean running speed."

Proof: Let fig. 163 be the tractive resistance curve, let P be the point corresponding to the mean speed V and let F be the resistance at P. Draw a tangent to the curve at P and let α be its slope, or the tangent of the angle that it makes with the horizontal, and f the amount by which the ordinate of the curve at any point Q exceeds the corresponding ordinate of the tangent. Then α is positive as long as the mean speed exceeds the low value at which the tangent drawn is horizontal, and f is positive at all points, since the curve is convex towards

the positive axes. Then if the speed at Q is $V + v$ the train resistance at this point is $F + a\ v + f$. The work done

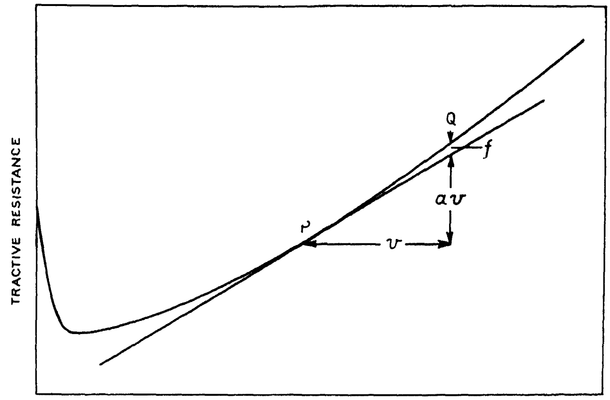


FIG. 163.—Tractive Resistance.

against train resistance in the total time T to which the mean speed V applies is :

$$F'VT = \int_o^T (F + av + f)\ (V + v)dt$$

$$= FVT + F\int_o^T vdt + aV\int_o^T vdt + a\int_o^T v^2dt + \int_o^T f(V + v)dt$$

the several integrals being taken through the range determined by the speed-time curve of the train for the interval of time T. As v is the deviation from the mean speed :

$$\int_o^T vdt = 0$$

The mean train resistance is therefore :

$$F' = F + \frac{a}{VT}\int_o^T v^2dt + \frac{1}{VT}\int_o^T f(V + v)dt \quad . \quad (7)$$

The quantities a, v^2, f, and $V + v$ are all positive at all times ;

thus the mean train resistance exceeds the train resistance at mean speed by the positive amount :

$$F' - F = \frac{a}{VT}\int_o^T v^2 dt + \frac{1}{VT}\int_o^T f(V + v)dt \quad . \quad (8)$$

The limitation imposed on V to make a positive is of no practical consequence, so that the theorem may be taken as established for all cases for which it is likely to be required. The practical effect of the theorem is that if the mean speed, excluding stops, is say 30 m.p.h., the mean train resistance should be estimated to correspond to 35 or 40 m.p.h. according to the extent of the variation of speed from the mean. Similar considerations would show that the average addition to tractive resistance due to wind is, for trains in both directions, positive.

Effect of Curves.—The additional resistance due to curves is, for most railways, so occasional in its incidence that its exact estimation is not a matter of importance. This is fortunate, as it is doubtful whether a reliable experimental determination is available, or indeed practicable unless a circular truck were especially laid for the experiment. If r is the radius of the curve, in traversing a distance l the train turns through angle l/r ; if b is the gauge of the track, the outside wheels go bl/r farther than the others ; if w is the weight of the train, supposed equally distributed between inside and outside wheels, μ the co-efficient of friction and F the tractive resistance due to the curve, in lbs. per ton, the work done is :

$$wFl = 2{,}240\,\mu\frac{w}{2}\;\frac{bl}{r}$$

$$\text{or } F = 1{,}120\,\mu b/r \quad . \quad . \quad . \quad (9)$$

This portion of the curve resistance accordingly varies inversely as the radius ; and this is the form in which the formula is usually presented. The actual values however do not agree with equation 10, for the constant 1,120 μb can hardly be greater than 1,500, whereas it is usually taken as about 6,000. Part of this discrepancy is perhaps due to flange friction, part to increased bearing friction and part to energy expended in giving angular velocity to the train and afterwards destroying

it. However, the subject is one on which further investigation is necessary before final conclusions can be expressed with propriety.

Energy-Output of Motors.—For every pound per ton tractive resistance the work done by the train motors per ton mile is 5,280 foot-lbs. or

$$\frac{5{,}280 \times 746}{33{,}000 \times 60} = 2 \text{ watt-hours approximately.}$$

It has been shown above (see equation 3) that the kinetic energy of the train when running with velocity v is:

$$T = \tfrac{1}{2} M' v^2.$$

If v is in miles per hour and the kinetic energy is expressed in watt-hours per ton actual weight:

$$T = \tfrac{1}{2} \times \frac{2{,}240}{32{\cdot}2} \times \left(\frac{5{,}280}{3{,}600}\right)^2 \times \frac{746}{33{,}000 \times 60} \frac{M'}{M} v^2$$

$$= 0{\cdot}0283 \frac{M'}{M} v^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The ratio M'/M usually varies between 1·08 and 1·12. If n is the number of stops per mile and v the speed at which brakes are applied, the whole output of the motors on straight and level track, reckoned in watt-hours per ton-mile, is practically:

$$W = 2 \times \text{mean train resistance (lbs. per ton)} + {\cdot}0283 \frac{M'}{M} v^2 n \quad (11)$$

Energy-Input.—In most actual cases, a number of the quantities involved in equation 11 will be mere estimates, and no very great accuracy is to be expected from the calculation. The equation is however useful, as showing in a general way how the energy is expended; and, used judiciously, is a valuable means for obtaining a rough approximation to the energy consumption. For the latter purpose, a suitable figure for the mean efficiency of the equipment is required. The true efficiency cannot be measured, as there are no means of determining the true output of the motors in service. It can however be computed from the results of stand tests, as will be shown in the next chapter. For suburban service and continuous current motors, its true value is usually of the order of 80 per cent. However, it is generally more convenient to use a false efficiency in this connection, in order to make

allowance for certain circumstances and contingencies unavoidable in service, of which it would be tedious to take direct account. The appropriate figure depends on the purpose for which the calculation is required; but in general a rough method such as this would only be used to obtain an approximate idea of the energy consumption in service. The curvature of the track increases the tractive resistance above that appropriate to straight and level track, and the wind has a like effect: the gradients also often increase the energy consumption somewhat. These factors may conveniently be taken into account in the efficiency. The loading of the train is often conjectural, and the tonnage is conveniently taken as that of unloaded trains, although equation 11 implicitly assumes the actual load. Unscheduled stops and signal slacks require further energy, and this is again increased when the consequent loss of time has to be made up. It is convenient for purposes of rapid calculation to assume figures in equation 11 to correspond with schedule runs on straight or level track, and to assume a false efficiency, of the order of 70 per cent., in order to allow for unaccounted energy consumption. It will be realized, however, that from the nature of the case no figure appropriate to all circumstances can be given for this false efficiency; since it depends on the nature of the traffic and on many local circumstances which affect the manner of running.

The foregoing method is able to furnish a very good approximation to the energy consumption if a representative speed-time curve for the service can be obtained, otherwise it is difficult to form an estimate of the speed at which brakes are applied in the representative run, and error in this quantity will affect the results considerably when stops are frequent. Messrs. Del Mar & Woodbury * have discussed the problem on lines somewhat similar to the above, but have given greater precision to the calculation where no speed-time curve is available by transferring disputable quantities from the realm of estimate to that of average experience. In their discussion of the subject the output of the motors is estimated for the time that they are taking power only, which should of course be the same in total as above, but with a different distribution between work done and kinetic energy. Two constants are

* *Electric Railway Journal*, vol. 42, p. 1055.

introduced taken from average experience; one, Q, is the ratio of the distance between stops to the distance travelled with power on, and the other, K, is the ratio of the maximum speed (at which power is cut off) to the mean speed whilst running. Equation 11 for the output then becomes:

$$W = \frac{2}{Q} \text{mean tractive resistance (lbs. per ton)} + \cdot 0283 K^2 \frac{M'}{M} v^2 n \qquad \text{. . . . (12)}$$

and the energy consumption is deduced by dividing W by the appropriate efficiency figure. The velocity v which must be used in computing W is now the mean running speed of the train, a quantity which is of course known. Fig. 164 shows

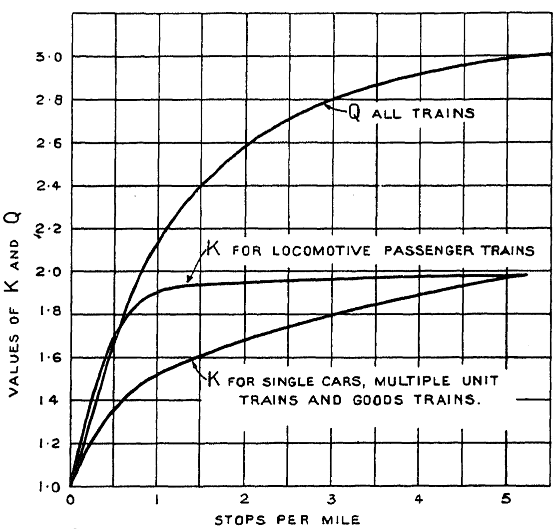


FIG. 164.—Del Mar and Woodbury's Factors.

the curves for K and Q, as given by Messrs. Del Mar & Woodbury. It may be mentioned that a quantity of the nature of Q representing the ratio of the distance between stops to the distance excluding braking is required by the reasoning in equation 11, but as this quantity differs little from unity its effect is inappreciable compared with the error introduced by uncertainty in the data employed.

HEATING OF MOTORS.—The losses that contribute to the heating of the motors amount in suburban service to some

7 per cent. of the total input to the driving equipment when continuous current motors are employed, and to·15 to 18 per cent. with single-phase commutator motors. With such motors as can be carried on the axles of motor coaches, the amount of heat that can be dissipated with an internal temperature rise in the neighbourhood of 65° C. is, for completely enclosed motors, from 2,000 to 3,000 watts, according to the size of the motor and the schedule speed of the train, whilst double this amount may be dissipated from ventilated motors with the same temperature rise. Thus a rough estimate of the heating of the motors in continuous service may be made to correspond with the estimated energy consumption.

The Principle of Equivalence.—Where nothing is known about a service other than the schedule speed and frequency of stops, the above methods are capable of giving as reliable results as are to be obtained ; and the method is valuable also in furnishing a means of comparison between different schedules as regards energy consumption ; at the same time emphasizing the fact that the energy consumption is chiefly a question of schedule, and only to a small extent a matter of equipment, assuming this adequate. Too much insistence cannot be laid upon the fact that it is not possible to make comparison between the efficiencies of different equipments merely by comparison of their energy consumption in radically different services ; much more than the schedule of the service must be known before a fair judgment can be made between the respective merits of the equipments. Where complete information is available of the schedule, the equipment and the circumstances of the service and route, the energy consumption can be computed with considerable accuracy and general methods for effecting this will be discussed in the next chapter ; but there frequently arise cases in which the available data consist of the general schedule of the service, the approximate weight of the train, and dynamical characteristic curves of the motor ; and in which there is desired a good representative figure for the energy consumption, taking account of the chief elements of data. This information can be obtained without the labour of individual calculation, by means of general curves which are computed without great difficulty to suit any particular type of motor, e.g. the continuous current series

motor. The underlying principle which makes this treatment of practical value is a certain principle of equivalence according to which all speed-time curves having the same shape can be treated together and represented by a single point on the curves of energy consumption.

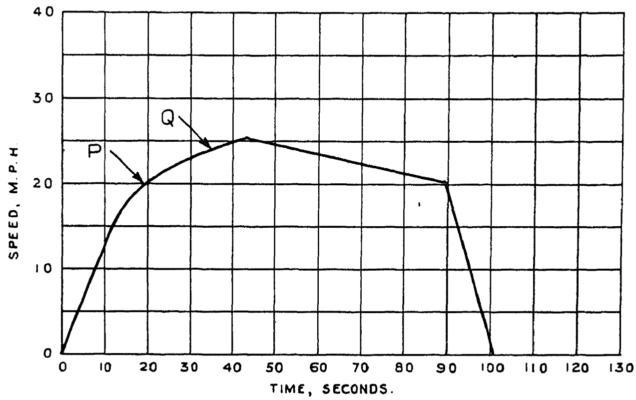


FIG. 165.—Schedule Run, illustrating principle of equivalence.

In order to demonstrate the principle of equivalence referred to, consider the two-speed time curves of figs. 165 and 166; these are similar in shape, every linear dimension in fig. 165

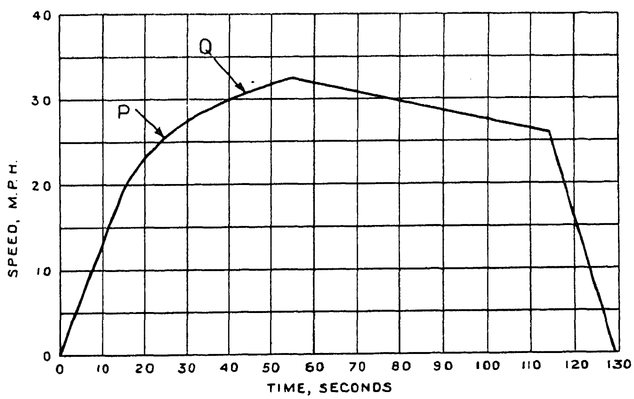


FIG. 166.—Schedule Run, illustrating principle of equivalence.

being increased in a certain proportion (x) to give fig. 166. If t is any time and s any speed in the first figure and t' s' the corresponding quantities in the second:

$$t' = xt. \qquad s' = xs.$$

If d is the distance run in the first figure, which is of course represented by its area, and n the number of stops per mile, d' and n' being the corresponding quantities in the second :

$$d' = x^2 d \text{ and } n'd' = nd = 1 \text{ mile.}$$

Hence :

$$x = \sqrt{\frac{n}{n'}},$$

and

$$t\sqrt{n} = t'\sqrt{n'}$$
$$s\sqrt{n} = s'\sqrt{n'}$$

Thus, if instead of using time as abscissa and speed as ordinate in figs. 165 and 166, the abscissa be taken as

$$\text{time} \times \sqrt{(\text{stops per mile})}$$

and the ordinate as

$$\text{speed} \times \sqrt{(\text{stops per mile})},$$

the two curves reduce to one.

ENERGY CONSUMPTION.—Consider any point P in fig. 165 and the corresponding point, P, in fig. 166. The slope of the curves is the same at these corresponding points, and therefore the rate of acceleration is the same, and also the accelerating tractive effort per ton. Since the train resistance is assumed the same, the total tractive effort is, to the assumed degree of approximation, the same at corresponding points. The power per ton at corresponding points, varying as tractive effort and speed, is therefore in the ratio 1 to x in the two curves, whilst the increment of energy between corresponding points PQ and P′Q′, varying as the power and as the increment of time, has the ratio 1 to x^2. This however is the ratio of the total distances represented, and thus, dividing by distance, the energy consumption per ton mile is the same in the runs represented by the two curves, since the efficiency is assumed the same at corresponding points. Thus all speed-time curves of definite shape consume the same energy per ton mile, to the degree of approximation determined by the accuracy of the assumptions involved.

In this, as in any such general treatment of the subject, a number of quantities are assumed at representative average values, and to the extent that these values differ from the

appropriate ones for the problem under consideration the method is imperfect, although correction can usually be applied to the results to allow for any known deviation. In the following example of the method, which is intended to apply particularly to continuous current motors in suburban service, the shape of the characteristic curves is that given in fig. 44. By this is implied, not necessarily that the motor to which these curves apply is used, but that it is possible to choose scales of ordinate and abscissa which would make the curves coincide with those of this figure. This is substantially justified for any normal motor of the kind that would be employed for the service in question. The tractive resistance of the train is taken at 14 lbs. per English ton throughout, and the coasting resistance at 20 per cent. higher figure, viz. 16·8 lbs. per English ton; the rate of braking is taken at 1·5 m.p.h. per second; the effective weight of the train is taken as 10 per cent. in excess of the actual weight. The data supplied for a problem are usually the following: (1) The distance between stops, or the number of stops per mile; (2) the time from start to stop, or the mean running speed; (3) the weight of the train per motor. The usual problem is to find a suitable motor; but in some cases the motor may be specified and the appropriate schedule sought. The shape of any speed-time curve according with the assumptions, depends upon three factors, which may be taken as: (1) the tractive effort during acceleration on rheostat; (2) the speed at which the speed curve is reached; (3) the proportion of the total time that power is on. Only a certain range of shapes however represent a given schedule, viz. those which, when plotted to represent a run of the correct distance, make the run also in the correct time. These may be considered to be distinguished one from another by the margin whereby the time of running the distance exceeds the minimum possible time, that is, the time of running it when power is kept on until brakes are applied.

Energy Consumption Horse-power and Tractive Effort.—If an equipment is capable of making a certain schedule, the consequent energy consumption will be found, except in extreme cases, to vary but little with the margin of time in hand. It is therefore possible to give the energy consumption

in the form of a single set of curves (fig. 167), without greater error than is introduced by the unavoidable defects of the method. Assuming the mean running speed given, the variable parameter determining the energy consumption is taken as the tractive effort during acceleration on rheostat. It is here assumed, however, that the equipment is capable of performing the service with sufficient margin; whether this is the case or not can be determined from the curves of figs.

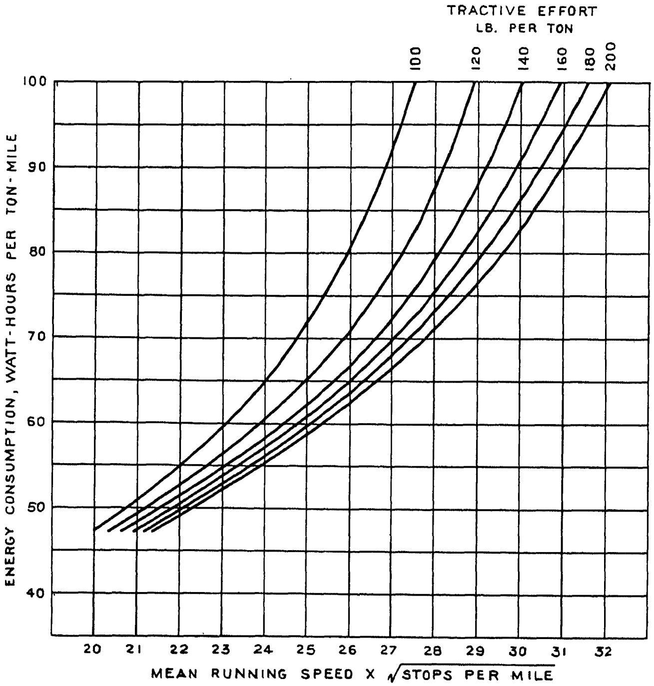


FIG. 167.—Energy Consumption, General Curves.

168, 169 and 170, which give the speed of reaching the speed curve for margins of 5 per cent., $7\frac{1}{2}$ per cent. and 10 per cent. respectively. The curves are given with both tractive effort during acceleration, and with horse-power on reaching the speed curve as variable parameters; so that a solution may be found with the data given in various forms. In many cases the accelerating horse-power here considered is not very different from the rated horse-power of the motors; accordingly, if the motor rating is given, the problem corresponding to any

margin can be solved approximately. It will be seen from the curves that when the accelerating power of the motors is given, no very wide range of mean running speed is practicable with change of gears, within the practicable limits of variation.

Use of Curves.—The use of the curves presents no difficulty, but a typical example may nevertheless be useful. Let the average distance between stations be 3,000 feet, and let it

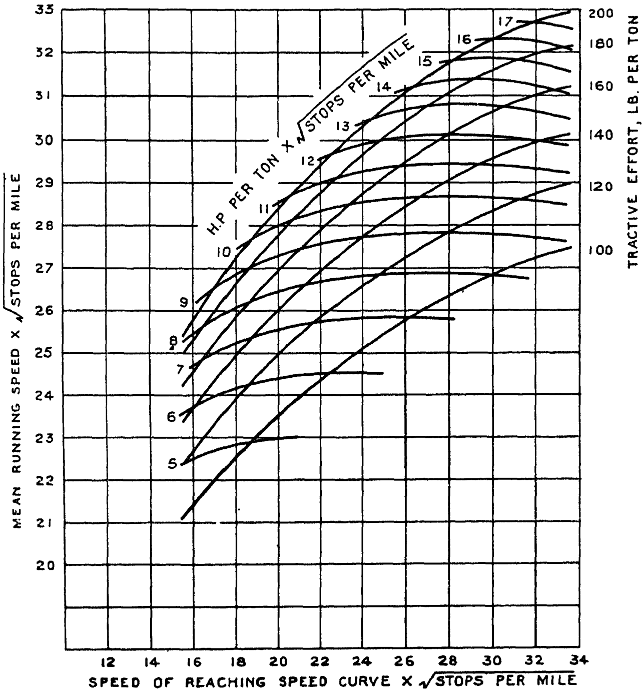


FIG. 168.—Capacity of Motor, 5% Margin, General Curves.

be required to run at a schedule speed of 18 m.p.h., including stops of 20 seconds at all stations; let the weight of the train be 20 tons per motor. The number of stops per mile is 5,280/3,000 or 1·76. The time of running 3,000 feet, including the stop, is: 3,000 × 3,600/18 × 5,280 or 113·6 seconds; and excluding the stop, 93·6 seconds. The mean running speed is therefore 3,000 × 3,600/93·6 × 5,280, or 21·9 m.p.h., which multiplied by $\sqrt{1{\cdot}76}$ gives 29 m.p.h. With $7\frac{1}{2}$ per cent. margin, the accelerating horse-power per ton × $\sqrt{1{\cdot}76}$ is from

11·5 to 12. Thus the actual accelerating horse-power per motor is 175 to 180. With the motor of greater power, the accelerating tractive effort approximates to 200 lbs. per ton, and the energy consumption on schedule run to 77 watt-hours per ton-mile; with the motor of less power, the accelerating tractive effort is about 150 lbs. per ton, and the energy consumption about 85 watt-hours per ton-mile. The choice of

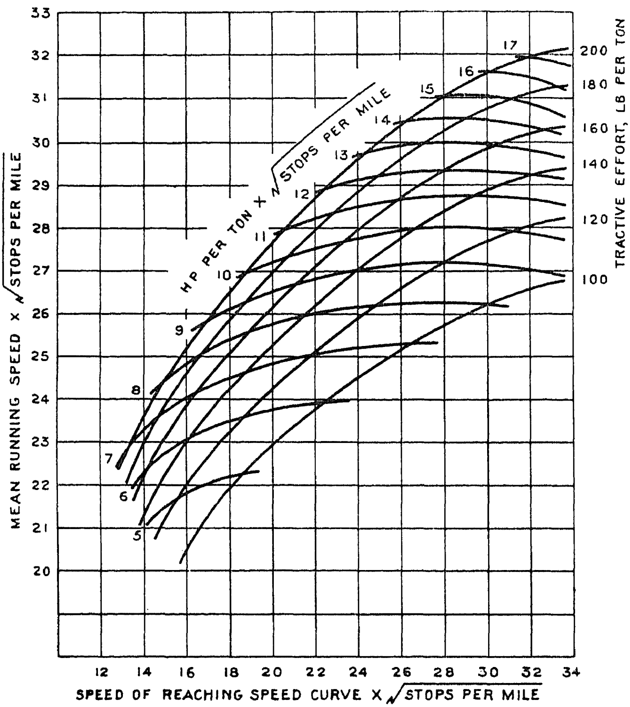


FIG. 169.—Capacity of Motor, 7½% Margin, General Curves.

motor and appropriate gear is not made from consideration of energy consumption alone; for as regards the gear, there is a mechanical lower limit to the number of pinion teeth; and in any case the weight on driving wheels limits the permissible tractive effort. In the present instance, a tractive effort of 200 lbs. per ton, in a multiple-unit train, would require a weight of about 12 tons per driving axle, which should be reckoned without load; and this is somewhat excessive for British coaching stock.

Such a set of curves as the above is capable of giving much valuable information concerning the general performance of electric trains. For instance, consider a train accelerating at 120 lbs. per ton to a speed of 24 m.p.h. A run of one mile is made by the train at a mean speed of 25·8 m.p.h., with $7\frac{1}{2}$ per cent. margin (fig. 169) and with energy consumption 70 watt-hours per ton (fig. 167). A run of two miles accelerating to equivalent speed ($24/\sqrt{2}$ or) 16·95 m.h.p. is made at a mean

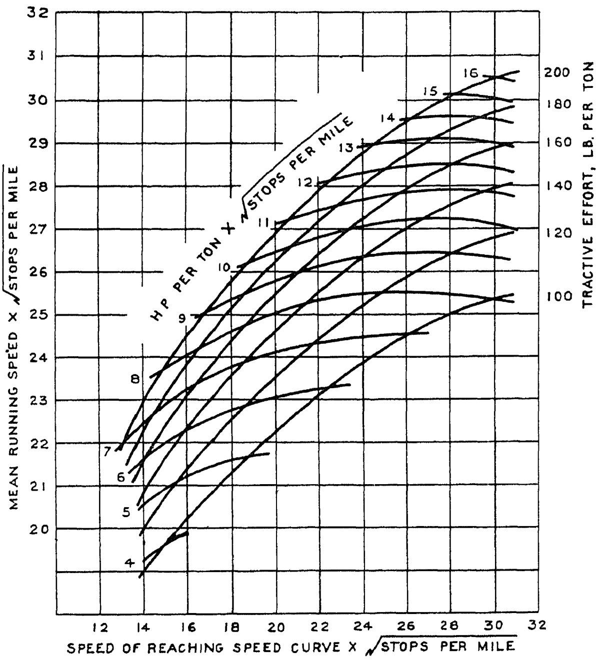


FIG. 170.—Capacity of Motor, 10% Margin, General Curves.

equivalent speed of 22·6 m.p.h. or an actual mean speed of 32·0 m.p.h., with energy consumption 55 watt-hours per ton-mile. If now a distance of three miles is made in two runs, one of one mile and the other of two miles, the time for the one-mile section is 139·5 seconds, and the energy consumption per ton 70 watt-hours, and for the two-mile section the time is 225 seconds, and the energy per ton 110 watt-hours. Thus the total running time for the three miles is 364 seconds, and the energy consumption 180 watt-hours per ton. The mean

running speed for the three miles is therefore 29·6 m.p.h., whilst the energy consumed amounts to 60 watt-hours per ton-mile. Referring again to figs. 169 and 167, in order to determine the running of the train on one and a half mile sections, the acclera-tion is now taken to equivalent speed ($24\sqrt{2}/\sqrt{3}$ or) 19·6 m.p.h., which yields a mean equivalent running speed of 24 m.p.h. or an actual mean speed of 29·4 m.p.h., with energy consumption 60·5 watt-hours per ton-mile. Thus the figures for mean speed and energy consumption per ton-mile deduced from the two sections of one mile and two miles are substantially in agreement with those for the mean section of one and a half miles. This is a particular case of a very important practical principle, to which reference has already been made, viz. that if a distance is covered by an electric train in a number of runs of different lengths, the average speed and energy consumption per mile are substantially those pertaining to a run of the mean length made with the same train.

The curves given furnish good average results for suburban service with continuous current motors, and similar curves could readily be constructed for other service or type of motor. The results taken from the curves may be corrected approxi-mately if the conditions do not exactly agree with the assumed data. Thus if the mean train resistance be estimated at other than the figure given the difference in energy consumption may be allowed for at the rate of about 2·7 watt-hours per ton-mile for each pound per ton difference in train resistance. Again, if the motor efficiency differs largely from that given in fig. 44, a corresponding change can be made in the energy consumption. The curves give results which correspond with runs made strictly in schedule time by a trained driver on straight and level track, and some allowance should be made for the exigencies of actual service, the delays, the inefficiency of the driving, the gradients, etc. This may add to the energy consumption anything from 5 per cent. to 15 per cent., according to the nature of the line and the traffic conditions. The curves as they are given are suitable for comparative purposes, but appropriate allowance should be made where they are to be used to estimate the energy consumed in actual service.

The equivalence of similar speed-time curves with regard to energy consumption per ton-mile seems to have been first

noticed by the late Mr. E. H. Anderson, who as long ago as 1901 constructed certain curves involving this principle from which the energy consumption and heating of tramway motors for any schedule could be deduced approximately. Mr. Anderson's curves were computed for a distance of 1,000 feet; and times and speeds were reduced to correspond with this distance, by dividing the actual times and speeds by the square root of the distance in thousands of feet. Instead of the margin of time in hand, the time coasted, as a percentage of the running time, was chosen to distinguish the manner of running.

CHAPTER IX

MECHANICS (*continued*)

In the preceding chapter, the factors which affect the dynamical performance of a train were discussed together with methods for making approximate estimates of energy consumption and motor heating. It is the purpose of the present chapter to develop more exact methods for determining the performance of a given train, driven by a motor equipment whose characteristics are known, and for ascertaining whether the equipment is suitable for a specified service, both from the dynamical and thermal points of view. These calculations are apt to be at once the commonest and most important that the engineer who has occasion to deal with the fundamental problems of the subject is called upon to make. They constitute the chief means by which the most suitable equipment for a particular service is determined, and enable the power requirement of the service to be estimated. Thus they provide the materials on which a discussion of the most satisfactory and economical manner for working the traffic can be founded.

Point-to-Point Method. The first method to be discussed is the point-to-point method—a simple and straightforward application of elementary mechanical principles, involving little of the element of estimation, and thus providing little opportunity for the exercise of bias. It applies equally well to any system of operation, and is capable of taking account of all essential features of a problem. The point-to-point method can be employed for practically any problem of train movement, but it is particularly applicable to the discussion of the average schedule of a service, and to the comparison of equipments on the basis of average schedules.

The present subject differs from most in that a comparison of performances requires greater accuracy in calculation than

does an absolute determination, for the latter is affected by uncertainty arising from causes which may be considered to influence both elements of a comparison to an equal extent. Thus great accuracy is wasted in a mere determination of performance, but is necessary if a true comparison between performances is to be made. If, for instance, a run of a mile were under discussion and a first approximation were found to represent a distance exceeding the mile by 50 feet, practically no greater certainty as to the actual performance would accrue from closer investigation, but a comparison affected by an excess of 50 feet in one element might lead to quite an erroneous conclusion as to the merits of the things compared. Refinements of calculation are accordingly justified in a comparison, which are quite unnecessary in the estimation of actual performance. On the other hand, the fundamental data need not be determined with such nicety, for as long as they approximate to the facts a comparison will not usually be sensibly affected by their inaccuracy.

The usual problem that it is the object of the calculation to solve is that of finding the most economical compromise among the varieties of train and equipment that can be used to meet certain conditions. This is far too complex a problem to permit of solution by any direct method, and the final determination has necessarily to be made by some method of trial. Whatever be the ultimate object of the calculation therefore, the immediate problem is that of determining what a given train, driven by an electrical equipment whose characteristics are completely known, is capable of; whether for instance it is able to run to a given schedule, and if so to determine the characteristics of the running, the speed, the margin of time in hand, the energy consumption, and the temperature of the motors; or what schedule it is expedient to impose on the train, allowing a suitable margin of time in hand to provide against contingencies not taken into account in the calculation.

The characteristic feature of the point-to-point method of solution is the adoption of speed as independent variable, and the calculation of increments of time, distance, energy consumption and motor loss to correspond with given increments of speed. The smaller the increments of speed are taken the more accurate will be the computed values and curves, but for practical purposes half-a-dozen increments between rest

and free-running speed will usually be found sufficient. If the calculation is made systematically, it nowhere presents difficulty or unusual chance of error. In fact, although mistakes are very undesirable in any calculation, a method of computation by increments is less likely than most to yield results grossly in error, for an arithmetical mistake usually affects a few increments only and does not of necessity affect the resulting sum greatly.

Data concerning Train and Service.—The following data concerning the train and its equipment are required in order that the general performance may be determined: (1) the weight of the train, including load and equipment; (2) its effective weight whilst accelerating; (3) the weight on driving wheels, with and without load; (4) such information concerning its size, nature and general composition as will enable the train resistance to be estimated for all running speeds; (5) the normal braking force or the average rate of braking retardation; (6) the average line voltage; (7) the number of motors; (8) the speed of the train for any motor current and running condition; (9) the corresponding motor speed, or the gear reduction and size of wheels; (10) the corresponding tractive effort as measured at the tread of the driving wheels; (11) the negative tractive effort due to the friction of motors and gears when the train is coasting; (12) the method of control; (13) the resistance of the several motor windings, the core losses, the load losses, the friction losses, and such other information as will enable the total loss in each motor under any running condition to be determined; (14) the thermal dissipation curves of the motor. The data necessary concerning the service when a general investigation of average performance is desired are: (1) the total distance to be run on schedule; (2) the total time allowed if the train has to run to a specified schedule, or (3) the margin of time required in hand if the schedule has to be made to suit the train; (4) the number of intermediate stops; (5) the time of each stop; (6) the layover not included in the schedule; (7) the general nature of the route with regard to curves, gradients, etc. If a full investigation of the manner of running is desired, a complete profile and map of the route becomes necessary, but the point-to-point method, although quite able to deal with the problem, becomes

unduly tedious in this case, and the analytical method discussed hereafter is preferable.

Effective Mass of the Train.—The tractive effort of a motor is reckoned as measured at the rims of the driving wheels when the motor is running at constant speed. The counter-torque due to friction in the motor, gears and axle bearings is accordingly taken from the armature-torque in obtaining the characteristic curves, and does not enter into the calculation of train motion. When the speed is varying, however, part of the motor-torque is used in accelerating the armature, and as this does not pass through the gears, there is a slight change in efficiency, which although quite negligible may be mentioned for the sake of logical completeness. The effect of rotating masses of armatures, wheels and gears was discussed in the last chapter from the energy point of view, and it was there shown that the force required to accelerate the train was proportional to its effective mass M′, instead of the actual mass M, where using the notation of the last chapter:

$$\mathrm{M}' = \mathrm{M} + \Sigma\frac{\mathrm{I}}{r^2} + \Sigma\frac{i}{r^2} + \Sigma\frac{\mathrm{I}'\gamma^2}{r^2} \qquad . \qquad . \qquad (1)$$

The effective mass of the train is a quantity which occurs whenever the subject of accelerating the train is in question, and some discussion of it may well be given here. The excess of the effective mass over the actual mass of the train is:

$$\Sigma\frac{\mathrm{I}}{r^2} + \Sigma\frac{i}{r^2} + \Sigma\frac{\mathrm{I}'\gamma^2}{r^2}$$

the summations extending over all wheels, gears and armatures in the train. The value of a moment of inertia can readily be obtained experimentally by suspending the part by means of a suitable bifilar suspension and noting the time of oscillation. In the case of the wheels it can be computed from the drawings without great difficulty. Where details are lacking, however, the following approximations may be useful.

Useful Approximations.—The radius of gyration of a steel railway wheel is generally about 0·77 of the radius of the wheel, so that approximately:

$$\frac{\mathrm{I}}{r^2} = \frac{mk^2}{r^2} = 0{\cdot}6m \qquad . \qquad . \qquad . \qquad (2)$$

In the case of a multiple unit train the order of this quantity is usually about a quarter of a ton for each wheel, or a ton per 4-wheeled bogie truck. The radius of gyration of a commutating railway motor armature is usually about 0·7 of the radius of the armature, so that approximately:

$$\frac{I'\gamma^2}{r^2} = \frac{m'k'^2\gamma^2}{r^2} = 0{\cdot}5\, m'\gamma^2 \frac{r'^2}{r^2}$$

$$= 0{\cdot}5\, m'\gamma^2 \left(\frac{d'}{d}\right)^2 \quad . \quad . \quad . \quad (3)$$

d and d' being the diameters of driving-wheel and armature respectively. Thus if:

$$m' = 2{,}430 \text{ lbs.}, \; d' = 20 \text{ in.}, \; d = 42 \text{ in.}, \; \gamma = \frac{70}{22}$$

$$\frac{I'\gamma^2}{r^2} = 2{,}800 \text{ lbs. approximately.}$$

The gears are of smaller consequence, adding perhaps a tenth of a ton each to the effective weight of the train.

The point-to-point method is most easily elucidated by the discussion of an example. Consider therefore a multiple unit train of four coaches, weighing complete with load and equipment 188 English tons, having a length of 253 feet and a cross sectional area of 115 square feet. Let the equipment consist of four motors of the type whose characteristics are given in fig. 52, and let the line voltage, gear reduction ratio, and size of wheel be that for which the curves of fig. 52 are drawn. Let the braking retardation be taken as 1·5 m.p.h. per second. Let the control be series-parallel with an average accelerating current of 275 amperes per motor; and with the field automatically tapped in parallel when the current falls to 125 amperes. Let the average distance between stations be 8,250 feet, and let the train be required to run this distance with an allowance of time in hand of about 7½ per cent. of the running time. Let the station-stop have an average duration of 60 seconds. The additional weight due to rotating masses is approximately 14 tons, made up as follows:

For 4 armatures at 2,800 lbs. each	11,200 lbs.
For 32 wheels at 600 lbs. each	19,200 lbs.
For 4 gears at 250 lbs. each.	1,000 lbs.
	31,400 lbs.

Calculation of Speed, Time and Distance.—The weight of the train is accordingly 47 English tons per motor, and the effective weight 50·5 tons per motor. The tractive resistance may be computed according to the methods suggested in the last chapter, which give the resistance curve of fig. 171. Tables 11 and 12 give the schedule of the calculation corresponding to the run in question. The beginning of the calculation, that is until the speed curve at running voltage is

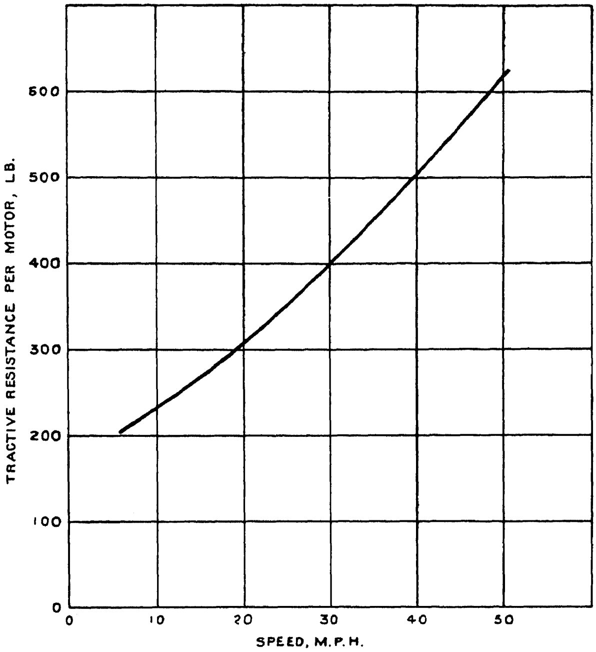


FIG. 171.—Tractive Resistance Curve.

reached, is somewhat abnormal and depends on the method of control in use. This portion might have been calculated in detail as was done in fig. 101, but it is difficult to conceive of a problem in which such refinement would be justified; and it is generally sufficient to assume that the accelerating tractive effort per motor is maintained constant at its mean value throughout the accelerating period, unless it is desired to take account of a lingering on the first point, which is sometimes necessary in the single-phase system in order to reach a certain speed with weak field. The first column in table 11 gives a

series of arbitrarily chosen currents beginning with the accelerating current and proceeding with diminishing decrements nearly to the free running current. The second column gives the speed, and the third column the tractive effort corresponding to the current in the first column, these being taken from fig. 52. The fourth column gives tractive resistance per motor from fig. 171. The fifth column is the difference between the third and fourth, and represents the accelerating tractive effort. The sixth column gives the mean accelerating tractive

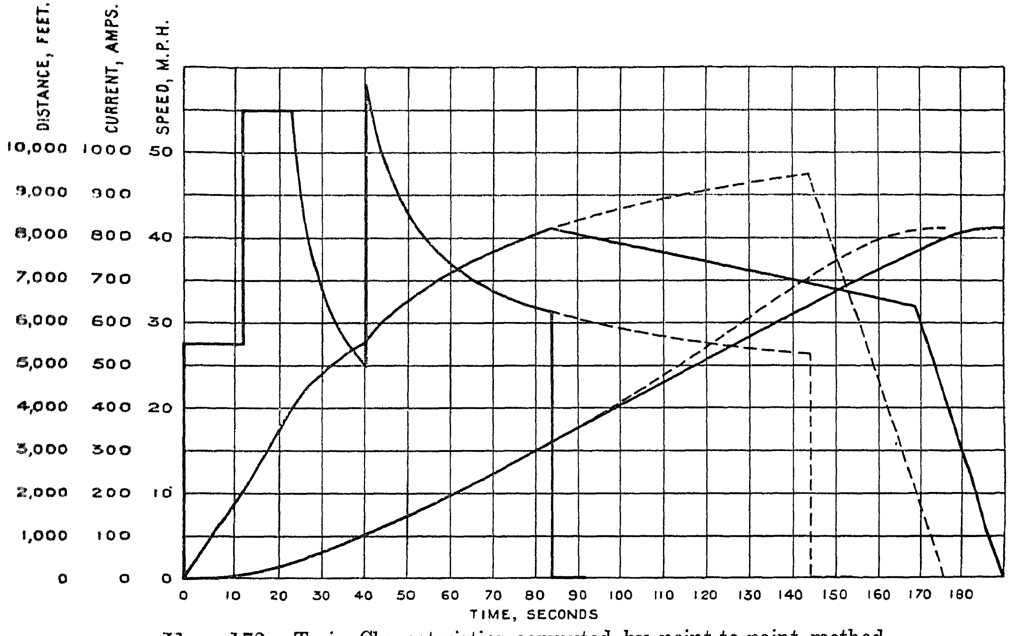


FIG. 172.—Train Characteristics computed by point-to-point method.

effort, each item being the mean between a pair of consecutive items in column five. The seventh column is the increment of speed, taken from column two. The eighth column is the corresponding increment of time, being the figure given in the seventh multiplied by 5,150 and divided by the figure in the sixth. The figure 5,150 is a constant for the train, being 102 times the effective weight per motor in English tons, and representing the tractive effort required to give the train an acceleration of 1 m.p.h. per sec. The ninth column gives the time, being the sum of the increments given in the eighth column. The tenth column is the mean speed, deduced from

column 2. The eleventh column is the increment of distance, being the product of mean speed and increment of time, reduced to feet by means of the multiplier 22/15. The twelfth column is the distance, or the sum of the increments of the eleventh. There is a discontinuity in the table where the motor-fields are tapped by means of relays, but the procedure at this point presents no difficulty. Fig. 172 gives train characteristics plotted from this table, being curves of current, speed and distance, (columns 1, 2 and 12) plotted against time (column 9).

TABLE 11.

1	2	3	4	5	6	7	8	9	10	11	12
Amps.	Speed	Tract. Effort	Tract. Resist.	Acc. T.E.	Mean Acc. T.E.	Incr. Speed	Incr. Time	Time Secs.	Mean Speed	Incr. Dist.	Distance feet.
Full field											
275	0	4,790	310	4,480	—	—	—	0	—	—	0
—	—	—	—	—	4,480	20·0	23·0	—	10·0	337	—
275	20·0	4,790	310	4,480	—	—	—	23·0	—	—	340
—	—	—	—	—	3,650	2·5	3·5	—	21·25	109	—
200	22·5	3,150	330	2,820	—	—	—	26·5	—	—	450
—	—	—	—	—	1,995	5·4	13·9	—	25·2	515	—
125	27·9	1,550	380	1,170	—	—	—	40·4	—	—	960
Tapped field											
290	27·9	3,640	380	3,260	—	—	—	40·4	—	—	960
—	—	—	—	—	2,610	4·3	8·5	—	30·05	374	—
220	32·2	2,380	420	1,960	—	—	—	48·9	—	—	1,330
—	—	—	—	—	1,450	7·3	25·9	—	35·85	1,360	—
165	39·5	1,440	500	940	—	—	—	74·8	—	—	2,690
—	—	—	—	—	715	5·5	39·6	—	42·25	2,456	—
140	45·0	1,050	560	490	—	—	—	114·4	—	—	5,150
—	—	—	—	—	355	4·4	63·8	—	47·2	4,415	—
125	49·4	830	610	220	—	—	—	178·2	—	—	9,570

COASTING AND BRAKING PERIODS.—The above calculation deals only with the time during which the motors are taking power from the line, and, in order to complete the curves, the portions corresponding with the coasting and braking periods have to be determined. These must be so drawn that the total area enclosed by the speed-time curve represents the

specified distance; and an allowance of time, in excess of the minimum possible, should be kept in hand sufficient to satisfy the exigencies of the service. It is usual to complete by a method of trial; but this is apt to be found a tedious procedure, and more direct methods of computation are usually preferable.

MINIMUM TIME OF RUNNING.—Consider first the determination of the minimum time in which the given distance can be run, a necessary preliminary to determining the margin of time in hand. The problem is that of adding a braking curve to the speed-time curve of fig. 172, having the specified slope, and drawn in such position as to include an area representing the correct distance. In the figure, let the speed at any time t_1 be s_1, and the distance D_1. If power be cut off and brakes applied at this instant, the rate of braking retardation being b, the time t_2 required to bring the train to rest is s_1/b, and the distance D_2, covered in the time $11s_1^2/15b$. If D is the required total distance, the defect, d, is $D-(D_1+D_2)$, which may of course be either positive or negative. If a is the rate of acceleration at time t_1, the additional time t_3 to be allowed is connected with d by means of the equation:

$$t_3^2 + 2\frac{a+b}{ab}s_1t_3 = \frac{15}{11}\frac{a+b}{ab}d$$

or

$$t_3 + \frac{a+b}{ab}s_1 = \frac{a+b}{ab}s_1\sqrt{\left\{1+\frac{15}{11}\frac{ab}{a+b}\frac{d}{s_1^2}\right\}} \quad . \quad (4)$$

If the point selected in the first instance is reasonably near to the correct point of cut off, this equation reduces to:

$$t_3 = \frac{15}{22}\frac{d}{s_1}\left\{1-\frac{15}{44}\frac{ab}{a+b}\frac{d}{s_1^2}\right\} \quad . \quad . \quad (5)$$

Thus the minimum time for the run is:

$$t = t_1 + \frac{s_1}{b} + \frac{15}{22}\frac{d}{s_1}\left\{1-\frac{15}{44}\frac{ab}{a+b}\frac{d}{s_1^2}\right\} \quad . \quad (6)$$

In the example under discussion $D = 8{,}250$ feet and $b = 1{\cdot}5$ m.p.h. per second. If now t_1 be taken at 114·4 seconds, so that $s_1 = 45$ m.p.h., $D_1 = 5{,}150$ feet, $D_2 = 990$ feet, $d = 2{,}110$ feet, $a = {\cdot}095$ m.p.h. per second; from equation 6:

$$t = 114{\cdot}4 + 30{\cdot}0 + 31{\cdot}0 = 175{\cdot}4 \text{ seconds.}$$

If the speed-time curve to correspond with a definite schedule

be sought, the time of running the specified distance will be known; and if it be required rather to find the schedule which it is dynamically practicable or advisable to run with the given equipment, a suitable margin upon the minimum time as above determined must be kept in hand, and the total time of running is therefore still known. In either case, accordingly, the position of the braking curve is known, and the next problem becomes that of placing a coasting curve, sloping at the appropriate angle, which, with the motor-accelerating and the braking curves already located, gives to the whole speed-time curve the area required to represent the specified distance.

Coasting Curve.—It was explained in the last chapter that the resistance to the motion of the train when coasting is greater than the true tractive resistance, inasmuch as the motor-friction now acts as a retarding force. Using the coasting resistance as the retarding tractive effort, the true coasting curve could be computed in the same manner as the accelerating curve with power on was computed in table 11. It is however generally quite sufficient to assume a mean value for the coasting resistance, inasmuch as the variation is comparatively small and uncertain. This value should be taken to correspond with a speed somewhat greater than the mean coasting speed, for reasons given in the last chapter. The coasting curve in fig. 172 is therefore assumed to be a straight line.

Completion of Speed-time Curve.—The solution of the problem of completing the train characteristic curves is usually sought by a method of trial; but the following direct method for effecting the object will present no difficulty to those accustomed to making calculations. Let the point for cutting off power be guessed as approximating to a point Q (fig. 173), which may usually with advantage be taken as one of the points determined in the tabular calculation of the curve. Let the time to this point be t_1, and the distance D_1. Let the speed at Q be s_1 and the rate of acceleration at this point, a; let the tangent at Q—whose slope is a—meet the braking curve, already located, at the point T, and let s_2 be the speed at T. Then if t is the total running time:

$$\frac{s_2 - s_1}{a} + \frac{s_2}{b} = t - t_1 = t_2$$

or

$$s_2 = \frac{b}{a+b}(s_1 + at_2) \qquad . \qquad . \qquad . \qquad (7)$$

Taking the fictitious speed-time curve so constructed as representing distance $D_1 + D_2$, D_2 is given by:

$$D_2 = \frac{22}{15}\left\{\frac{s_1 + s_2}{2}\frac{s_2 - s_1}{a} + \frac{s_2^2}{2b}\right\}$$

$$= \frac{11}{15a}\left\{\frac{b}{a+b}(s_1 + at_2)^2 - s_1^2\right\} \qquad . \qquad . \qquad (8)$$

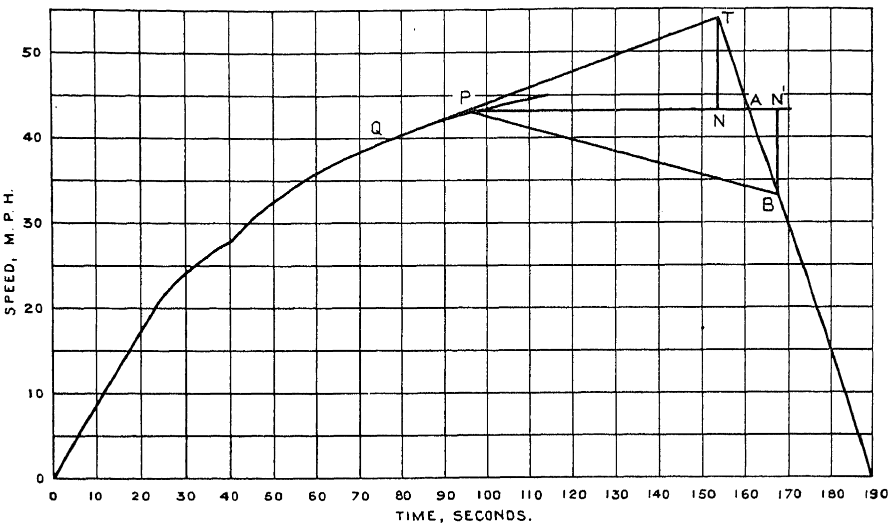


FIG. 173.—Speed-Time Curve accompanying Calculation.

Let the distance, $D_1 + D_2$, exceed the specified distance, D, by d. The problem is now therefore to draw in the coasting curve at the appropriate slope, c, in such manner as to cut off from the figure an area representing distance, d. Let the coasting curve cut the speed curve at P, and the braking curve at B, the speed at B being $s_2 - s$. Draw PA horizontal, cutting the braking curve at A, and TN, BN′ vertical. Writing $PA = x$, and $TN + BN' = s$; the area cut off by the coasting curve:

$$d = \tfrac{1}{2} \cdot \frac{22}{15} xs$$

Now:

$$x = \mathrm{TN}\left(\frac{1}{a} + \frac{1}{b}\right) = \mathrm{BN}'\left(\frac{1}{c} - \frac{1}{b}\right)$$

or

$$\mathrm{TN} = \frac{ab}{a+b}\,x,\ \mathrm{BN}' = \frac{bc}{b-c}\,x$$

Hence:

$$s = \frac{b^2(a+c)}{(a+b)(b-c)}\,x$$

and

$$d = \frac{11}{15}\,\frac{(a+b)(b-c)}{b^2(a+c)}\,s^2$$

or

$$s^2 = \frac{15}{11}\,\frac{b^2(a+c)}{(a+b)(b-c)}\,d \qquad (9)$$

This equation determines the point B, and therefore the position of the coasting curve.

Continuing the discussion of fig. 172, since the minimum time of running the distance has been found to be 175·4 seconds, and the margin of time to be kept in hand is $7\frac{1}{2}$ per cent., the total running time (t) should be 190 seconds. Assume as a first approximation, that power is cut off at $t = 74{\cdot}8$ seconds (see table 11) and take coasting resistance as 555 lbs. per motor. Then:

$$a = 940/5{,}150 = {\cdot}182,\ b = 1{\cdot}5,\ c = 555/5{,}150 = 0{\cdot}108,$$

$$\begin{array}{lll} t_1 = 74{\cdot}8, & s_1 = 39{\cdot}5, & \mathrm{D}_1 = 2{,}690 \\ t_2 = t - t_1, & s_2 = 54{\cdot}0, & \mathrm{D}_2 = 6{,}850 \\ \quad = 115{\cdot}2, & \text{(eqn. 7)} & \text{(eqn. 8)} \end{array}$$

$$d = \mathrm{D}_1 + \mathrm{D}_2 - \mathrm{D} = 9{,}540 - 8{,}250 = 1{,}290 \text{ feet.}$$

From equation 9:

$$\begin{aligned} s^2 &= 490 \\ s &= 22{\cdot}1 \\ s_2 - s &= 31{\cdot}9 \text{ m.p.h.} \end{aligned}$$

The above method is capable of giving the distance correctly, within a few feet, unless the selected point, Q, be very wide of the mark. If Q is chosen beyond the correct point, the distance represented tends to be in excess of the specified distance.

The point B having been determined, the train characteristics of fig. 172 can now be completed.

Calculation of Energy Consumption and Losses in Motors.—The relation between current and time having been determined in table 11, and the instant when power should be cut off having now been found, the energy consumption and motor losses can readily be calculated since the system of control is known. In the continuous current system with series-parallel control, the mean power input during acceleration on rheostat is approximately three-fourths of the parallel power, whilst in series, series-parallel, and parallel control it is eleven-sixteenths of the parallel power. In the single-phase system it is often sufficient to take the mean between the power which gives the full accelerating tractive effort with motor stationary and that at which the running speed curve is reached with the same tractive effort. In the three-phase system with cascade-parallel control, the mean power during acceleration may be taken at three-fourths of the parallel power. In estimating the heating of the motors, the accelerating current may be assumed constant until the running curve is reached, except in some forms of single-phase motor in which the field is weakened and the armature current increased for acceleration. The appropriate methods to apply will be obvious when the characteristics of the motor and control are known. The period of acceleration to the speed curve is therefore treated in the calculation as if the number of controller points were infinite and the rheostats adjusted to give uniform accelerating current per motor, whereas actually a limited number of controller points are employed, resulting in as many peaks of current. With rheostats adjusted to give approximately uniform peaks, however, this deviation from actual conditions is without appreciable effect on the speed, energy consumption, or loss in motors, so that there is no need to complicate the calculation by making assumptions nearer to actual conditions in this respect.

Table 12 gives a schedule of calculation of energy input and motor-loss for the typical run considered. The first two columns are repeated from table 11, being the first and eighth columns of this table, but finished in accordance with fig. 172. The third column gives the mean current. The fourth column

TABLE 12.

1	2	3	4	5	6	7	8	9	10
Amps.	Incr. time	Mean Amps.	Incr. Energy	Amps. squared	Mean Amps. squared	Incr. Res. Loss	Core Loss	Mean C.L.	Incr. C.L.
Full	field								
275	—	—	—	75,700	—	—	0	—	—
—	23·0	275	1,020	—	75,700	247,700	—	2,810	64,700
275	—	—	—	75,700	—	—	5,620	—	—
—	3·5	237·5	179	—	57,850	28,800	—	5,020	17,600
200	—	—	—	40,000	—	—	4,420	—	—
—	13·9	162·5	487	—	27,800	55,000	—	4,085	56,800
125	—	—	—	15,600	—	—	3,750	—	—
Tap	ped fi	eld							
290	—	—	—	84,100	—	—	6,330	—	—
—	8·5	255	467	—	66,250	62,100	—	5,950	50,600
220	—	—	—	48,400	—	—	5,570	—	—
—	25·9	192·5	1,072	—	37,850	108,000	—	5,500	142,300
165	—	—	—	27,300	—	—	5,430	—	—
—	9·2	160	317	—	25,650	26,000	—	5,460	50,200
155	—	—	—	24,000	—	—	5,490	—	—
		Sum	3,542	watt-hours		527,600	watt-secs.		382,200 watt-secs.

gives the increment of energy input per motor, being, except for the first item, the product of the voltage by the items of the second and third columns, the whole divided by 3,600. The first item is three-fourths of this, to provide for series-parallel control being used. The fifth column is the square of the motor current, and the sixth the mean square. The seventh column gives the increment of resistance loss in the motor windings in watt-seconds, being the product of items in second and sixth columns by the motor resistance. The eighth column gives core-loss, including load loss, from fig. 56. The ninth column gives the mean core-loss, and in this connection it may be mentioned that the mean core-loss during the period of acceleration on resistance is practically 50 per cent. of full voltage core-loss. The tenth column gives the corresponding energy loss, being the product of items of second and ninth columns.

The sum of the increments in the fourth column in table 12 gives the energy input as 3,540 watt-hours per motor for the distance of 8,250 feet, from which is deduced the energy con-

sumption of 9·07 kilowatt-hours per train mile or 48·2 watt-hours per ton mile. The sum of the increments in the seventh column gives the resistance loss per motor as 528,000 watt-seconds for the run, which, divided by the total time, including station-stop (250 seconds) gives an average resistance loss of 2,110 watts. Similarly the sum of the increments in the tenth column gives the total core-loss for the run as 382,000 watt-seconds, or an average of 1,530 watts. The brush-resistance loss may be obtained from column 4, increasing the first item by a third; this gives a total of 3,880 × 2 × 3,600/775 = 36,000 watt-seconds or an average of 150 watts, approximately. The average armature speed is 573 r.p.m. and the corresponding average friction loss 1,830 watts. Thus the total computed average loss in the motor during schedule running is about 5,620 watts. From fig. 55 it is seen that the motor, when running at 573 r.p.m., dissipates 128 watts per degree rise: accordingly the temperature rise to be expected with 5,620 watts loss in continuous running on the testing stand is 44° C. As the motor is of the self-ventilated type, this may also be taken as the approximate rise in the service considered. Actual service on the railway would probably be somewhat more severe, as no allowance has been made for untoward circumstances. The estimation of the heating of the motor in service is of equal importance with the determination of its dynamical capability or energy consumption; and comparisons based on dynamical characteristics alone are often quite misleading.

The Use and Value of the Method.—The point-to-point method has two principal uses: it is the simplest method for making accurate comparisons, whether between the performances of a given train in different services, or between the performances of different trains in the same service; it is also the most satisfactory means of obtaining a general idea of the performance of a train. It has the advantage of being a direct application of first principles, so that errors in the calculation are readily detected. The average performance which it is appropriate to discuss by this method is of great value, for although the investigation is confined to the average schedule and to level track, it corresponds in general very fairly with the average performance on the actual schedule. Of course, if the line is, like the Central London Railway, graded

; to dip in each direction from every station, the schedule be run with less powerful equipments and with lower ;y consumption than the calculation would indicate, but 'e the stations are promiscuously arranged with reference :adients, the average energy consumption for runs which 'n to the starting-point is not usually more than a few per greater than it would be if the track were level every- 'e—the extra power required at one place being practic- given back at another. Small as is the usual effect of ients on the average performance, the effect of variation stance between stations is even smaller—the performance he average distance agreeing very closely with the average)rmance for the actual distances. The effect of curves, tions, etc., unless such as to impose restriction of speed, is l and sufficiently well met by taking a somewhat high e for train resistance.

ADEQUACY OF METHOD FOR DEALING WITH ACTUAL :DULE.—Altogether the computed train characteristics of verage schedule run are of great value and significance, :sponding closely with the general performance of the .. Sometimes however a final choice of the equipment calculated to perform a given service has to be made, or, :hoice having been made, it is desired to determine what t the special features of the particular route have on the)rmance of the train, and what is approximately the best of handling it, under the various circumstances that arise. :ch cases the discussion of the average schedule is obviously ficient, and the gradients, distances, etc., incident to the ıl service must be introduced into the calculation. The ;-to-point method could be used for this purpose without ulty,—for the effect of a gradient is only to change the ;ance to the motion of the train ; but since the independent ,ble is the speed, and that this has no simple relation to listance, a calculation by methods of trial is necessary ever the gradient changes, and the point-to-point method nes very tedious.

ıalytical Method.—The method to be discussed next leveloped by the author some years ago,* and is particu-

,ee *Transactions of the American Institute of Electrical Engineers,* ;2, p. 113.

larly applicable to such problems as indicated in the last paragraph, in that it enables the distance to be taken as independent variable, and identifies the speed-distance curve on any gradient with one of a universal system of speed-distance curves which have been determined once for all, providing at the same time a simple formula to connect time and distance. The method, which is an analytical one, depending on the determination of a formula to connect speed and tractive effort, is applicable to any motor having series characteristics, whether continuous current or single-phase. It may here be mentioned that motors having approximately constant speed characteristics, such as the polyphase induction motor, present little difficulty in the determination of their performance in service; for, the changes of speed being small, the train very quickly reaches a steady condition on any gradient, and accordingly the methods now under discussion are unnecessary for such motors.

Speed-Tractive Effort Curve of Motor.—The method assumes that the curve between speed and tractive effort for the train equipment in any running condition has a particular form which is capable of being expressed in terms of three constants. The assumption however, arbitrary as it appears, is nearer to the facts than the nature of the problem actually requires; for it is within the limits of determinancy of the data. The total resistance to the motion of the train is chiefly composed of the gradient and tractive resistances. Of these the latter is a function of the speed and it is accordingly convenient to subtract it from the tractive effort of the motors throughout the range of speed curve running, leaving only resistances which are independent of speed to be taken into account. The motor curve, therefore, from which the performance of the train is to be deduced, is that between speed and the residual tractive effort after the requirements of ordinary tractive resistance have been satisfied. It is assumed that the portion of this curve which is employed in service takes the form of a rectangular hyperbola having its asymptotes parallel to the axes. Thus if F is the tractive effort and S the speed, the equation of the curve is of the form:

$$(F + F_o)(S - S_o) = KS_o \quad . \quad . \quad . \quad (10)$$

where F_o, S_o and K are constants. The determination of the

constants is a matter of no great difficulty in any particular case. Three speeds are taken, one at about the accelerating current, one about free-running speed, and one intermediate between these, and, the corresponding tractive efforts having been determined, the three constants in equation 10 are found as the solution of three simultaneous equations. Thus from figs. 52 and 171 the following table is deduced :

TABLE 13

Current	275	170	125
Speed (S)	28·7	38·6	49·3
Tractive effort	3,380	1,520	830
Tractive resistance	390	490	610
Residual tractive effort (F)	2,990	1,030	220

Hence :

$$\begin{aligned} 28{\cdot}7\ F_o - 2{,}990\ S_o + 85{,}810 &= \\ 38{\cdot}6\ F_o - 1{,}030\ S_o + 39{,}760 &= \\ 49{\cdot}3\ F_o - 220\ S_o + 10{,}850 &= (K + F_o)S_o \end{aligned}$$

$$\therefore \quad \begin{aligned} 9{\cdot}9\ F_o + 1{,}960\ S_o - 46{,}050 &= 0 \\ 10{\cdot}7\ F_o + 810\ S_o - 28{,}910 &= 0 \end{aligned}$$

$$\therefore \quad S_o = 15{\cdot}95 \text{ m.p.h.}, \ F_o = 1{,}493 \text{ lbs.}, \ K = 3{,}580$$
$$= 23{\cdot}4 \text{ f.p.s.}$$

Accuracy of Curve.—In order to show how nearly the equation so determined follows the speed-tractive effort curve of the motor over the useful range, fig. 174 has been deduced from figs. 52 and 171, and on it are shown a few points plotted in accordance with equation 10, using the constants determined above. Such an equation is required for each running curve of the motor. If series-parallel control is used, for instance, there will be two equations, corresponding with series and parallel running respectively, and with field-control there will be others corresponding to the various field strengths. With single-phase commutator motors one or other of several curves may be employed as a running curve, according to the requirements of the service. It will be found possible to determine an equation of the form of equation 10 which will represent any of these curves with substantial accuracy. In many cases, however, the normal running is confined to one curve, and the abnormal running is insignificant in amount; as when series running is used in passing over special work

outside a terminus and the like, and the acceleration is continued to full parallel elsewhere. In such a case it may be deemed sufficient for the purpose in view to take the series curve as giving a half the speed of the parallel curve for the same tractive effort, so that its equation, as deduced from that of the parallel curve, is :

$$(F + F_o)(S - \tfrac{1}{2}S_o) = \tfrac{1}{2}KS_o \quad . \quad . \quad (11)$$

Where however there is much series running, the correct curve should be determined.

NOTATION.—In the following discussion, the symbol for a

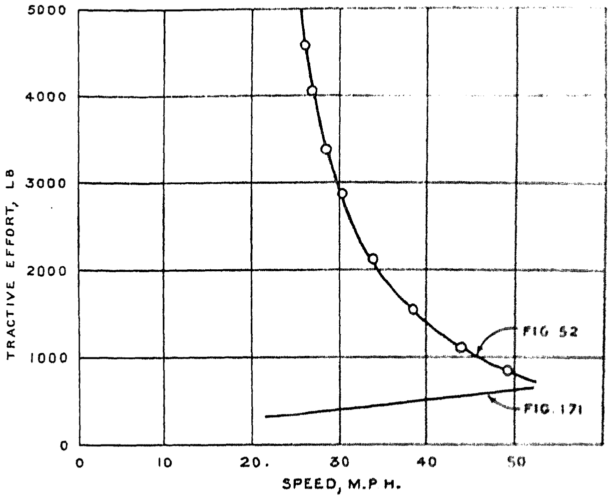


FIG. 174.—Motor Speed Curve.

variable, if written without distinguishing mark, denotes that it refers to the current value of the variable ; if dashed, it denotes that it refers to the free running condition corresponding to the particular gradient : the suffix 1 attached to a symbol denotes that it refers to the initial condition of speed-curve running on the gradient, whilst the suffix 2 denotes that it refers to the final condition actually attained. Capital letters are used to denote quantities expressed in normal units, whilst the corresponding small letters denote these quantities when expressed in certain special units to be defined later.

Free-running Speed.—When the free-running condition has been reached, the tractive effort as given by equation 10

is equal to the residual resistance which, as has been explained, is that of the gradient only. Thus it is appropriate to write F′ for the grade-resistance, and the free running speed is S′, given by :

$$(F' + F_0)(S' - S_0) = KS_0 \quad . \quad . \quad (12)$$

As has been indicated above, special units of time and distance, suited to the train and its equipment, are to be used ; but it may here be mentioned that the new unit of speed is S_0 ; if s is written for the speed in terms of this unit, that is :

$$s = \frac{S}{S_0} \quad . \quad . \quad . \quad . \quad (13)$$

equation 10 becomes :

$$(F + F_0)(s - 1) = K \quad . \quad . \quad . \quad (14)$$

and equation 12 becomes :

$$(F' + F_0)(s' - 1) = K \quad . \quad . \quad . \quad (15)$$

Equations of Motion and Determination of Time and Distance.—Let force be expressed in gravitational units, g being the value of the acceleration due to gravity. The general equation for the motion of the train whose effective mass is M′ on a grade whose resistance is F′ is :

$$M'\frac{dS}{dt} = (F - F')\,g$$

or

$$M'S_0\frac{ds}{dt} = (F - F')\,g \quad . \quad . \quad . \quad (16)$$

For speed-curve running accordingly, from equations 14, 15 and 16 :

$$M'S_0\frac{ds}{dt} = Kg\,\frac{s' - s}{(s - 1)(s' - 1)} \quad . \quad . \quad (17)$$

Thus, the time of speed-curve running between speeds S_1 and S_2 is :

$$T = \frac{M'S_0}{Kg}\int_{s_1}^{s_2}\frac{(s - 1)(s' - 1)}{s' - s}\,ds$$

$$= \frac{M'S_0}{Kg}(s' - 1)\left[(s' - 1)\log\frac{s' - 1}{s' - s} - (s - 1)\right]_{s_1}^{s_2} \quad (18)$$

The corresponding distance of running is:

$$D = \int_{s_1}^{s_2} S dT$$

$$= S_o \int_{s_1}^{s_2} s dT$$

$$= \frac{M'S_o^2}{Kg} \int_{s_1}^{s_2} \frac{(s'-1)s(s-1)}{s'-s} ds \quad . \quad (19)$$

Let the units of time and distance be taken as:

$$T_o = \frac{M'S_o}{Kg}, \qquad D_o = \frac{M'S_o^2}{Kg} \quad . \quad . \quad (20)$$

Universal Speed-time and Speed-distance Curves.—Write, consistently with foregoing:

$$\frac{T}{T_o} = t \qquad \frac{D}{D_o} = d \quad . \quad . \quad (21)$$

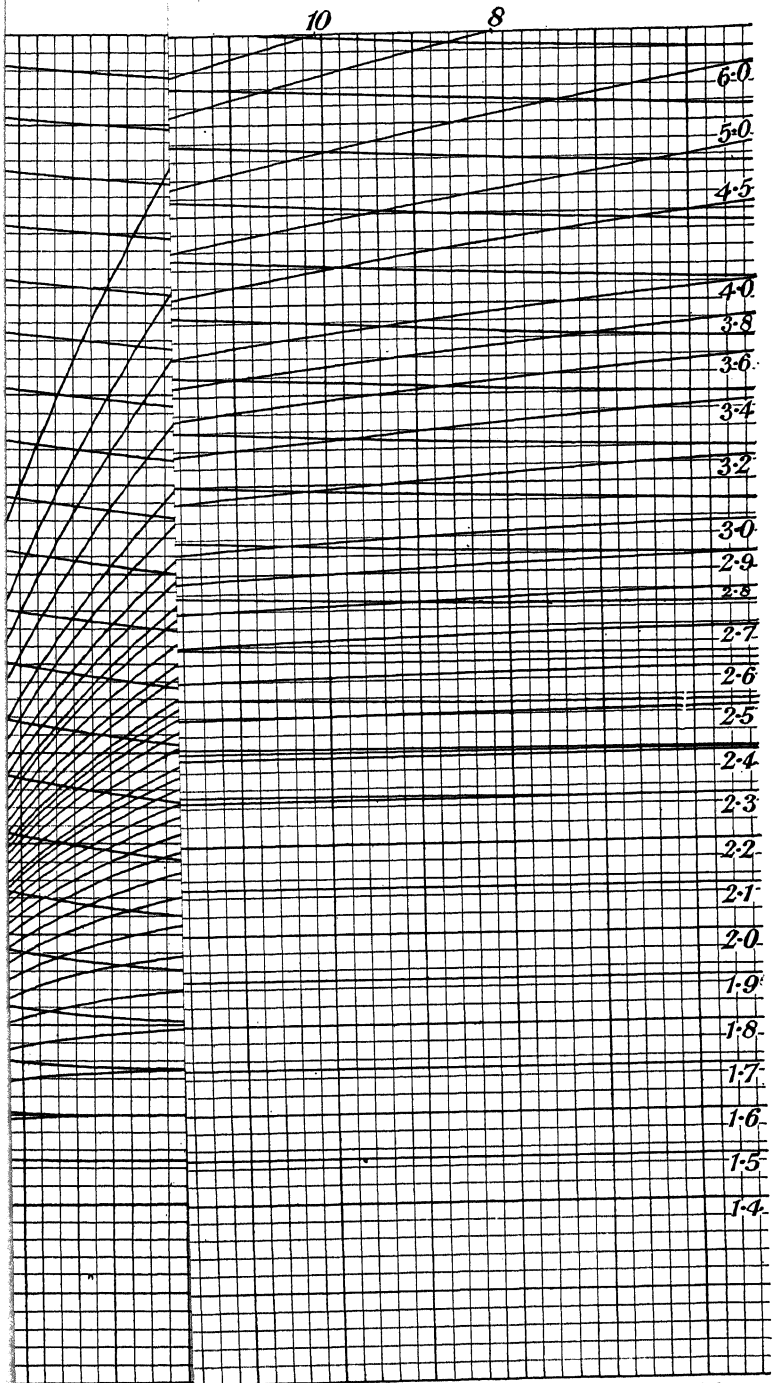
Equation 18 becomes:

$$t = (s'-1)\Big[(s'-1)\log\frac{s'-1}{s'-s} - (s-1)\Big]_{s_1}^{s_2} \quad . \quad (22)$$

Equation 22 shows that t is the difference between the abscissae at points, whose ordinates are $y = s_1$ and $y = s$, on the curve whose equation is:

$$x = (s'-1)\Big[(s'-1)\log\frac{s'-1}{s'-y} - (y-1)\Big] \quad . \quad (23)$$

Equation 23 represents, therefore, a general speed-time curve corresponding to the period of running on the motor curve. As long as the equipment, train, and gradient combine to give a certain value, s', to the free running speed, the speed-time curve, with the units of speed and time taken as directed above, is a portion of the curve represented by equation 23, in which s' has the value determined by equation 15. If therefore the free running speed s' be taken as a variable parameter and a complete system of speed-time curves plotted in accordance with equation 23, these will represent to a certain scale the motor-curve portions of all possible speed-time curves that can be obtained with series motors. Similar remarks apply to the speed-distance curves to be considered next, and these from the present point of view are of greater importance, since


10
8
6·0
5·0
4·5
4·0
3·8
3·6
3·4
3·2
3·0
2·9
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2·1
2·0
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3
17
18
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21
22

distance is the natural co-ordinate in which to express the limits of a gradient, of a curve, or other constant circumstance.

From equations 19 and 21 :

$$d = \int_{s_1}^{s_2} \frac{(s' - 1)s(s - 1)}{s' - s}\, ds$$

$$= s' \int_{s_1}^{s_2} \frac{(s' - 1)(s - 1)}{s' - s}\, ds - \int_{s_1}^{s_2} (s' - 1)(s - 1) ds$$

$$= s't - \tfrac{1}{2}(s' - 1)[(s - 1)^2]_{s_1}^{s_2} \quad . \quad . \quad . \quad . \quad (24)$$

The general equation of the speed-distance curves is accordingly :

$$x = (s' - 1)\left\{ s' \left[(s' - 1) \log \frac{s' - 1}{s' - y} - (y - 1) \right] - \tfrac{1}{2}(y - 1)^2 \right\} \quad (25)$$

The system of speed-distance curves derived from this equation for a succession of values of s' is given in fig. 175. These are the universal curves by means of which the performance of any train on any route can be computed. It will be noticed that each of these curves has two branches, the upper of which is derived from the integral when s is greater than s' : this branch is for use when a gradient is reached at a speed greater than the free running speed for the gradient, so that the train slows down as it progresses, but no variation in the method of treatment is involved thereby.

In using the present method, the distance is taken as independent variable, the corresponding speed is taken from the appropriate curve of fig. 175, and the time is computed from equation 24, which may be written :

$$s't = d + \tfrac{1}{2}(s' - 1)(s_2 - s_1)(s_2 + s_1 - 2) \qquad (26)$$

Accelerating Period.—Before discussing the method of calculation in detail, it is necessary to consider the period of acceleration by which the normal running curve of the motor is reached. It is usually sufficient for this purpose to assume a uniform tractive effort during the period in question, and to take train resistance as that at the speed of reaching the motor curve. Thus the tractive effort available for overcoming the resistance of the gradient and accelerating the train is connected with the speed of reaching the motor curve by equation

10, and writing F_1 for the tractive effort, the speed is given by:

$$(F_1 + F_o)(s_1 - 1) = K \quad . \quad . \quad . \quad (27)$$

If a is the constant rate of acceleration (see equations 16 and 17):

$$\begin{aligned} M'a &= (F_1 - F')g \\ &= Kg \frac{s' - s_1}{(s' - 1)(s_1 - 1)} \quad . \quad . \quad (28) \end{aligned}$$

If T_a is the time of acceleration:

$$\begin{aligned} T_a &= \frac{S_1}{a} \\ &= T_o \frac{(s' - 1)s_1(s_1 - 1)}{s' - s_1} \quad . \quad . \quad (29) \end{aligned}$$

Thus expressing the time in terms of the unit T_o as before:

$$t_a = \frac{(s' - 1)s_1(s_1 - 1)}{s' - s_1} \quad . \quad . \quad . \quad (30)$$

The distance of the acceleration is:

$$D_a = \tfrac{1}{2}S_1T_a = \tfrac{1}{2}S_oT_os_1t_a = \tfrac{1}{2}D_os_1t_a$$

In terms of the unit D_o:

$$d_a = \tfrac{1}{2}s_1t_a = \tfrac{1}{2}\frac{(s' - 1)s_1^2(s_1 - 1)}{s' - s_1} \quad . \quad . \quad . \quad (31)$$

Change of Gradient during Acceleration.—If the gradient changes during the acceleration period, after distance d_{1a}, the constant s' changing from s_1' to s_2' the speed s_{11} at the instant of the change is given by:

$$d_{1a} = \frac{(s_1' - 1)(s_1 - 1)}{s_1' - s_1} \frac{s_{11}^2}{2} \quad . \quad . \quad (32)$$

The total time is given by:

$$t_a = t_{1a} + t_{2a} = \frac{(s_2' - 1)(s_1 - 1)}{s_2' - s_1}s_1 + \frac{(s_2' - s_1')(s_1 - 1)^2}{(s_2' - s_1)(s_1' - s_1)}s_{11} \quad . \quad (33)$$

The total distance is given by:

$$d_a = d_{1a} + d_{2a} = \frac{(s_2' - 1)(s_1 - 1)}{s_2' - s_1}\frac{s_1^2}{2} + \frac{(s_2' - s_1')(s_1 - 1)}{(s_2' - s_1)(s_1' - 1)}d_{1a} \quad (34)$$

Acceleration from Speed.—When the uniform accelera-

tion, instead of starting from rest, starts from some known speed s_{11}, as when power is applied after a slow-down, the equations for accelerating time and distance are:

$$t_a = \frac{(s' - 1)(s_1 - 1)}{s' - s_1}(s_1 - s_{11}) \quad . \quad . \quad (35)$$

$$d_a = \frac{(s' - 1)(s_1 - 1)}{s' - s_1}\,\frac{s_1^2 - s_{11}^2}{2} \quad . \quad . \quad (36)$$

The speed of reaching the motor curve after acceleration is usually comparatively low, as expressed in universal units, and there is considerable difficulty in determining the corresponding abscissa in fig. 175, on account of the multiplicity of converging curves at low speeds. The abscissa can however be determined directly from equation 25, which may, if preferred, be expanded in the form:

$$d_1 = \tfrac{1}{2}(s_1 - 1)^2\left\{1 + 2s'\left[\tfrac{1}{3}\left(\frac{s_1 - 1}{s' - 1}\right) + \tfrac{1}{4}\left(\frac{s_1 - 1}{s' - 1}\right)^2 + \tfrac{1}{5}\left(\frac{s_1 - 1}{s' - 1}\right)^3 + \right]\right\} \quad . \quad . \quad . \quad (37)$$

A few terms of this expansion are generally sufficient for the purpose.

Method of Making Calculations.—When the data appropriate to a particular train and equipment have been determined, their application to the problem of computing the dynamical performance of the train in given service is comparatively simple. The necessary integration having been carried out once for all, and incorporated in the curves of fig. 175, the computation as regards any particular gradient is reduced in most cases to the labour of looking out one abscissa and one ordinate on the curve appropriate to the train and gradient in fig. 175, and making a few simple calculations from the results. The abscissa in question is that corresponding with the speed of entering on the gradient. To this should be added the distance travelled on the gradient, giving a new abscissa; and the corresponding ordinate is the speed of leaving the gradient and of entering on the next. The time on the gradient is then obtained from equation 26. The power on entering or leaving the gradient may be deduced from the motor characteristics; and the energy used may be estimated therefrom; or the energy may be computed directly by methods

to be given later. The motor losses may be treated in a similar manner. The passage from one motor speed curve to another usually involves looking up another point in fig. 175, but presents no feature of difficulty. The change in the unit of time and distance should however be kept in mind.

Tables 14 and 15 show in detail how the calculation may be made. In these the first two columns give constants of the speed curve in use (equation 14). The third, fourth and fifth columns give appropriate units of speed, time and distance respectively. The sixth column gives the gradient, reckoned positive when against the train. The seventh column gives the tractive resistance, F′, due to the gradient only. This figure may be taken to include the resistance due to curves also, but for the purpose in view it is perhaps easier to include an average value for this in the ordinary tractive resistance and make allowance for it in the equation of the speed curve; in the table, F′ is the gradient ratio multiplied by the weight per motor, in lbs. The eighth column gives the free running speed on the gradient (equation 15). The ninth column gives distances, in feet; and the tenth the corresponding distances in terms of the unit D_0. The eleventh column gives the speed at which the appropriate curve of fig. 175—determined by s' in the eighth column—is reached. The twelfth column gives the abscissa of fig. 175 corresponding to the ordinate given in the eleventh. The thirteenth column is the sum of the items in the tenth and twelfth. The fourteenth column gives the ordinate corresponding to the abscissa of the thirteenth and represents the speed of leaving the particular curve; except where the units are changed it also represents the speed of reaching the succeeding curve (column 11). The fifteenth column gives the interval of time, computed from equation 26. The sixteenth column gives the same interval in seconds. The seventeenth column gives the integral time in seconds; the eighteenth the speed (columns 14 and 3), and the nineteenth the integral distance (from column 9) corresponding. The curves of figs. 176 and 177 are plotted from the three last-mentioned columns.

The acceleration periods are somewhat special, and in order to make the examples more generally instructive, a speed restriction, compelling series running, is supposed to exist for the first 150 yards of fig. 176 (table 14). The speed of reaching

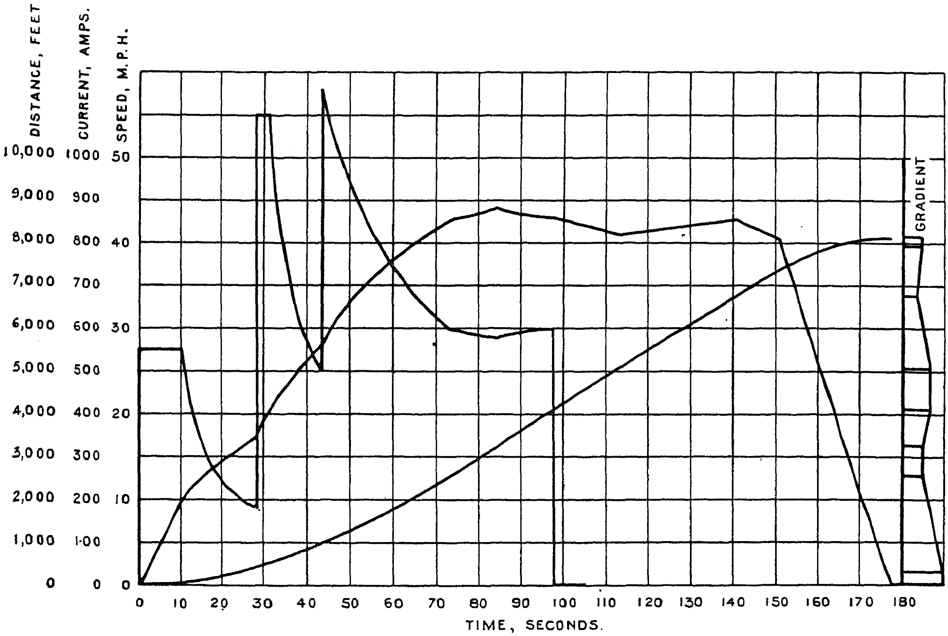

FIG. 176.—Train Characteristics, 4 coach train: Outward Journey.

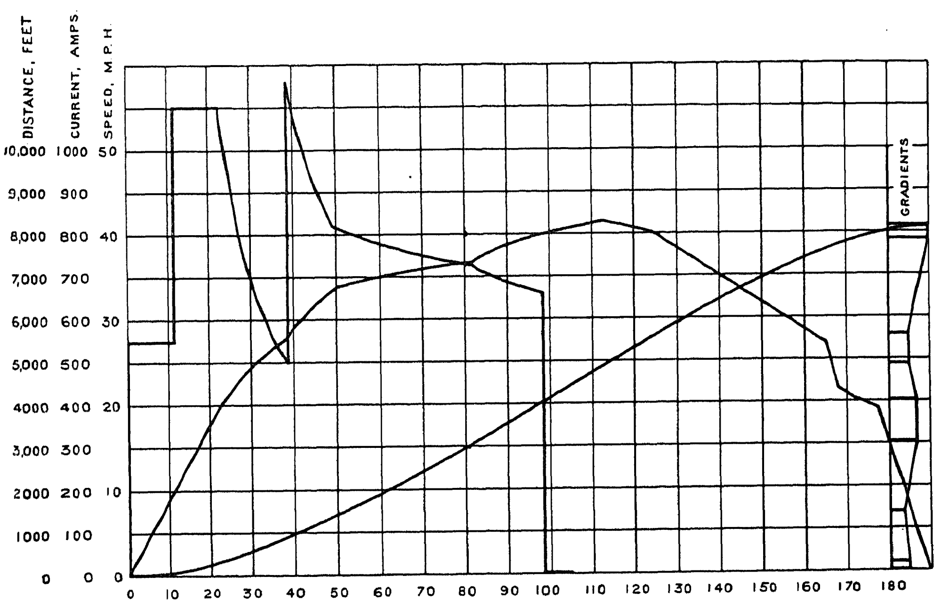

FIG. 177.—Train Characteristics, 4 coach train: Return Journey.

TABLE 14

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
K	F_o	S_o	T_o	D_o	Gradient	F′	s'	D	d	s_1	d_1	d_2	s_2	t	T	Time secs.	Speed m.p.h.	Distance feet	a	P_o	P′ (or P_1)	P′T	$aKT_o(s_2-s_1)$
																0	0	0					
3120	1650	9·72	10·95	106·5	—1/200	—526	3·775	75	·71	0	0	·71	1·51	·945	10·35				·018	27	107·5	1112	
																10·4	10·0	75					
,,	,,	,,	,,	,,	—1/200	—526	3·775	225	2·11	1·51	·20	2·31	2·285	1·07	11·70				,,	,,	17·5	205	
																22·1	15·1	300					
,,	,,	,,	,,	,,	—1/95	—1107	6·75	150	1·41	2·285	1·83	3·24	2·59	·58	6·35				,,	,,	7·1	45	665
																28·4	17·2	450					
3120	1650	19·43	21·9	426	—1/95	—1107	6·75	72	·17	1·295	—	·17	1·51	·12	2·63				·036	54	215	565	
																31·0	20·0	520					
,,	,,	,,	,,	,,	—1/95	—1107	6·75	443	1·04	1·51	·19	1·23	2·105	·565	12·37				,,	,,	14·1	174	1462
																43·4	27·9	965					
3580	1493	23·4	23·0	538	—1/95	—1107	10·28	1585	2·95	1·75	·45	3·40	2·67	1·29	29·70				·042	88	41·5	1231	
																73·1	42·6	2550					
,,	,,	,,	,,	,,	L	0	3·40	700	1·30	2·67	6·33	7·63	2·755	·485	11·15				,,	,,	88	982	
																84·3	44·0	3250					
,,	,,	,,	,,	,,	1/100	1051	2·405	860	1·60	2·755	—1·46	·14	2·675	·585	13·45				,,	,,	132·2	1780	3195
																97·7	42·7	4110					
					L	0		960							15·65							6094	5322
																113·4	40·8	5070					
					—1/110	—956		1670							27·33							11416kw.sec. =3·18 kw. h.	
																140·7	42·6	6740					
					1/266	396		610							10								
																150·7	40·6	7350					
					1/266	396		580							12·9								
																163·6	20·2	7930					
					L	0		200							13·5								
																177·1	0	8130					

TABLE 15

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
K	F_o	S_o	T_o	D_o	Gradient	F′	s'	D	d	s_1	d_1	d_2	s_2	t	T	Time secs.	Speed m.p.h.	Distance feet	a	P_o	P′ (or P_1)	P′T	$aKT_o(s_2-s_1)$
																0	0	0					
3120	1650	19·43	21·9	426	L	0	2·89	200	·47	0	0	·47	1·16	·81	17·72				·036	54	215	3610	
																17·7	15·4	200					
,,	,,	,,	,,	,,	—1/266	—396	3·49	128	·30	1·16	—	·30	1·51	·225	4·93				,,	,,	215		
																22·6	20·0	328					
,,	,,	,,	,,	,,	—1/266	—396	3·49	584	1·37	1·51	·20	1·57	2·105	·735	16·10				,,	,,	39·8	640	1460
																38·7	27·9	912					
3580	1493	23·4	23·0	538	—1/266	—396	4·26	478	·89	1·75	·50	1·39	2·10	·455	10·45				·042	88	71·4	745	
																49·2	33·5	1390					
,,	,,	,,	,,	,,	1/110	956	2·46	1670	3·10	2·10	2·50	5·60	2·30	1·40	32·20				,,	,,	128·2	4130	
																81·4	36·7	3060					
,,	,,	,,	,,	,,	L	0	3·40	960	1·78	2·30	2·60	4·38	2·51	·735	16·90				,,	,,	88	1485	2620
																98·3	40·0	4020					
					—1/100	—1051		860							14·4								
																112·7	41·3	4880					
					L	0		700							11·7								
																124·4	40·0	5580					
					1/95	1107		1980							40·3							10610	4080
																164·7	27·0	7560					
					1/95	1107		120							3·3								
																168·0	21·4	7680					
					1/95	1107		150							4·95								
																173·0	20·0	7830					
					1/200	526		130							4·55								
																177·5	19·2	7960					
					1/200	526		170							12·0								
																189·5	0	8130					

$\underbrace{10610 \quad 4080}_{}$ 14690kw.sec. = 4·08 kw. h.

the motor-curve may be taken from this curve, or computed from the accelerating tractive effort by means of equation 27.

The time of acceleration (column 15) is given by equation 30, and the distance (column 13) by equation 31. In the fourth row, the acceleration to full parallel is performed, the intervals of time and distance being given by equations 35 and 36. On the return journey (fig. 177, table 15) there is a break of gradient during acceleration, and equations 32, 33 and 34 are employed. In the eighth line of table 14 it should be noted that the gradient is reached at a speed higher than the free running speed, and the figures in columns 12 and 14 are got from the upper branch of the curve of fig. 175. The coasting and braking periods, having no reference to the motor characteristics, are computed directly in terms of ordinary units.

Speed-time, Distance-time and Power-time Curves.—Having determined both the speed and the time at which the limits of each gradient are reached, the speed-time curve and the distance-time curve can be plotted approximately. It is usually of little importance that intermediate speeds are not accurately determined, since the distance is allowed for correctly, but if desired these intermediate speeds could be obtained from the speed-distance curves and equation 26. The power being known at each speed, the power-time curve can be put in approximately, and thus the complete train-characteristics are determined.

Energy Consumption.—The integral of the power-time curve gives the energy consumption; but, when a schedule calculation is being made, this is more easily deduced analytically. The motor characteristics that have been given for series motors show the tractive effort as almost a straight line function of the current, at any rate over the most used portion of the curves. If the residual tractive effort, after taking away the tractive resistance, be plotted against current, the result is even more nearly a straight line. Hence the relation between power (P) and residual tractive effort (F) approximates closely to:

$$P = \alpha F + P_0 \quad . \quad . \quad . \quad (38)$$

Thus for the figures of table 13, $\alpha = \cdot 042$, $P_0 = 88$, P being in kilowatts.

Between any two speeds s_1 and s_2, on a particular gradient, the energy consumption is:

$$\int_1^2 \mathrm{P}d\mathrm{T} = \int_1^2 (\mathrm{P} - \mathrm{P}' + \mathrm{P}')d\mathrm{T}$$

$$= \int_1^2 a(\mathrm{F} - \mathrm{F}')d\mathrm{T} + \mathrm{P}'(\mathrm{T}_2 - \mathrm{T}_1)$$

$$= \frac{a\mathrm{M}'}{g}(\mathrm{S}_2 - \mathrm{S}_1) + \mathrm{P}'(\mathrm{T}_2 - \mathrm{T}_1) \quad . \quad . \quad (39)$$

(see equation 16).

The energy consumption in speed-curve running accordingly consists of two portions, one $\mathrm{P}'(\mathrm{T}_2 - \mathrm{T}_1)$, (where $\mathrm{P}' = a\,\mathrm{F}' + \mathrm{P}_o$), pertaining to the gradient, and the other $a\mathrm{M}'(\mathrm{S}_2 - \mathrm{S}_1)/g$, pertaining to the motor-speed curve. The energy consumption during acceleration by controller is deduced from the power $\mathrm{P}_1 = a\mathrm{F}_1 + \mathrm{P}_o$ with appropriate allowance for the method of control.

In tables 14 and 15 the twentieth column gives a and the twenty-first P_o. The twenty-second column gives P' (or P_1 when the train is being accelerated by controller). The twenty-third column gives $\mathrm{P}'\mathrm{T}$ (or $\mathrm{P}_1\mathrm{T} \times$ suitable constant), from columns 16 and 22. The twenty-fourth column gives $a\mathrm{M}'(\mathrm{S}_2 - \mathrm{S}_1)/g$ (see column 18), or $a\mathrm{KT}_o(s_2 - s_1)$ (see columns 1, 4, 11, 14 and 20). In this column it is of course unnecessary to compute the increment for each gradient; but only to consider the complete change of speed on each speed curve. The sum of the items in the twenty-third and twenty-fourth columns gives the energy consumption for the run, in kilowatt-seconds.

Use of these Methods.—The motor losses can, if desired, be deduced by similar methods, but in most cases it is sufficient to base these on the average schedule run, as the uncertainty of the data employed in computing the resulting temperatures under service conditions renders the more detailed calculation of small practical value; and a large allowance to cover this uncertainty has in any case to be made. The analytical method here expounded is chiefly valuable in the treatment of such problems of dynamical performance as are exemplified above. Minor variations in the method may be necessitated

by the circumstances of particular cases, but these in general present no features of difficulty.

In the initial stages of the discussion of an electrification scheme, it is natural for the available data to be of a general nature only, and the methods of the last chapter then suffice for the discussion of the subject. When such preliminary discussion, in its effect on revenue and expense, has indicated the most suitable train and schedule, the point-to-point method may be used to determine the most desirable equipment for the purpose, from the point of view both of dynamical capability and of heating. Finally when this has been settled, the performance of the train under the actual conditions, and the appropriate mode of operation should be investigated by the analytical method. The information so obtained forms a sure basis on which to found an estimate of the expense involved in electrification, and a judgment of the advantages to be expected from it. Although considerable labour must be expended in deriving the information it is repaid by the economies that naturally result from certain knowledge.

CHAPTER X

POWER SUPPLY

Factors which Determine Power Requirements.—In the two preceding chapters, methods for determining the performance of individual trains have been discussed in considerable detail; and it has also been shown that the position of the train, with its power consumption at any time, can be deduced with substantial accuracy from the motor characteristics, and the particulars of the train and service. This work is of the greatest importance; for it ascertains exactly what is to be expected of particular trains under normal circumstances, or under such abnormal circumstances as it may be considered desirable to investigate. It moreover forms one of the elements necessary for the subsequent determination of the equipment needed to supply the power to the trains, of the load factor, of the total energy required to operate the service, and of the consequent cost of operation. The other element involved is the distribution of the trains on the railway at all times, and is expressed by a time-table of the service. The two elements are interconnected through the train-characteristics, and together form the complete expression of the traffic, from the engineer's point of view.

THE GRAPHICAL TIME-TABLE.—The time-table, devised to conform with the normal performance of the individual trains on the actual route, is usually in the first place laid out graphically. It consists of approximate distance-time curves for all trains, using the section under consideration (see fig. 178). In the preparation of a graphical time-table for the service on an existing railway, the times of the various trains have to be adjusted to suit the physical limitations of the route; so that, for instance, the faster moving traffic may overtake the slower only where the necessary tracks are available to permit it to

pass. Where so radical a change of methods is introduced as that from steam to electrical working, it is natural that change in the conditions of the route is sometimes desirable. This is matter for the consideration of the electrical engineer, and for discussion with the departments interested; as is also the arrangement of the traffic so as to suit the peculiarities of electrical working and take advantage of its strongest features. The graphical time-table as finally decided, is from the present point of view principally of interest as showing the frequency of service and normal location of all scheduled trains at all times. Where the trains run at sensibly equal speed, as is often the case on lines employed entirely for suburban traffic, the preparation of a time-table is not difficult, although trains may be frequent. It is where trains run irregularly and at different speeds on the same lines that considerable adjustment is necessary in the time-table, in order to secure the service which, in view of the physical limitation of the railway, promises the greatest measure of advantage. For the present purpose it is desirable to prepare two time-tables, one to correspond with the heaviest expected, contemplated, or possible service—for use in determining the amount of plant required,—and the other to correspond with the expected average service—for use in determining the expense of operation of the plant.

Determination of Substation and Distribution Systems.—The layout of the substation and distribution systems is in general a subject for the exercise of judgment and the comparison of the various practicable arrangements from all points of view, rather than a matter to be settled by definite rules. The problem of devising the most satisfactory scheme is that of determining the most economical, subject to certain limitations on the allowable voltage drop in the track conductors. If the distance between substations is made unduly great, the distribution conductors required to give a certain voltage drop with the given service of trains become heavy and expensive. In fact, where a rail return is employed, a sudden and considerable increase in expense accrues when the distance which gives limiting voltage drop in the unassisted rails is exceeded. On the other hand, increase in the distance between substations improves their individual load factors and

efficiencies, reducing the total plant in service as well as the standby plant, and making it somewhat cheaper to instal and to operate. With plant and arrangements of given type, accordingly, there is a certain spacing of substations which yields the most economical results for a certain route and traffic.

The voltage drop that may be allowed in line conductors depends on considerations of energy loss, of satisfactory acceleration, and—in the case of passenger trains—of train lighting. The permissible voltage drop in the track rails is principally governed by considerations of the possibility of interference with neighbouring communication circuits, and of electrolytic corrosion in pipes and other buried metal work. The permissible drop need not be the same throughout a system; in fact, the dictates of economy usually require greater drop on unimportant branch lines than on lines of heavier traffic, and greater drop on goods lines than on passenger lines. In systems of operation employing unattended substations, it is advisable to work with smaller drop in the distribution lines than in systems in which attendants are available to restore power if the automatic switches should cut it off from any cause; for in the former it is expedient to allow for the possibility of substations being out of operation for more or less extended periods. The varying circumstances of actual operation sometimes concur in producing a voltage drop much in excess of anticipation based on scheduled traffic, but it is not usually necessary to take account of such abnormal events except from the point of view of the ability of the substation plant to stand the load without harm.

Distance Apart of Substations.—In order to make a start on the determination of the most suitable number and location of substations, the average current taken per mile of route should be computed for different parts of the system, from the average current per train and the density of the trains. From this, the voltage drop in line conductors should be determined, assuming such conductors and feeders as would be considered normal for the system of operation used. For a continuous current system, let C be the average current and R the resistance of line conductors and feeders, both reckoned per mile of route; let $2y$ be the distance between substations

and x the distance of a point from the centre point, so that its distance from the nearer substation is $y - x$, and from the farther, $y + x$. The current entering the rail within the space dx produces a voltage drop at the point determined by x given by:

$$\frac{de'}{R(y-x)} + \frac{de'}{R(y+x)} = C\,dx$$

or

$$de' = RC\frac{(y-x)(y+x)}{2y}\,dx$$

The voltage drop at the central point between substations is:

$$dv = \frac{y}{y+x}\,de' = \tfrac{1}{2}RC(y-x)\,dx.$$

The current from a length of rail dx situated at $-x$, produces an equal drop of voltage at the centre point, so that the whole drop at this point is:

$$v = RC\int_0^y (y-x)\,dx = RC\frac{y^2}{2} \qquad . \qquad . \qquad (1)$$

or

$$y = \sqrt{\frac{2v}{RC}} \qquad . \qquad . \qquad . \qquad (2)$$

Single-phase Case.—The calculation becomes a little more intricate in the case of the single-phase alternating current system. Equation 1 however still holds for the voltage drop in phase with the current, and equation 2 therefore determines the spacing for given energy loss. This is usually all that is required at this stage; but if the total voltage drop is desired, its maximum value is given, with sufficient accuracy, by equation 1, in which however R is now the impedance per mile of the track conductors, instead of the resistance per mile.

If the current C be estimated for a time of heavy traffic, when the voltage drop is at its greatest and the distribution of current approximates most closely to uniformity, and if moreover the voltage v be taken as from 40 to 60 per cent. of the normally permissible drop at the part of the system considered, the relation between the spacing of substations and the resistance of line conductors given by equation 2 will in

general be found a good first approximation to the result of the more accurate survey, which should in any event be undertaken later. In the case of the continuous current system, the calculation should be performed for the combined positive and negative line conductors ; and again for the track return, if this is used, in order to determine which tends to limit the substation spacing. For the alternating current system the voltage drop in the track return has no definite significance : moreover, little of the current is carried by the track.

Terminal Substations.—The distance that a terminal substation can be permitted to feed in the direction of the dead end is less than a half of the normal distance between adjacent substations. It is clear that if the current collection were distributed uniformly along the route, as assumed above, the voltage drop at the end of the line would be the same as midway between substations if the distance to the end were a half of the distance between substations. If, however, a single train were taking current midway between substations, half the current would traverse half the distance, and thus the voltage drop would be the same as that in a train taking the same total current at a quarter of the distance between substations towards a dead end. The appropriate distance of a substation from a terminal is accordingly between a quarter and a half of the distance between adjacent substations, the proportion being greater the nearer the distribution of load along the track approaches to uniformity ; it being borne in mind, however, that the terminal is necessarily a starting place where heavy currents are taken.

Amount and Cost of Plant.—When the appropriate substation spacing at different parts of the system has been provisionally determined, to correspond with certain line conductors, it is desirable to form an idea of the nature and amount of substation plant required, and to make a first estimate of its cost, with that of the line conductors. The approximate operating and maintenance costs should also be determined, together with the value of wasted energy, and the interest on first cost. This should be repeated for other cross sections of line conductors, and the annual expenses compared, in order that the spacing finally chosen may approximate to the most economical.

In a system of any extent, there are usually a number of junctions from which several lines radiate, and which accordingly form natural distributing points. These may be at once noted as particularly desirable locations, in arranging the substation system. Substations should, where practicable, be located near railway stations, for the convenience of attendants, inspectors and visiting engineers, and to facilitate the delivery of small supplies. Having formed an approximate idea of the correct spacing at different parts of the railway, a system of substations should be laid out provisionally, due attention being paid to the important considerations just mentioned. When this has been done, a more detailed survey of the voltage drop in the line conductors should be undertaken, the graphical time-table and other circumstances of the traffic being very carefully studied, with a view of finding the worst conditions likely to arise in normal operation. In this investigation, special consideration should be given to places where a heavy draught of current is to be expected, such as railway stations, and gradients against which heavy trains may be required to operate; and future developments should be kept well in mind and allowed for as far as practicable, for the layout of the substation system is not a thing that can readily be changed at a later date. Having therefore determined the drop in the assumed line conductors under conditions of probable operation, with the substations thus provisionally located, the size of these conductors and the location of the substations should be further adjusted in accordance with the information so obtained, and the use of additional feeders and track-draining boosters considered, until the whole distribution and substation system promises the greatest measure of satisfaction having regard to operation, to investment, to annual expense, and to probable changes or extensions. On a system with many ramifications, it is often found impracticable to prevent substations crowding one another somewhat at certain places, but with care the layout can usually be arranged so that there is no very great loss in economy from this cause.

Superposition of Substation Loads.—In computing the load of a substation, the principle of superposition is applicable; each train may be assumed to divide its load between substations as if no other trains were using the distribution

system. In fig. 179 let P and Q be substations supplying power at voltages V and V′; and let A and B be trains taking currents c and c' respectively. Let the resistance of the distribution system between P and Q be R, between A and Q, r, and between B and Q, r', and let the currents in the sections of line PA, AB, BQ be c_1, c_2 and c_3 respectively. The voltage

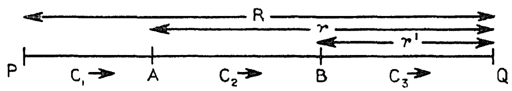

FIG. 179.—Substation Load Calculation.

at A is $V - (R - r)c_1$, at B, $V - (R - r)c_1 - (r - r')c_2$, and at Q:

$$\begin{aligned} V' &= V - (R - r)c_1 - (r - r')c_2 - r'c_3 \\ &= V - Rc_1 + r(c_1 - c_2) + r'(c_2 - c_3) \\ &= V - Rc_1 + rc + r'c' \end{aligned}$$

$$c_1 = \frac{V - V'}{R} + \frac{r}{R}c + \frac{r'}{R}c' \qquad . \quad . \quad . \quad (3)$$

The first term on the right-hand side of equation 3 is the circulating current, due to the difference in voltage of supply at the substations. The second term is the load due to the train at A, computed as if no other load were present; and the third term is the load due to the train at B, similarly computed. The equation shows that the substation load is the sum of these partial loads; and this is true in general.

Calculation of Actual Substation Loads.—In the preceding chapter is shown how the position of a train and the power it takes at any time can be computed from the data of the train and its equipment. With a known distribution system, the ratio r/R may be supposed known at every point of the route. Thus the curve of substation load and time for a single train can be deduced, for any substation, from the power-time curve of the train, by multiplying the ordinate at each point by the appropriate value of r/R. The sum of the ordinates so obtained for all trains taking power from a particular substation gives the load curve of the substation. Figs. 180 and 181 show respectively, in broken lines, power-time curves of a train in each direction on a route between

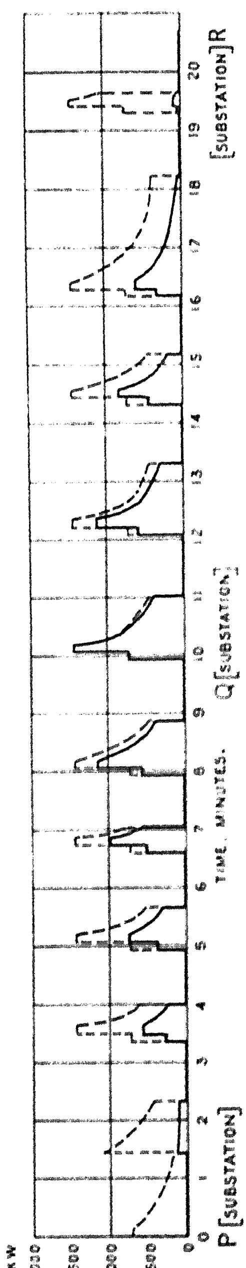

FIG. 180.—Power taken by Train, and Substation Load.

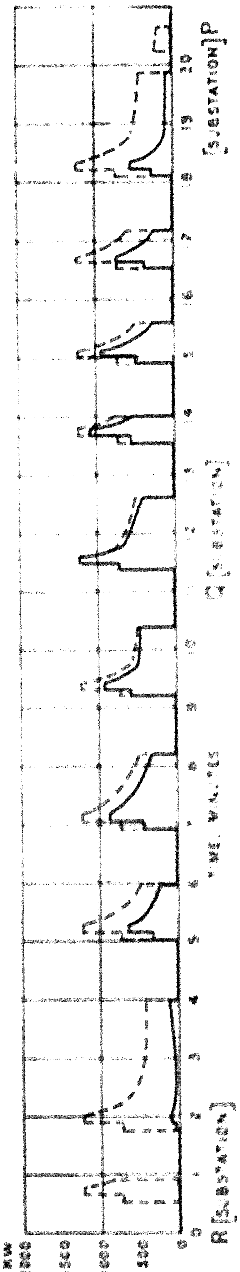

FIG. 181. Power taken by Train, and Substation Load.

substations P and R, and in full lines the load on the intermediate substation Q. Fig. 182 shows a load curve for the substation Q corresponding to a five-minute service of trains in each direction. The task of deducing such curves as fig. 182 for the whole day and for all substations is a tedious one; but when train characteristics, substation layout and distribution

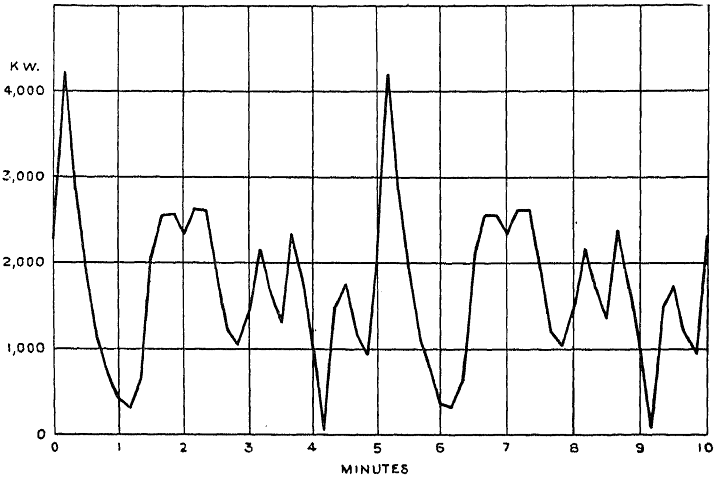

FIG. 182.—Substation Load Curve.

system have been provisionally determined, the work is quite straightforward, and presents no features of difficulty.

Determination of Substation Plant.—When the location of the substations and the appropriate distribution lines have been finally settled, the precise capacity and nature of suitable substation plant may be determined. For this purpose an estimate of the load curve of each substation should be made by methods indicated above. It is usually sufficient in the first place to treat the trains as each taking their mean current throughout; and having determined the general shape of the load curve in accordance with this assumption, to pick out for more detailed consideration the portions of the curve likely to affect the choice of plant. The shape of the substation load curve depends on the system of operation used, particularly as affecting the length of route supplied from the substation. With a high potential system, in which each substation supplies

power for a large amount of traffic, the load curve may be but little more irregular than that of the generating station. On the other hand, where a substation provides for a comparatively small fraction of the whole traffic, its load curve is likely to be exceedingly irregular, particularly at times of light traffic, when, between the peaks of power due to one or two trains, there may be more or less extended intervals of no load whatever. The load factor of such a substation may easily be less than 20 per cent.

The determination of the most suitable substation plant, besides depending on electrical conditions, as shown in chapter VI, depends also on the nature of the traffic. Passenger traffic for instance is substantially regular from day to day, and, in attended substations, extra plant is brought into operation in time to meet the needs of the diurnal peaks of loads. Goods traffic on the other hand is likely to be much more irregular, and to require a greater surplus of plant capacity to be kept in operation continuously. Where it is practicable to vary the amount of substation machinery in operation to accord with the variation of the load, it is desirable to plot the detailed load curves of the several substations for a normal time of light traffic. The result of this investigation has considerable influence in the determination of the most suitable capacity for the substation units ; for the momentary overload capacity of the machinery is likely to be more severely taxed at times of comparatively light traffic when few units are in operation than at time of heavy traffic when the accelerating peaks form a smaller proportion of the load. The height of the load curve during the period of heaviest traffic determines the number of units necessary in each substation ; but since these periods do not usually last for more than one or two hours at most, the capacity required in the plant may, where the limitations of commercial design permit, be met in part by overload capacity in the units; thus the average peak load may without detriment be greater than the plant in operation would bear continuously with a conservative temperature rise.

In general, the capacity of the substation plant should be based on immediate or contemplated requirements, but buildings may with advantage be constructed with a view to more problematical future developments, to be met by the installation of further plant as occasion arises. The determin-

ation of the substation plant required under certain circumstances is largely a matter for the exercise of judgment, on which divergent opinions may be held. The following rule may however serve as a general guide in determining the amount of plant: Assuming the nature of the machinery to be such as to stand short period overloads of 200 per cent. without detriment, determine the highest peak that the contemplated traffic would justify, and take this as 75 to 100 per cent. overload on the running plant; thus leaving a further 100 per cent. of the rated capacity at least, exclusive of standby plant, in order to meet emergencies. It is expedient to be liberal in the provision of plant, with respect both to capacity and to number of units; for circumstances of practical operation sometimes combine very adversely to call for greater power than normal operation would indicate. The number of different sizes and types of unit should for economy be made as small as possible, and it is generally more economical to use a few units of large capacity than many units of small capacity. All units in a substation should preferably be of the same capacity, and at least one extra unit should be provided therein to act as standby in case part of the plant is inoperative from any cause. Since the load factor of a substation is generally much poorer than that of the generating station, the total capacity of the substation plant is usually greater than that of the generating plant; although, since it is practicable to make the overload capacity of the units much greater than that of generating units, the excess is less than might be expected. The excess of rated capacity may amount to some 50 or 60 per cent., but these figures really convey little information, since, as indicated above, they are involved with the kinds of machinery used and the method of rating.

Inasmuch as the substation and distribution systems are features of comparative permanence in the electrification scheme, it is labour well spent to investigate the various possibilities in considerable detail, in order to secure the greatest measure of efficiency on economical terms. Not only should the substations be located with great care, to secure satisfactory distribution, and the substation plant chosen to suit the requirements of the load; but the relative economy of using machinery of higher or lower efficiency should be considered. It will usually be found that the extra cost of

securing reasonably high operative efficiency will be repaid in the saving of energy resulting from the reduction in the losses.

Site of Generating Station.—The most suitable site for a generating station is often determined by natural conditions, for the economic advantage accruing from favourable conditions is usually much greater than can result from selection based on the distribution of the load. This is obviously so in the case of water-power stations, whose location is prescribed within narrow limits; but, even in the case of steam-power stations, the advantages of a plentiful supply of condensing water and good facilities for obtaining fuel will be found to outweigh the disadvantages of a considerable length of transmission line. All available sites should therefore be considered in choosing the most suitable location. Where a site is remote from the centre of the substation system, it is advisable to consider transmission to one or more distributing stations; and thence—usually with reduction of pressure—to the several substations. The distributing centres can often be located at substations with advantage, but the objection to bringing very high tension transmission lines into large towns and the cost of a right of way for them in such localities is often the determining factor in their location. It is in accordance with good practice to run the transmission lines in duplicate to each substation as a precaution against serious breakdown, and where the generating station is considerably removed from the substation system, a saving both in cost of transmission lines and in energy loss is likely to accrue if the energy is first transmitted at high pressure to a more central point.

Preferred Location.—Where several central sites are available for the location of a generating station or distributing station, there is a general impression that, from the point of view of annual cost of transmission, including interest on the expenditure on lines, the site nearest to the centre of gravity of the load is the most advantageous. That this is an erroneous impression can be seen from the consideration that if there are two substations only, the smallest cost of transmission lines and the smallest energy loss will clearly accrue if the generating station is located at the larger of the two. If d be the distance of a substation from the power station, and a the cross section of copper in the transmission lines, the annual value of capital

expenditure and maintenance in so far as it varies may be taken as proportional to ad, and the annual value of the losses to $\frac{d}{a}$ C^2, where C^2 is the mean square value of the current. Accordingly from the point of view of economy in the transmission system, a quantity of the form:

$$X = p\Sigma ad + q\Sigma \frac{d}{a} C^2 \quad . \quad . \quad . \quad (4)$$

should be made a minimum, the summation being extended to all substations. Hence, varying a:

$$\frac{C}{a} = \sqrt{\frac{p}{q}} \quad . \quad . \quad . \quad . \quad (5)$$

or the cross section should be chosen proportional to the current whatever the distance of the substation. This in its present application implies that the cross section of the lines to any substation should be made proportional to the anticipated ultimate capacity of the substation. If this condition is satisfied, X is proportional to ΣdC. Here d is a function of two independent variables, and if θ be the angle which the line joining power station to substation makes with any fixed direction, the first conditions to be satisfied in order to make X a minimum are:

$$\Sigma C \cos \theta = 0 \text{ and } \Sigma C \sin \theta = 0 \quad . \quad . \quad (6)$$

Thus with two substations:

$$C_1 \cos \theta_1 + C_2 \cos \theta_2 = 0 \text{ and } C_1 \sin \theta_1 + C_2 \sin \theta_2 = 0 \quad (7)$$

If $C_1 = C_2$ these are satisfied by making $\theta_2 = \theta_1 + \pi$, and all points on the line joining the substations are equally favourable locations for the power station. If, however, C_1 and C_2 are different, the equations 7 are inconsistent; which may be interpreted that the power station should be at a substation, or that one of the angles is indefinite. The conditions of maximum economy show that the chosen substation should be the greater, as is otherwise evident. Where any number of substations are ranged along an extended line, which forms the right of way for the transmission line, the most favourable location for the power station, from the point of view of transmission, is that which divides the substation capacity equally on the two sides of it, whatever the distance of the

several substations apart; the location so determined will usually be at a substation. The conclusion is true even if the line of substations is branched. Thus if the substation system for a railway is as shown in fig. 183, the figures by the several substations representing their relative capacities, the most economical transmission results from locating the power station at the substation marked PH.

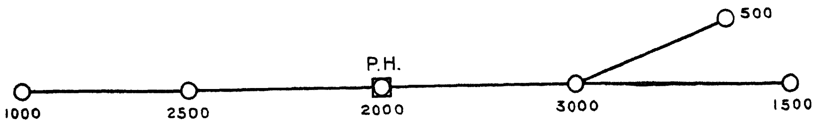

FIG. 183.—Location of Generating Station.

A case of some interest is that in which there are three substations and unrestricted wayleaves in all directions. If it is found possible to draw a triangle with sides proportional to their capacities, and also possible to determine a point at which the line joining any two substations subtends an angle equal to the external angle between the corresponding sides of this triangle, the point so determined is the most favourable location for the

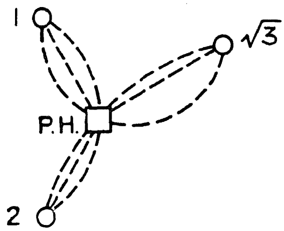

FIG. 184.

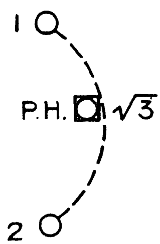

FIG. 185.

Location of Generating Station.

power station from the point of view of the transmission system. If however the capacity of one of the substations is equal to or greater than the sum of the other two, the greatest substation is the most favourable location; whilst if any of the segments of circles which contain the external angles mentioned above contains also the third substation, the construction becomes impossible, and this substation is the most favourable location. For instance, if the loads are in the ratio $1 : \sqrt{3} : 2$, the external angles are 90°, 120° and 150°; thus in fig. 184 the most favourable location for a power station is

at PH, whilst in fig. 185 it is at the substation of capacity $\sqrt{3}$. With four substations of equal capacity placed so that it is possible to join them together by lines which form a convex quadrilateral, it is clear that the intersection of the diagonals is the most favourable location for the power station. More general cases might be analysed with increasing complexity, but as already explained, other considerations than expense incidental to the transmission system have the preponderating influence in determining the site of the power station, and general analysis is not justified by its value, particularly as the assumption of unrestricted right of way for the transmission lines is not usually admissible. The discussion given above is intended rather to emphasize the fact that the centre of gravity of the load is not the most economical location for the power station, even from the sole point of view of transmission expense, and that all available sites should therefore be considered, inasmuch as the expense of transmission is seldom the controlling factor in the determination.

Efficiency of Transmission, Conversion and Distribution.—The next step in the determination of the scheme is to estimate the average combined efficiency of the distribution system, substations and transmission lines, from train back to metering point, under the load conditions resulting from the traffic considered. There is usually little difficulty in deducing this with sufficient accuracy for the purpose of estimating the total energy consumption. The various efficiencies tend in a measure to compensate one another and lead to a combined efficiency more uniform than its constituents. For the lines are running at lowest efficiency at times of heavy traffic, when the substations are well loaded and therefore operate at high efficiency, and vice versa. Representative figures for the average efficiency of the several elements which are interposed between the prime movers and train wheels are given in the table 16, as compiled by Mr. C. E. Eveleth.* The table is based on American experience, and is for the most part estimated to accord with the circumstances of specific roads. It will be seen that not all classes of service are represented, and in fact the table is practically confined to such as it is expedient to work by the various systems. Mr. Eveleth's comments concerning the several services are given on page 381.

* *General Electric Review*, vol. 16, p. 812.

TABLE 16. AVERAGE ALL DAY EFFICIENCIES—PER CENT.

Number	1	2	3	4	5	6	7	8	9	10	11	12
Source of Power	Three-Phase	Three-Phase	Three-Phase	Three-Phase	Three-Phase	Three-Phase	Single-Phase	Single-Phase	Three-Phase	Single-Phase	Single-Phase	Three-Phase
Frequency	25	25	60	60	25	60	25	25	60	25	25	60
System, i.e. Locomotives	D-C.	D-C.	D-C.	D-C.	Three-Phase	Three-Phase	Split-Phase	Split-Phase	Split-Phase	Single-Phase	Single-Phase	Single-Phase
Gearing, etc.	Gearless	Geared	Gearless	Geared	Geared	Geared	Geared	Geared Side Rod	Geared	Geared	Geared	Geared
1. Generator	93	93	93	93	92	93	89	89	93	89	89	93
2. Step-up transformers	97	97	97	97	97	97	96	96	97	96	—	97
3. Step-up auto transformer							—	—	—	—	97	—
4. High tension transmission	95	95	95	95	95	95	95	95	95	95	97	95
5. Step-down transformers	97	97	97	97	96	97	96	96	97	96	—	97
6. Frequency changers						85	—	—	84	—	—	84
7. Motor-generator			85	85				—	—	—	—	—
8. Rotary converter	91	91						—	—	—	—	—
9. 2nd step-up transformers						97		—	—	—	—	—
10. 2nd step-up auto transformers								—	98	—	—	98
11. Secondary transmission						97	97	97	97	—	—	97
12. 2nd step-down transformers						96	—	—	—	—	—	—
13. Trolley auto transformers							97	97	97	—	97	97
14. Contact conductor	90	90	90	90	95	95	97	97	97	97	97	97
15. Locomotive transformers					96	96	96	96	96	96	96	96
16. Phase converter							92	92	92	—	—	—
17. Control, blowers, etc.	94	94	94	94	95	95	95	95	95	96	96	96
18. Motors	93	91	93	91	91	91	91	91	91	88	88	88
19. Tractive equalizer					97	97	97	97	97	—	—	—
20. Gearing, side rods		95		95	94	95	95	90	95	95	95	95
21. Weight	98	100	98	100	100	100	98	98	98	98	98	98
Combined all day efficiencies	58	55	54	51	59	46	49	46	43	57	59	47
Efficiency transmission, conversion and distribution only	75.5	75.5	70.5	70.5	86.5	67	83	83	69	88	91	69

Systems 1 to 4 inclusive cover direct current installations with geared and gearless motors. Frequencies of 25 and 60 cycles are assumed, as either may be most desirable depending upon the location of the project under consideration. Generally in the eastern part of the United States either frequency may be considered, but in the West there is practically nothing but 60 cycles, and ordinarily power would be purchased at this frequency, or at least provision would be made to connect with a 60 cycle power system in case of emergency. By the use of 60 cycle rotary converters instead of motor-generator sets for systems 3 and 4 their efficiencies would be identical with systems 1 and 2, or vice versa. System 1 is in a general way similar to the New York Central and system 4 similar to the Butte, Anaconda & Pacific installation.

Systems 5 and 6 contemplate the use of 25 cycle, three-phase motors on the locomotives. System 5 is similar to that used in the electrification of the Cascade Tunnel of the Great Northern Railroad.

Systems 7 to 9 inclusive, designated as "split-phase," contemplate the use of 25 cycle polyphase induction motors on the locomotives, taking power through transformers and a phase converter from a single-phase trolley. The secondary transmission here considered is deemed desirable on account of the higher cost of stepping down directly from the high tension transmission line to the trolley with substation spacing of ten miles or less, such as has been found necessary to mitigate telephone and telegraph disturbances. Items 11 and 13 might be eliminated, increasing the combined efficiency from 49 to 52 and from 46 to 49. (Systems 7 and 8.)

System 8 is identical with 7 with the exception of the mechanical structure of the locomotive, the latter having side rods and gears and the former having gears only.

Systems 10 to 12 contemplate the use of 25 cycle compensated commutator single-phase motors on the locomotives, taking single-phase power from the trolley through the requisite step-down transformers.

System 11, which has distribution connections similar to the New York, New Haven & Hartford Railroad, has a limited application on account of the moderate distance over which power can be transmitted, particularly if applied to single track lines. It therefore necessitates power houses at no great distance apart.

Item 19, called "tractive equalizer," is a value inserted to represent the necessary average loss due to provision on each locomotive equipped with induction motors to permit the operation of locomotives in the same trains which have different diameters of driving wheels. The inherent characteristics of induction motors are such that 2 or 3 per cent. difference in speed makes the difference between full tractive effort and no tractive effort. Locomotives with same diameter of driving wheels will divide the load properly, but as the wheels on some locomotives are decreased in diameter, due to wear, these locomotives will not take their share of the load if coupled with machines having full-sized wheels. To equalize the load this necessitates the insertion of an artificial resistance or other provision for adjustment of the motor slip to obtain identical characteristics for motors in locomotives with different diameters of driving wheels. If 6 per cent. variation were allowed between new wheels and old, the loss due to equalization of draw bar pull would be about 6 per cent. in the locomotive with new wheels adjusted to divide the load properly with a locomotive having worn wheels; the latter not having any artificial loss, as its adjusting resistance would be all cut out. The average loss for conditions where locomotives with old and new wheels are operated miscellaneously is estimated to average 3 per cent. due to the tractive equalizers.

Item 21, called "weight efficiency," is a value inserted to equalize the difference in average train weight due to difference in weight of the locomotives.

The table has been carried from prime movers to locomotive

wheels, for all intermediate elements have their effects on working costs. It is here assumed however that the special features of the power station on the one hand and of the train equipments on the other, are taken into account elsewhere, so that for the present purpose the combined efficiency required is that of transmission lines, substations and distribution lines only; that is, the combination of items 4 to 14 inclusive. This efficiency has been added to the table.

Efficiency of the Rotary Converter Substation. The rotary converter substation merits particular attention, for,

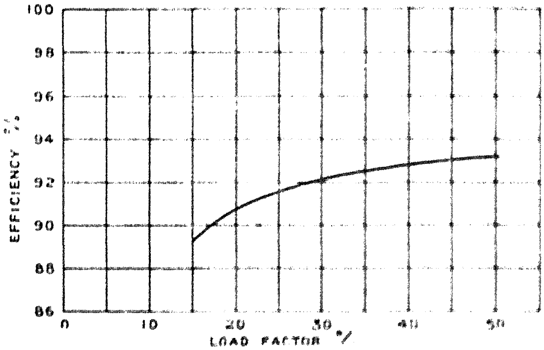

FIG. 186.—Efficiency of Rotary Unit as function of Load Factor.

being that usually employed in connection with suburban electric railways, it is at once the commonest and best known

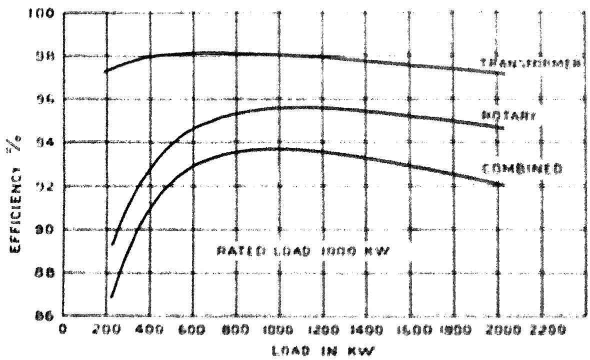

FIG. 187.—Efficiency of Rotary and Transformer as function of Load.

form. Fig. 186 gives a representative curve of the average overall efficiency of rotary converter units, expressed as a function of the load factor. The curve has been based on

typical modern plant, the transmission voltage being of the order of 20,000 volts and the distribution voltage 600 to 1,000 volts. Fig. 187 shows the efficiency of the unit as a function of the load. The frequency was actually taken at 25 cycles, but the substation efficiency does not change greatly with frequency, the rotary and transformer efficiencies varying in opposite directions and having a sensibly constant product. However, the all-day efficiency is not properly a function of the load factor alone, depending on the shape of the whole load curve. On this account fig. 186 has greater value as representative of its class than as data.

The Generating Station Load-Curve.—In computing the output of energy from the generating station, for the purpose of estimating its cost, or of determining the plant necessary for its generation, there is usually a factor of uncertainty to be introduced to take account of inefficient driving, signal checks, station delays, unscheduled running, etc. Accordingly a representative figure for the efficiency from generating station busbars to train under the conditions actually existing is usually sufficiently accurate for the purpose of the estimate in question. The average power input to the several trains having been determined, by methods given in the last two chapters, a suitable allowance should be made for the uncertainty referred to, and the result divided by the estimated efficiency between train and generating station busbars. The result is the power requirement of the trains. From the power requirement so estimated, and the time-table of the service, the load curve of the generating station can be determined without difficulty, by the addition of the loads of the several trains. The work may conveniently be carried out in the following manner: Let tables be prepared for the several services giving in one column the times of commencing journeys, in a second, the times of finishing, and in a third, the mean power requirement of the train; let another table be prepared, in the first column of which all the times given in all other tables are given in order of their occurrence, in the second column should be given the corresponding power, with a positive sign if the time is that of commencement and a negative sign if that of finish of a journey; in the third column the algebraic sum of the power given in the second should be

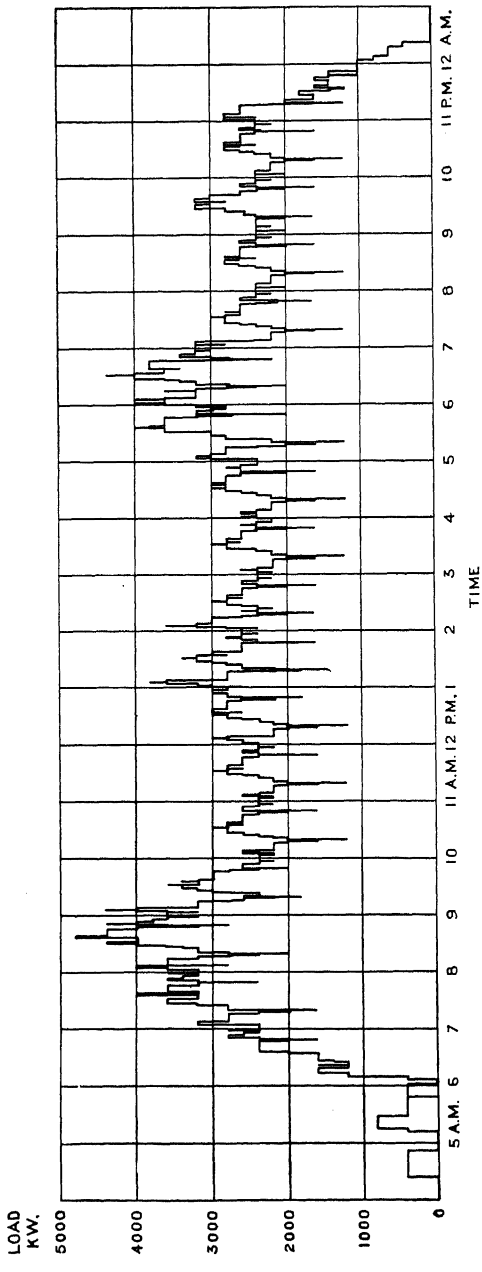

FIG. 188.—Load Curve of Generating Station.

formed for each time, by successive addition; this gives the load to be plotted against the time given in the first column. Tables 17 and 18 give the first few lines of such tables, corresponding to the time-table of fig. 178, the complete load curve being shown in fig. 188.

TABLE 17

Principal Services.						Other Service.		
Up.			Down.			Up and Down.		
a.m.		kw.	a.m.		kw.	a.m.		kw.
5.15	5.28	400	5.13	5.48	400	4.25	4.52	400
6.6	6.19	400	6.10	6.22	400	—	—	—
6.36	6.49	400	6.13	6.48	400	—	—	—
6.51	7.4	400	5.50	6.2	400	—	—	—
6.10	6.43	400	6.43	7.18	400	6.25	6.54	200
6.20	7.19	400	6.58	7.33	400	6.27	6.56	200
7.21	7.37	400	6.50	7.48	400	—	—	—
6.35	7.8	400	7.5	7.17	400	—	—	—
6.50	7.49	400	7.28	8.3	400	—	—	—
7.5	7.38	400	8.9	8.15	400	—	—	—
7.51	8.7	400	7.20	8.18	400	—	—	—
7.20	8.19	400	7.35	7.47	400	7.25	7.54	200
7.35	8.8	400	7.58	8.33	400	7.27	7.56	200
8.21	8.37	400	7.50	8.48	400	7.39	8.6	400

TABLE 18

Time a.m.	Increment Load.	Load.	Time a.m.	Increment Load.	Load.
4.25	+ 400	400	6.6	+ 400	400
4.52	— 400	0	6.10	+ 800	1,200
5.13	+ 400	400	6.13	+ 400	1,600
5.15	+ 400	800	6.19	— 400	1,200
5.28	— 400	400	6.20	+ 400	1,600
5.48	— 400	0	6.22	— 400	1,200
5.50	+ 400	400	6.25	+ 200	1,400
6.2	— 400	0	6.27	+ 200	1,600

Determination of Generating Plant.—The mean load curve so obtained is of very great value in the determination of the most suitable generating plant for the work, and also has a large influence on generating costs. In fact, in a scheme of electrification involving numerous trains, it is generally hardly necessary in a first estimate to look beyond the mean curve, provided the plant has reasonable overload capacity; it is only where the scheme is a restricted one, or where some special condition, such as a long steep gradient, exists, or where the generating plant is not of a nature to stand overloading, that the variation from the mean load curve becomes of special importance. In order to make a final determination of the number and capacity of generating units best suited to the needs of the case, however, it is usually advisable to make an estimate of the greatest peaks of power likely to be experienced at a normal time of heavy load, and also to make a similar estimate for a normal period of light load. In the case of steam turbo-generators capable of standing considerable overload for short periods, the greatest anticipated peaks during the light load period should hardly exceed the rating of the units in service. During the heavy load period, the highest estimated peaks may be permitted to overload the running plant to the extent of 33 per cent., or even more in suitable circumstances, provided the duration and value of the mean load are not such as would lead to deleterious heating in the generators. Water power plant is not generally designed to allow such overloading, and it is not usually desirable to estimate on the normal railway peaks exceeding the rated capacity of the running plant. The same is true also of gas-driven plant, which is usually rated near to its ultimate capacity.

The plant capacity required during the normal period of light load indicates the smallest size of unit that it is advisable to instal, for it is generally economical to employ units of large capacity and similar to one another; the peak load then indicates the number of running units that must be provided. One or two units should be installed in excess of the requirements, indicated by the load curve, to act as standby for use in case of emergency; it is usually expedient also to provide floor space for additional units, to be installed as traffic develops.

Generating Costs and Load Factor.—The shape of the load curve by its influence on the amount of machinery required to be held in running order as compared with the average amount actually running, has considerable effect on the cost of energy generation. The load factor, which may be taken as the ratio of the mean of the maximum ordinate in fig. 188, affects not only capital charges, but also the efficiency of the operating staff, and to a somewhat smaller extent the amount of coal and other supplies used. Whilst therefore it is proper and expedient to base an estimate of generating cost on results obtained from the operation of other similar undertakings, the effect of difference in load factor should be taken into account as well as the effects of difference in wages and cost of supplies.

APPENDIX

Table of Locomotive Statistics

Railway	Baltimore and Ohio. Baltimore Belt-line Tunnel	Grand Trunk St. Clair River Tunnel	Aroostook Valley
Locomotives:			
Installed	1903	1908	1912
Number	2	6	1
Service	Goods	Mixed	Goods
Type and Dimensions:			
Type, main frame	Two unit, rigid frames	Rigid frame	Bogie truck, rigid bolsters
Wheel arrangement	0–8–0+0–8–0	0–6–0	4–0–4
No. driving axles	8	3	4
Type of drive	Individual, geared	Individual, geared	Individual, geared
Total wheel base	43 ft. 7 in.	16 ft. 0 in.	22 ft. 4 in.
Rigid wheel base	14 ft. 7 in.	16 ft. 0 in.	6 ft. 10 in.
Diam. driving wheels	42 in.	62 in.	36 in.
Diam. running wheels	—	—	—
Length over buffers, or between knuckles	58 ft. 7½ in.	29 ft. 5½ in.	31 ft. 1 in.
Cab	Full length of each unit	Over whole length	Centre
Width	9 ft. 5 in.	—	9 ft. 6 in.
Height	13 ft. 8 in.	—	11 ft. 9 in.
Weight, lbs.:			
Complete loco.	320,000	132,000	82,000
On drivers	320,000	132,000	82,000
Electrical part	95,000	58,400	28,600
Motor and gear	—	15,660	4,700
Power equipment:			
System	C.C.	S.P.	C.C.
Frequency, cycles	—	25	—
Line voltage	625	6,600	1,200
Motor voltage	625	235	600
No. of motors	8	3	4
Type	G.E.–65–B	W–137 compensated series	G.E.–206–A
Rated load per motor, h.p.	200	240	100
Rated speed, m.p.h.	8·6	10·8	14
Gearing, type	—	Twin	—
Gear reduction	81/19	85/16	65/17
Control	Electro-magnetic 27 notches	Electro-magnetic (storage battery) 20 notches	Electro-magnetic 6 ser. 4 ser. par. notches
Remarks and references	—	—	—

Oregon Electric	North Eastern Quayside Line, Newcastle-on-Tyne	Metropolitan, London, Main Line	Southern Pacific	Chicago, Milwaukee and St. Paul
1912	1904	1907	1911	1916
6	2	10	15	
Goods	Shunting	Passenger	Goods	Shunting
Bogie truck, floating bolsters	Bogie truck, swinging and floating bolsters	Bogie truck, swinging and floating bolsters	Bogie truck	Bogie truck
4–0–4	4–0–4	4–0–4	4–0–4	4–0–4
4	4	4	4	4
Individual, geared	Individual, geared	Individual, geared	Individual, geared	Individual, geared
26 ft. 8 in.	27 ft.	24 ft. 6 in.	25 ft.	30 ft. 4 in.
7 ft. 2 in.	6 ft. 6 in.	7 ft. 6 in.	7 ft. 4 in.	8 ft.
37 in.	36 in.	38 in.	36½ in.	40 in.
—	—	—	—	—
37 ft. 4 in.	37 ft. 11 in.	33 ft. 6 in.	35 ft.	41 ft. 5 in.
Centre	Centre	Whole length of loco.	Centre	Centre
9 ft. 6 in.	8 ft. 8 in.	8 ft. 7 in.	—	10 ft. 1 in.
11 ft. 11 in.	11 ft. 9 in.	12 ft. 3½ in.	—	14 ft. 3 in.
120,000	125,000	105,000	120,000	140,000
120,000	125,000	105,000	120,000	140,000
36,000	34,000	36,000	44,200	55,000
6,200	5,500	6,200	6,740	7,200
C.C.	C.C.	C.C.	C.C.	C.C.
600 and 1,200	550/600	550	600 & 1,500	3,000
600	500/600	550	750	1,500
4	4	4	4	4
G.E. 212	G.E. 55 2 turn	G.E.69–B.	W. 308–D. 3	G.E. 255
200	90	200	225	135
15·8	8·6	19·5	17·6	12
—	—	—	—	—
65/18	59/18	64/19	57/16	64/17
Electro-magnetic	Electro-magnetic	Electro-magnetic	Pneumatic Cam	Electro-magnetic
7 ser., 5 ser. par. notches	5 ser., 4 par. notches	5 ser., 4 par. notches.	19 notches	
	See fig. 189, p. 392	See fig. 190, p. 392	See fig. 191, p. 393	See fig. 192, p. 393

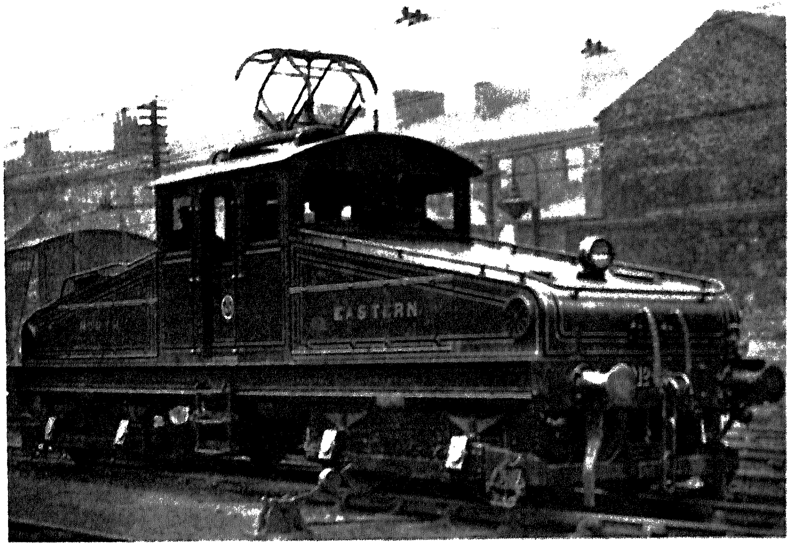

FIG. 189.—North-Eastern Railway (Quayside Line) Locomotive.

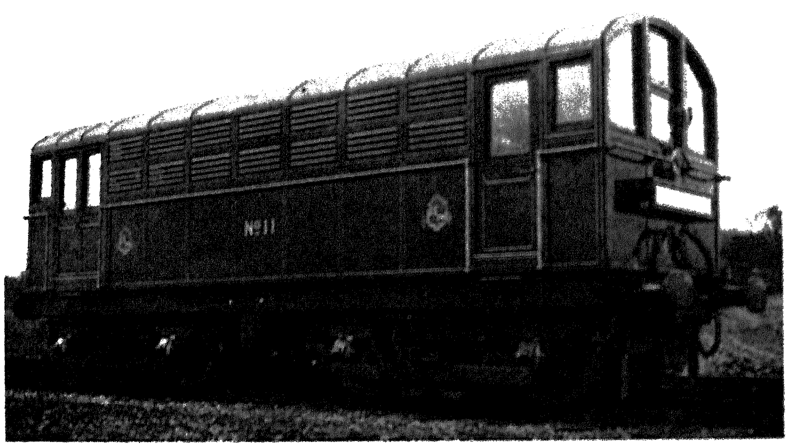

FIG. 190.—Metropolitan Railway (Main Line) Locomotive.

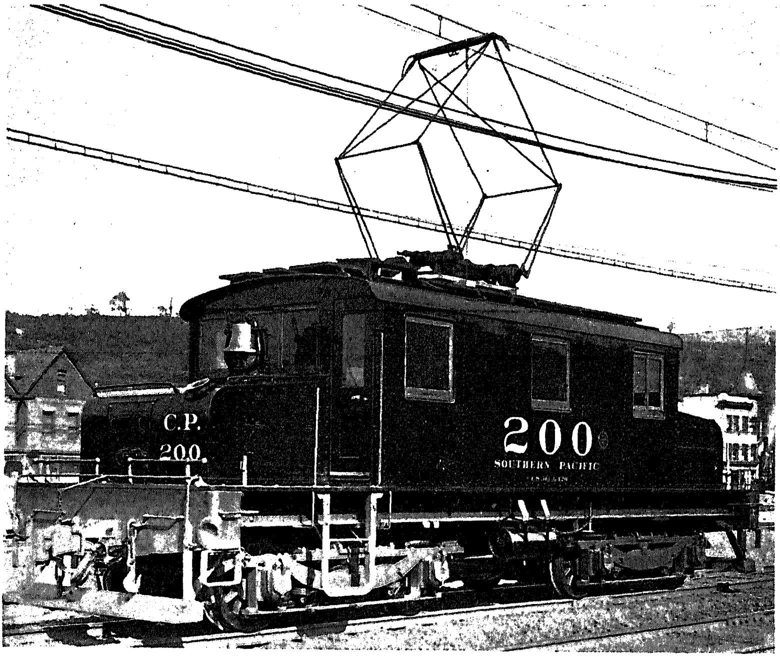

FIG. 191.—Southern Pacific Railway Goods Locomotive.

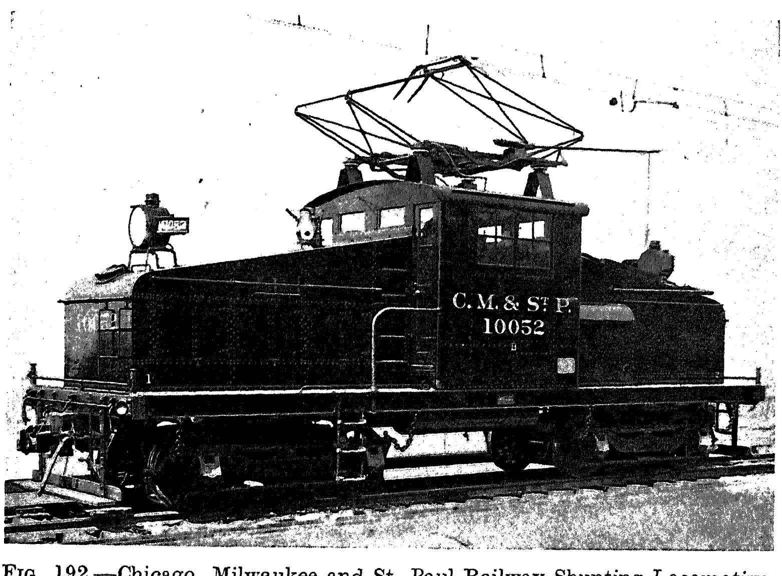

FIG. 192.—Chicago, Milwaukee and St. Paul Railway Shunting Locomotive.

Railway	Butte, Anaconda and Pacific	Michigan Central (Detroit River Tunnel)	Great Northern (Cascade Tunnel)
Locomotives :			
Installed	1913	1910	1909
Number	19	6	4
Service	Goods	Mixed	Mixed
Type and dimensions :			
Type, main frames	Two truck, hinge articulation	Two truck, hinge articulation	Two truck, hinge articulation
Wheel arrangement	0 4 4 0	0 4 4 0	0 4 4 0
No. driving axles	4	4	4
Type of drive	Individual, geared	Individual, geared	Individual, geared
Total wheel base	26 ft. 0 in.	27 ft. 6 in.	31 ft. 9 in.
Rigid wheel base	8 ft. 8 in.	9 ft. 6 in.	11 ft. 0 in.
Diam. driving wheels	46 in.	48 in.	60 in.
Diam. running wheels			
Length over buffers, or between knuckles	37 ft. 4 in.	39 ft. 6 in.	44 ft. 2 in.
Cab	Over whole length	Centre	Over whole length
Width	10 ft. 0 in.	10 ft. 0 in.	10 ft. 0 in.
Height	15 ft. 6 in.	12 ft. 4 in.	14 ft. 3 in.
Weight, lbs. :			
Complete loco.	160,000	200,000	230,000
On drivers	160,000	200,000	230,000
Electrical part	60,000	64,000	108,000
Motor and gear	8,800	11,600	
Power equipment :			
System	C.C.	C.C.	Three phase
Frequency, cycles			25
Line voltage	2,400	600	6,000/6,600
Motor voltage	1,200	600	500
No. of motors	4	4	4
Type	G.E. 229 -A	G.E. 209	Induction
Rated load per motor, h.p.	320	275	500
Rated speed, m.p.h.	15·4	11·8	15
Gearing, type	Twin	Twin	Twin
Gear reduction	87/18	83/19	81/19
Control	Electro-magnetic 10 ser., 9 ser. par. notches	Electro-magnetic 9 ser., 8 ser. par. and 7 par. notches	Electro-magnetic
Remarks and references	See fig. 193, p. 396	See figs. 11 & 195, pp. 39 & 397	See fig. 194, p. 396

North Eastern (Shildon-Newport Branch)	Chicago, Milwaukee and St. Paul	Chicago, Milwaukee and St. Paul	New York, New Haven and Hartford	New York, New Haven and Hartford (New York Terminal)
1915	1916	1919/20	1911	1911/12
10	30	10	16	36
Mineral traffic	Goods	Passenger	Shunting	Mixed
Two truck, draw bar articulation	Two unit, 4 main and 2 bogie trucks, hinge articulation	Two main trucks, draw bar articulation	Two truck, draw bar articulation	Two truck, draw bar articulation
0–4–4–0	4–4–4–4–4–4–	4–6–2–2–6–4	0–4–4–0	2–4–4–2
4	8	6	4	4
Individual, geared	Individual, geared	Individual, geared quill, twin motors mounted above axles	Individual, geared quill, motors mounted above axles	Individual, geared quill, twin motors mounted above axles
27 ft.	102 ft. 8 in.	79 ft. 10 in.	23 ft. 6 in.	40 ft. 6 in.
8 ft. 9 in.	10 ft. 6 in.	16 ft. 9 in.	7 ft. 0 in.	8 ft. 0 in.
48 in.	52 in.	68 in.	63 in.	63 in.
—	36 in.	36 in.	—	33 in.
39 ft. 4 in.	112 ft.	88 ft. 7 in.	37 ft. 0 in.	50 ft. 0 in.
Centre	Full length of unit	Full length of double unit	Centre	Over whole length
8 ft. 4 in.	10 ft. 0 in.	11 ft. 0 in.	—	10 ft. 3 in.
13 ft. 2 in.	16 ft. 8 in.	14 ft. 6 in. over cab	—	13 ft. 10 in.
166,660	576,000	550,000	160,000	220,000
166,660	450,000	336,000	160,000	168,000
54,320	—	—	71,700	92,100
8,200	—	—	—	—
C.C.	C.C.	C.C.	S.P.	S.P. and C.C.
—	—	—	25	25
1,500	3,000	3,000	11,000	11,00 ac.–600 cc.
750	1,500	1,500 (750 per armature)	190	275
4	8	6	4	8
Series	G.E.253—A	Twin armatures permanently in series	Compensated series	Compensated series twin mounted
275	430	700	188	213
20	15·25	23·8	12	36
Twin	Twin, spring	—	Twin	—
4·5	82/18	89/24	101/17	92/22
Electro-magnetic	Electro-magnetic, regenerative, 17 ser., 2 transfer, 12 ser. par., 1 tapped field.	Electro-magnetic, individual and cam 33 resistance, 9 running notches	Electro-magnetic, 10 notches	Electro-pneumatic, 11 ac.—19 cc. notches
Engineering, 26 May & 2 June, 1916	See fig. 197, p. 398	*Journ.A.I.E.E* April, 1920	See fig. 196, p. 397	See fig. 199, p. 402

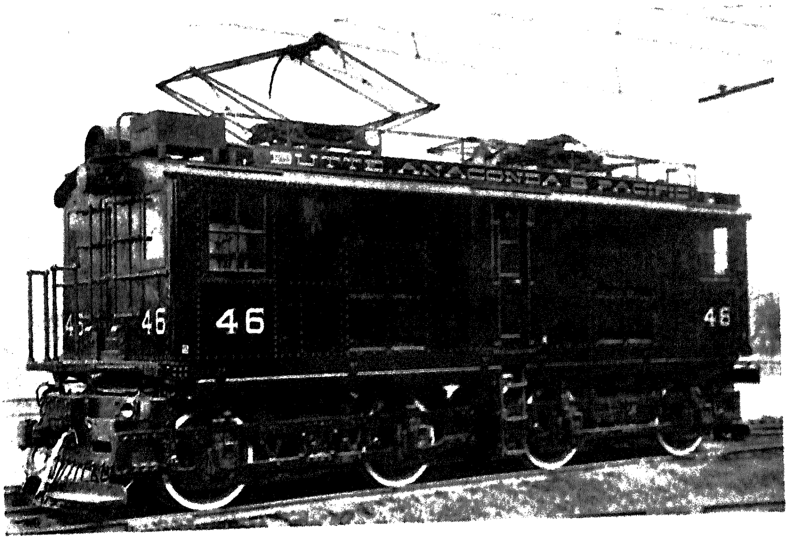

Fig. 193.—Butte, Anaconda and Pacific Railway Locomotive.

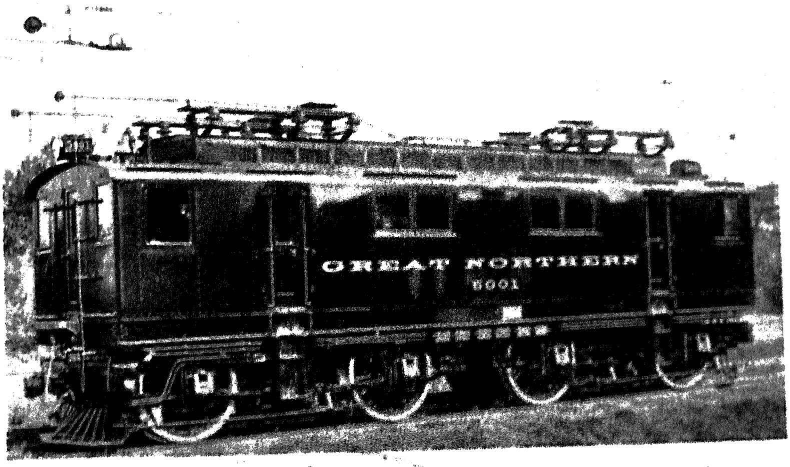

Fig. 194.—Great Northern Railway (Cascade Tunnel) Locomotive.

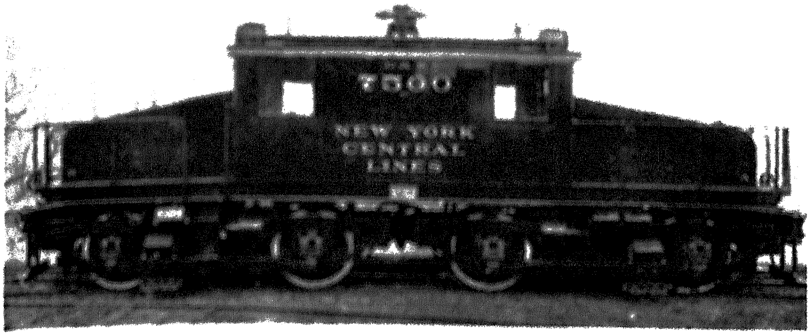

FIG. 195. Michigan Central (Detroit River Tunnel) Locomotive.

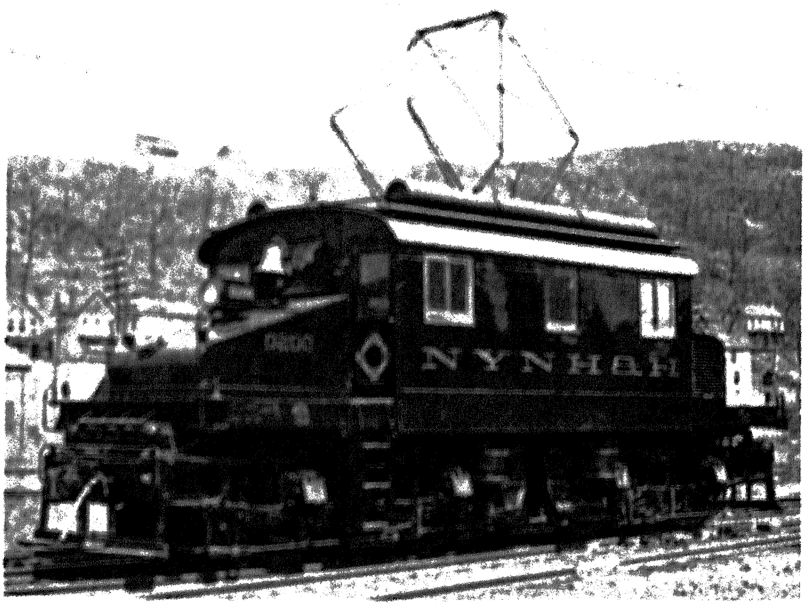

FIG. 196. New York, New Haven and Hartford Railroad Shunting Locomotive.

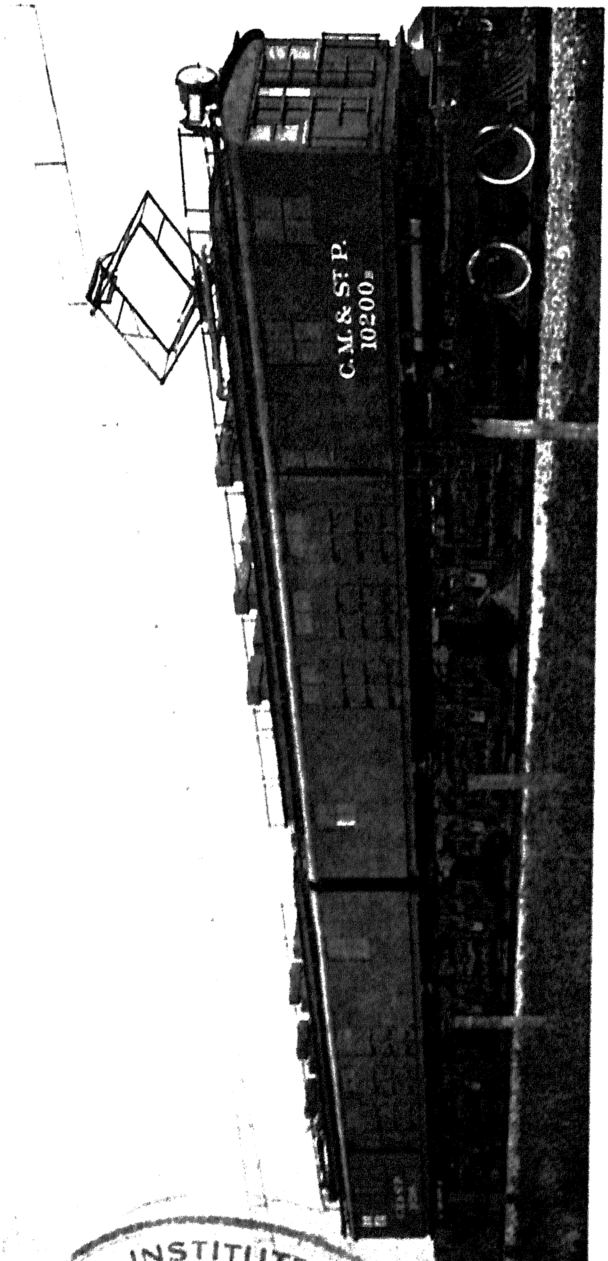

FIG. 197.—Chicago, Milwaukee and St. Paul (Rocky Mountains Divisions) Goods Locomotive.

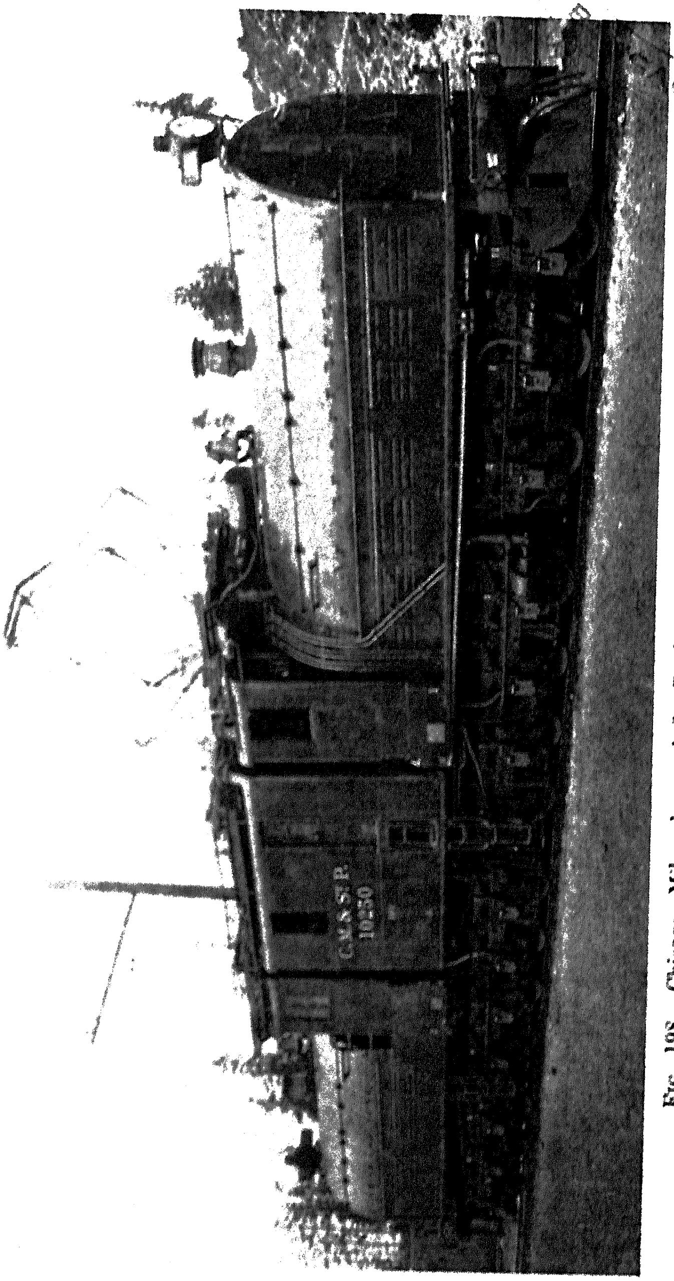

FIG. 198.—Chicago, Milwaukee and St. Paul (Cascade Division) Passenger Locomotive.

Railway	New York, New Haven and Hartford (New York Terminal)	Chicago, Milwaukee and St. Paul (Cascade Division)	New York Central and Hudson River (New York Terminal)
Locomotives:			
Installed	1907	1919	1906
Number	41	5	—
Service	Passenger	Passenger	Passenger
Type and Dimensions:			
Type, main frames	Modified bogie	Two unit frame each of two trucks hinge connection	Rigid main frame
Wheel arrangement	2 4 4 2	6 8 8 6	2-8-2
No. driving axles	4	12	4
Type of drive	Individual, gearless quill	Individual, gearless	Individual, gearless
Total wheel-base	30 ft. 9 in.	67 ft. 0 in.	27 ft. 0 in.
Rigid wheel-base	8 ft. 0 in.	13 ft. 9 in.	13 ft. 0 in.
Diam. driving wheels	62 in.	44 in.	44 in.
Diam. running wheel	33 in.	36 in.	36 in.
Length over buffers or between knuckles	37 ft. 7 in.	76 ft. 0 in.	37 ft. 0 in.
Cab	Over full length of platform	Compound, in three parts	Centre
Width	10 ft. 3 in.	10 ft. 0 in.	10 ft. 1 in.
Height	13 ft. 10 in.	14 ft. 11½ in.	13 ft. 9 in.
Weight, lbs.:			
Complete loco.	204,500	530,000	200,000
On drivers	162,000	458,000	140,000
Electrical part	110,400	235,000	60,000
Motor and gear	16,420		
Power equipment:			
System	S.P. and C.C.	C.C.	C.C.
Frequency, cycles	25		
Line voltage	11,000 A.C., 625 C.C.	3,000	625
Motor voltage	220	1,000	625
No. of motors	4	12	4
Type	Compensated series	G.E. 100 bipolar gearless	G.E. 84, bipolar gearless
Rated load per motor, h.p.	350	290	550
Rated speed, m.p.h.	54·5	36·2	40
Gearing, type		—	—
Gear reduction			
Control	Electro-magnetic 11 ac, 19 cc. notches	Electro-magnetic and pneumatic cam, 31 resistance, 8 operating notches	Electro-magnetic
Remarks and references		See fig. 198, p. 399	See fig. 5, p. 27

New York Central and Hudson River (New York Terminal)	New York Central and Hudson River (New York Terminal)	Pennsylvania Altoona Johnstown	Norfolk and Western (Bluefield Division)	Pennsylvania (New York Terminal and Tunnel)
—	1914/17	1917	1914	1910
12	16	—	12	33
Passenger	Passenger	Goods	Goods	Passenger
Rigid main frame	Hinge-articulated frame	Hinge-articulated trucks	Two unit draw bar articulation	Two unit trucks connected by draw bar
4–8–4	4–4–4–4	2–6–6–2	2–4–4–2 + 2–4–4–2	4–4–4–4
4	8	6	8	4
Individual, gearless	Individual, gearless	Collective, geared jack shaft, coupling rods	Collective, geared jack shaft, coupling rods	Collective, inclined connecting rods jack shaft and coupling rods
36 ft. 0 in.	46 ft. 5 in.	63 ft. 11 in.	94 ft. 10 in.	55 ft. 11 in.
13 ft. 0 in.	6 ft. 6 in.	13 ft. 4 in.	11 ft. 0 in.	7 ft. 2 in.
44 in.	36 in.	72 in.	62 in.	72 in.
36½ in.	—	36 in.	30 in.	36 in.
43 ft. 0 in.	56 ft. 10 in.	76 ft. 6¼ in.	105 ft. 8 in.	64 ft. 11 in.
Centre	Centre	Full length of loco.	Full length of each unit	Full length of unit
10 ft. 1 in.	10 ft. 0 in.	10 ft. 1 in.	10 ft. 3 in.	—
13 ft. 9 in.	14 ft. 6 in.	14 ft. 8 in.	14 ft. 9 in.	—
230,000	250,000	500,000	528,000	312,000
142,000	250,000	420,000	448,000	200,000
60,000	—	—	224,000	119,000
—	—	—	—	45,000
C.C.	C.C.	Split-phase	Split-phase	C.C.
—	—	25	25	—
625	625	11,000	11,000	600
625	625	850	725	600
4	8	4	8	2
G.E. 84, bipolar gearless	G.E. 91, bipolar gearless	Induction	3-phase induction, 4 and 8-pole windings	W–315–A
550	325	1,200	410	1,250
40	49	10 and 20	14 and 28	30·2 full field
—	—	Twin spring	Twin motors drive each jack shaft	—
—	—	106/21	85/18	—
Electro-magnetic	Electro-magnetic 24 notches	Cascade liquid rheostat	Electro-pneumatic liquid rheostat	Electro-pneumatic 31 notches field control
See fig. 200, p. 402	See fig. 201, p. 403	—	See fig. 203, p. 404	See fig. 204, p. 405

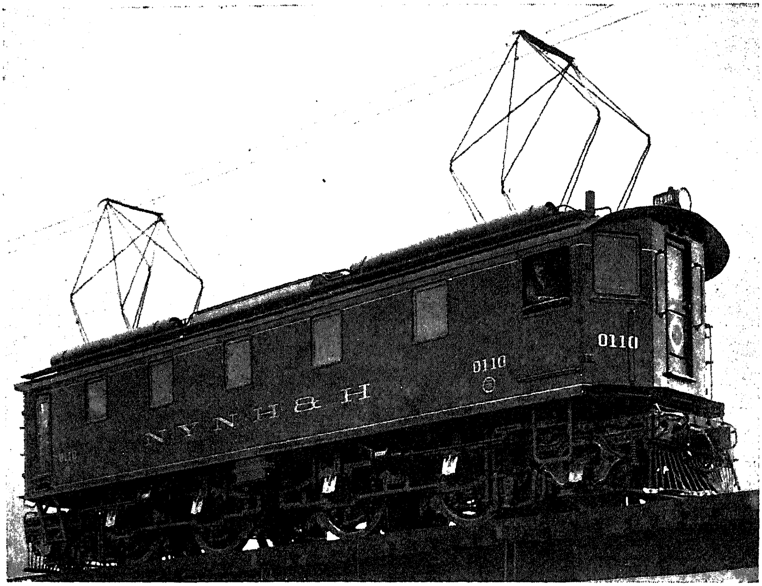

FIG. 199.—New York, New Haven and Hartford Railroad Locomotive.

FIG. 200.—New York Central and Hudson River (New York Terminal) Rigid Frame Passenger Locomotive.

FIG. 201. New York Central and Hudson River (New York Terminal) Articulated Frame Passenger Locomotive.

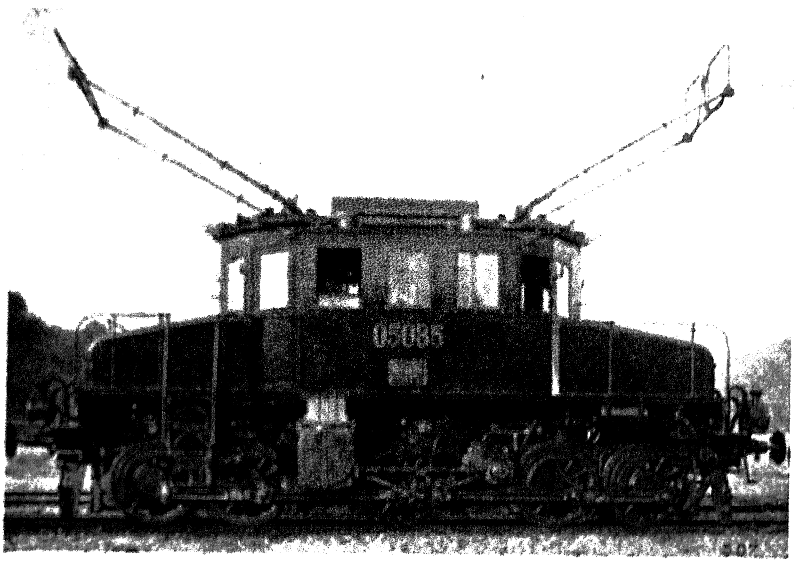

FIG. 202.—Italian State Railway (Giovi Line) Goods Locomotive.

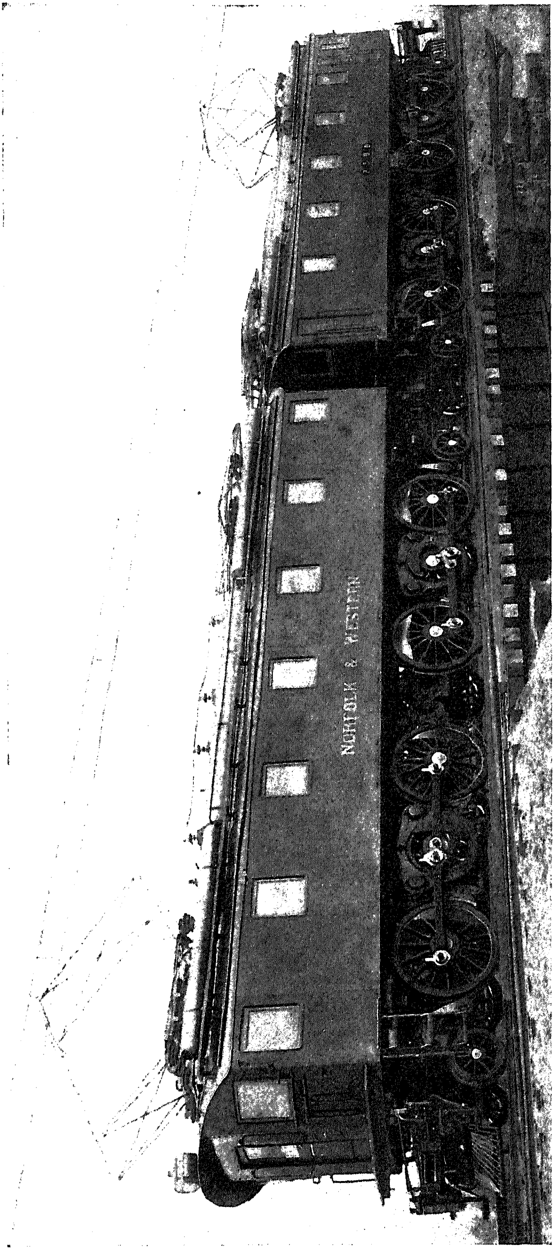

FIG. 203.—Norfolk and Western Railway (Bluefield Division) Goods Locomotive.

FIG. 204.—Pennsylvania Railway (New York Terminal and Tunnel) Passenger Locomotive.

Railway	Bern-Lötschberg	Prussian State Dessau-Bitterfeld Line	Swedish State Riksgräns Line
Locomotives:			
Installed	1911	1911	1915
Number	1	1	13
Service	Passenger	Passenger	Goods
Type and Dimensions:			
Type, main frame	Double truck, connected through king pins and underframe	Rigid main frame	Two unit
Wheel arrangement	0–6–6–0	4–4–2	2–6–6–2
No. driving axles	6	2	6
Type of drive	Collective, one motor per truck, geared jack shaft and coupling rods	Collective, single motor, vertical connecting rods, jack shaft and coupling rods	Collective, inclined connecting rods jack shaft and coupling rods for each unit
Total wheel-base	35 ft. 1 in.	29 ft. 6 in.	47 ft. 7 in.
Rigid wheel-base	13 ft. 3 in.	9 ft. 10 in.	—
Diam. driving wheels	52·2 in.	63 in.	43·3 in.
Diam. running wheels	—	39½ in.	29·5 in.
Length over buffers or between knuckles	49 ft. 3 in.	41 ft.	61 ft.
Cab	Over full length of platform	Over whole length of platform	Over each unit
Width	9 ft. 9 in.	10 ft. 0 in.	—
Height	12 ft. 3 in.	12 ft. 6 in.	—
Weight, lbs.:			
Complete loco.	198,500	157,000	304,000
On drivers	198,500	68,500	231,000
Electrical part	97,000	—	132,000
Motor and gear	21,600*	32,000	29,000†
Power equipment:			
System	S.P.	S.P.	S.P.
Frequency, cycles	15	15	15
Line voltage	15,000	10,000	15,000
Motor voltage	420	—	200
No. of motors	2	1	2
Type	Oerlikon, compensated series	Winter-Eichberg	Compensated series 24 poles
Rated load per motor, h.p.	1,000	1,000	840
Rated speed, m.p.h.	—	—	24
Gearing, type	Citröen, triple helical	—	—
Gear reduction	3·25	—	—
Control	Electro-magnetic by contactors	Electro-magnetic	Electro-magnetic 18 notches
Remarks and references	See *Engineering*, Sept. 1, 1911	A.E.G. Publication E.D. 197	*L'Industrie Electrique*, Feb. 10, 1917
	* Transformer 12,100 lbs.		† Transformer 22,000 lbs.

Midi	Prussian State	Italian State, Giovi	Simplon Tunnel	Bern-Lötschberg
1911	1911	1913 seq.	1906	1913
1	1	130	2	13
—	Mixed	Goods	Mixed	—
Rigid main frame	Two unit, link connected	Rigid frame	Rigid frame	Rigid frame
2–6–2	2–4–4–2	0–10–0	–-6–2	2–10–2
3	4	5	3	—
Collective, inclined connecting rods, two jack shafts, coupling rods	Collective, nearly vertical connecting rods, jack shafts and coupling rods	Collective, Scotch yoke, gearless, coupling rods	Collective, Scotch yoke, gearless, coupling rods	Collective, Scotch yoke, geared, coupling rods
31 ft. 6 in.	40 ft. 10 in.	20 ft. 1 in.	31 ft. 10 in.	37 ft. 2 in.
11 ft. 10 in.	9 ft. 5½ in.	—	16 ft. 1 in.	14 ft. 9 in. driving 53 in.
51·6 in.	50 in.	42 in.	64½ in.	53 in.
33·5 in.	33·4 in.	—	33½ in.	33 in.
43 ft. 2 in.	51 ft. 8 in.	31 ft. 3 in.	40 ft. 6 in.	52 ft. 6 in.
Full length of platform	Over each unit accordian connected	Centre	Centre	Full length
10 ft. 4 in.	10 ft. 4 in.	—	9 ft. 6 in.	9 ft. 8 in.
13 ft. 11 in.	12 ft. 2 in.	—	12 ft. 1 in.	14 ft. 9 in.
187,500	205,000	134,000	137,000	236,000
119,000	150,000	134,000	97,000	172,000
—	108,000	65,000	61,800	130,000
27,300	31,000	—	23,700	30,400 ‡
S.P.	S.P.	Three-phase	Three-phase	S.P.
16⅔	15	16⅔	15	15
15,000	10,000	3,300	3,000/3,300	15,000
—	1,235	3,300	550	420
2	2	2	2	2
Winter-Eichberg	Winter-Eichberg	Induction 8 pole	Induction 6 and 12 pole	Oerlikon compensated series
800	800	1,000	12 pole 400 6 pole 550	1,250
34	—	14 and 28	22 and 44	31
—	—	—	—	Triple helical
—	—	—	—	2·23
Electro-magnetic		Cascade parallel, liquid rheostat	Pole changing and rheostatic	Motor operated drum, 12 notches
Light Railway and Tramway Journ., June 2, 1911	Originally made for Lötschberg Railway	See *Engineer*, Aug. 29, 1913 seq. See fig. 202, p. 403	*Engineering*, July 28, 1911	*Engineer*, Dec. 5, 1913 ‡ Transformer 16,500 lbs.

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